Information Approach for Calculating the Resolutions of Energy, Length and Information

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***Abstract*—In this paper, a possibility of the existence of connection between the Bekenstein information bound and a model's uncertainty of material object is verified. The uncertainty is defined by the information quantity inherent to the model. It is calculated according to a metric called comparative uncertainty, by which the discrepancy between the chosen model and the observed object is found. A value of comparative uncertainty depends on the chosen number of variables. The data are used to define the values: a minimum of energy resolution, the minimum attainable value of the sampling length and the minimum achievable amount of information.**

Keywords—Bekenstein bound; Information theory; Mathematical modelling; Uncertainty relations

I. INTRODUCTION

The harmonic construction of modern science is based on separate blocks which are still not joined together: quantum electrodynamics, cosmology, biology, thermodynamics, chemistry, computer science, information theory. At the same time, there have been numerous attempts to create a picture of the objective reality covering the whole body of knowledge. However, they do not lead to proper results. One from the main obstacles standing in the way of the is, perhaps, that researchers and scientists consider continuous space-time but there are still unresolved problems associated with the processes of observation and measurement. Although discrete and continuous features coexist in any natural phenomenon, depending on the scales of observation [1], one can suppose a deeper level of reality which exhibits some kind of discrete elementary structure.

A serious review paper considering many possibilities of something similar to a discrete quantum length scale is [2]. In [3] the authors demonstrated that, in non-perturbative quantum descriptions, the existence of a minimum uncertainty in physical time is not generally unavoidable when gravitational effects are taken into account. Minimum time and length uncertainty in rainbow gravity has been reported in [3-5]. Unfortunately, the absolute minimum values of variables under consideration are not introduced in the majority of studies.

Without going into the theoretical debate about the possibility of the existence or absence of the "firstborn" building blocks of the universe, this article represents our attempt to verify the possible values of the minimum achievable discrete quanta of energy, length and information quantity. We do this through the use of a universal metric of the model uncertainty and information theory. The metric is called the comparative uncertainty and it is caused only by the number of variables taken into account in the physical-mathematical model.

II. BASIS THESES

In [6] Bekenstein proved that a bound of a given finite region of space with a finite amount of energy contains the maximum finite amount of information required to perfectly describe a given physical system. In informational terms, this bound is given by

$$ϒ \le (2\cdot\pi\cdot R\cdot E)/(\hbar\cdot c\cdot \ln 2) \quad (1)$$

or

$$S_E \le (2\cdot\pi\cdot\boldsymbol{\kappa}\cdot R\cdot E)/(\hbar\cdot c) \quad (2)$$

where ϒ is the information expressed as number of bits contained in the chosen material object framed by a sphere; Entropy is S_E, the ln2 factor comes from defining the information as the logarithm to the base 2 of the number of quantum states, R is the radius of the sphere that can enclose the given system, E is the total mass-energy including any rest masses; ħ is the reduced Planck constant, **κ** is the Boltzmann constant, and *c* is the speed of light.

The results are purely theoretical in nature, although it is possible to find application of the proposed formula in medicine or biology. Factually, the act of the Bekenstein modelling process already implies an existence of the formulated physical-mathematical model describing the sphere under investigation. In this model the variables are taken into account from the system of primary variables (SPV) [7] such as SI (International system of units). The SPV is a set of dimensional variables (DL), both primary and those calculated on their basis, which are necessary and sufficient to describe the known laws of nature, as in the physical content and quantitatively [8]. In turn, SPV includes the primary and secondary variables used for descriptions of different classes of phenomena (COP). For example, in the study of mechanics the base units of SI are typically used: ***L***, ***M***, ***T*** (***LMT***). On analyzing recorded variable dimensions in [6], the model of the Bekenstein relation includes four primary dimensional (DL) variables in SI: the length ***L***, mass ***M***, time ***T***, and temperature ***Θ***. That is why, in the framework of SI, one can classify $COP_{SI} \equiv$ ***LMTΘ***. Here ≡ means that this class includes four above mentioned primary variables.

Generally the dimension of any secondary variable ***q***, can only express a unique combination of dimensions of main primary variables in different degrees [7]:

$$q \supset L^l \cdot M^m \cdot T^t \cdot I^i \cdot \Theta^{\Theta} \cdot J^j \cdot F^f \quad (3)$$

where *l*, *m*... *f* are the exponents of variables taking only integer values. The range of each has maximum and minimum value [9]:

$$-3 \le l \le +3,\ \ -1 \le m \le +1,\ \ -4 \le t \le +4,\ \ -2 \le i \le +2,$$
$$-4 \le \Theta \le +4,\quad -1 \le j \le +1,\quad -1 \le f \le +1. \quad (4)$$

So the number of choices of dimensions for each variable, according to (4) is the following:

$$e_l = 7;\ e_m = 3;\ e_t = 9;\ e_i = 5;\ e_\theta = 9;\ e_j = 3;\ e_f = 3. \quad (5)$$

It can be shown [10] that an amount of information, ΔA_e, about the observed modeled sphere is calculated according to

$$\Delta A_e \le \kappa \ln [\aleph_{SI} /(z'' - \beta'')], \quad (6)$$

where ΔA_e is measured in units of entropy [11], ***z***'' is the number of physical DL variables recorded in the mathematical model, and ***β***'' is the number of primary physical DL variables recorded. The number of possible dimensionless (DS) complexes (criteria), with **ξ** = 7 main DL variables for SI, is given by $\aleph_{SI}$ equals [10]:

$$\aleph_{SI} = (e_l \cdot e_m \cdot e_t \cdot e_i \cdot e_\theta \cdot e_j \cdot e_f - 1)/2 - 7 =$$
$$= (7 \cdot 3 \cdot 9 \cdot 5 \cdot 9 \cdot 3 \cdot 3 - 1)/2 - 7 = 38{,}265. \quad (7)$$

Here "-1" corresponds to the occasion when all exponents of primary variables in (3) are treated as having zero dimensions. Division by 2 means that there are both required and reverse variables, for example, the length L^1 and the running length L^{-1}. In other words the object can be judged, with knowledge of only one of its symmetrical parts, while others structurally duplicating this part may be regarded as information empty. There are seven primary variables ***L***,***M***,***T***,***Θ***, ***I***,***J***,***F***.

In this case, the mathematical theory of information does not cover all the wealth of information content, because it distracted from the semantic content side of the message. From the point of view of the theory, a phrase of 100 words, taken from the newspaper, Shakespeare or Einstein's theory, each contains the same amount of information.

In order to transform information quantity with entropy ΔA_e to bits ΔA_b, one should divide it by the abstract number **κ** ln2 [11]. Then

$$\Delta A_b = \ln [\aleph_{SI} /(z'' - \beta'')] / \ln 2 \quad (8)$$

The approach above has already been verified for heat- and mass-transfer processes, in the design of thermal energy storage systems [10] and for fundamental physical constants [12]. Therefore let's speculate about how we can apply it for analyzing the element structure of the existing universe.

III. Applications of $\aleph_{SI}$ – hypotheses

A. *Minimum Energy Resolution*

In the case of the Bekenstein bound, the information quantity contained in a sphere ϒ equals the information quantity ΔA_b obtained by the modelling process

$$\Upsilon = \Delta A_b \quad (9)$$

Or, taking into account (1) and (8),

$$(2\pi \cdot R \cdot E)/ (\hbar \cdot c \cdot \ln 2) = \ln [\aleph_{SI} /(z'' - \beta'')] / \ln 2 \quad (10)$$

So that

$$R \cdot E = \hbar \cdot c \cdot \ln [\aleph_{SI} /(z'' - \beta'')]/ 2\pi =$$
$$= 5.031726 \cdot 10^{-27} \cdot \ln [\aleph_{SI}/(z'' - \beta'')] \quad (11)$$

According to analysis of recorded variable dimensions, the Bekenstein model is classified by $COP_{SI} \equiv$ ***LMTΘ***.

In order to verify ***z***'' – ***β***'', we will use the definition of the comparative uncertainty [11] and its model expression $\varepsilon_{LMT\theta}$ introduced in [10]:

$$(\varepsilon_{LMT\theta}) = [(z'-\beta')/ \aleph_{SI} + (z''-\beta'')/ (z'-\beta')], \quad (12)$$

where $\varepsilon_{LMT\theta} = \Delta_{pmm}/M$, Δ_{pmm} is the absolute uncertainty in determining the DS theoretical field ***u***, which is "embedded" in a physical-mathematical model and caused only by its dimension. The DS interval of supervision of a field ***u*** is given by M. **z**' is the number of physical DL values in the selected COP_{SI}, and **β**' is the number of DL primary physical values in the selected COP_{SI}.

The conditions for achieving the minimum comparative uncertainty of a model $(\varepsilon_{min})_{LMT\theta}$, for $COP_{SI} \equiv$ ***LMTΘ***, can be formulated if one equates its partial derivative with respect to ***z***'- ***β***' to zero. Then we get:

$$[(\varepsilon_{min})_{LMT\theta}]'_{z'-\beta'} = [(z'-\beta')/ \aleph_{SI} + (z''-\beta'')/(z'-\beta')]' = \quad (13)$$

$$= [1/ \varkappa_{SI} - (\boldsymbol{z}''\text{-}\boldsymbol{\beta}'')/ (\boldsymbol{z}'\text{-}\boldsymbol{\beta}')^2],$$

$$[1/\varkappa_{SI} - (\boldsymbol{z}''\text{-}\boldsymbol{\beta}'')/ (\boldsymbol{z}'\text{-}\boldsymbol{\beta}')^2] = 0, \quad (14)$$

and

$$(\boldsymbol{z}'\text{-}\boldsymbol{\beta}')^2/ \varkappa_{SI} = (\boldsymbol{z}''\text{-}\boldsymbol{\beta}''). \quad (15)$$

Taking into account (5), one can calculate $\boldsymbol{z}'\text{-}\boldsymbol{\beta}'$:

$$\boldsymbol{z}'\text{-}\boldsymbol{\beta}' = (\boldsymbol{e}_l \cdot \boldsymbol{e}_m \cdot \boldsymbol{e}_t \cdot \boldsymbol{e}_\theta -1)/2 -4 = \\ = (7\cdot 3\cdot 9\cdot 9 -1)/2 -4 = 846, \quad (16)$$

where "-1" corresponds to all exponents of the primary variables in (3) having zero dimensions. Dividing by 2 means that there are both required and reverse variables, for example, the length L^1 and the running length L^{-1}. In other words, the object can be judged, knowing only one of its symmetrical parts, while others structurally duplicating this part, may be regarded as information empty. The 4 corresponds to four primary variables ***L***, ***M***, ***T***, and ***Θ***.

The minimum comparative uncertainty of a model $(\varepsilon_{min})_{LMT\theta}$, can be reached at condition (15). Then we get

$$(\boldsymbol{z}'' - \boldsymbol{\beta}'') = (\boldsymbol{z}' - \boldsymbol{\beta}')^2/ \varkappa_{SI} = \\ = 846^2/38{,}265 \approx 18.704194 \quad (17)$$

Taking into account (7), (11), and (17), the achievable value of $(R\cdot E)_{min}$ equals

$$(R\cdot E)_{min} = 5.031726\cdot 10^{-27}\cdot \ln [\varkappa_{SI}/(\boldsymbol{z}'' - \boldsymbol{\beta}'')] = \\ = 5.031726\cdot 10^{-27}\cdot \ln[38{,}265/18.704194]= \\ = 3.835958\cdot 10^{-26} \ (m^3\ kg\ s^{-2}). \quad (18)$$

$(R\cdot E)_{min}$ can be applied to verify the lowest energy uncertainty E_{min} indicating that the universe itself cannot distinguish energy levels lower than a special limit [13].

The age of universe T_{univ}, is about 13.7 ± 0.13 billion years, or $4.308595\cdot 10^{17}$ s [14]. Therefore, a radius of the universe is given by

$$R_{univ} = T_{univ} \cdot c = 1.291684 \cdot 10^{26} \ (m). \quad (19)$$

So, the minimum energy resolution, E_{min}, is

$$E_{min} = 3.835958\cdot 10^{-26} /1.291684\cdot 10^{26} = \\ = 2.969734\cdot 10^{-52} \approx 3\cdot 10^{-52}\ (m^2\ kg\ s^{-2}). \quad (20)$$

E_{min} is difficult to imagine and its lower value was introduced in [13] as: 10^{-50} J. At the same time, the value in (20) is of the same order as the ~10^{-45} ergs = 10^{-52} m^2 kg s^{-2} provided in [15]. E_{min} can be used, along with $\varkappa_{SI}$, and combining the thought experiment with field studies, for defining the uncertainty values of fundamental physical constants [12].

B. Minimum Energy Resolution

There are numerous concepts, approaches, methodologies and formulae proposed to calculate the minimum non-vanishing length in space-time R_{min}, or the resolution limit in any experiment [16]. Following on from ideas introduced in Section II, we have supposed that any our measurement has a certain intrinsic limited length about small scale physics. We will now calculate it.

t' Hooft [17] and Susskind [18] introduced the quantity S_{HS} that is the holographic entropy bound expressed in terms of the entropy

$$S_{HS} \leq \pi\cdot c^3\cdot R^2 / \hbar\ G \quad (21)$$

or

$$\Upsilon_{HS} \leq \pi\cdot c^3\cdot R^2 / (\hbar\cdot G\cdot \boldsymbol{\kappa}\cdot \ln 2). \quad (22)$$

Here Υ_{HS} is the information quantity expressed in bits and corresponding to S_{HS}, R is the radius of an object sphere expressed in meters, and G is the gravitational constant. Use c=299,792,458 $m^1 s^{-1}$, $\hbar$=$1.054572\cdot 10^{-34}$ $m^2 kg s^{-1}$, G=$6.67408\cdot 10^{-11} m^3 kg^{-1} s^{-2}$, $\boldsymbol{\kappa}\ln 2$ = $9.569926\cdot 10^{-24}$, and $\boldsymbol{\pi}$ = 3.141592.

Equating $\boldsymbol{\Delta A}_b$ (8) to (22) and using the known values of physical variables, we get

$$\pi\cdot c^3\cdot R^2/(\hbar\cdot G\cdot \boldsymbol{\kappa}\cdot \ln 2)= \ln[\varkappa_{SI}/(\mathbf{z}''-\boldsymbol{\beta}'')]/\ln 2, \quad (23)$$

$$1.256712\cdot 10^{93}\cdot R^2 = \ln [\varkappa_{SI} /(\mathbf{z}''-\boldsymbol{\beta}'')]/\ln 2, \quad (24)$$

$$R = 3.388203\cdot 10^{-47} \{\ln[\varkappa_{SI} /(\mathbf{z}''-\boldsymbol{\beta}'')]\}^{1/2}. \quad (25)$$

Taking into account (7), (17), and (25), the minimum achievable value of the length discretization or the universal, global standard of length, equals

$$R_{min} = 3.388203\cdot 10^{-47}\cdot [\ln (38{,}265/18.704194)]^{1/2} = \\ = 3.388203\cdot 10^{-47}\cdot [7.607852]^{1/2} = \\ = 3.388203\cdot 10^{-47}\cdot 2.758233 = \\ = 9.345455\cdot 10^{-47}\ (m) \quad (26)$$

It could be suggested that this quantum of space (26) is a concept that measures a "degree of distinguishability." In addition, maybe, the minimal length scale is not necessarily the Planck length. The scale of distance, just like the duration of time, turns out to be a property not of the world but of the models we employ to describe it [19].

C. Minimum Information Quantum Bit

Taking into account (20) and (26), let us calculate a possible minimum achievable amount of information Υ_q, in other words, an information quantum bit, or "qubit" [20]:

$$Y_q \leq (2\cdot\pi\cdot R_{min}\cdot E_{min})/(\hbar\cdot c\cdot \ln 2)= =0.79411\cdot 10^{-71}(\text{bit}). \quad (27)$$

Perhaps some readers of this article consider the three examples presented here to be a game of numbers. In his defense, the author reminds them of the attempts of Heisenberg and many other scientists to find the “firstborn” building blocks of the universe. The results presented are routine calculations from formulae known in the scientific literature. The author does not set himself the task of understanding the significance of the data for applications such as quantum electrodynamics or the theory of gravity; only experts in these areas can “separate the wheat from the chaff.” However, if the Bekenstein bound and $\varkappa_{SI}$-hypothesis have a physical explanation, perhaps, the discrete units of quanta of energy, length and information, can be used to study the universe.

IV. Conclusion

From the perspective of information theory and $\varkappa_{SI}$-hypothesis, the quantization of the energy, the length and the information medium is attempted.

Researchers can radically accelerate the speed of designing and delivering new models to industry and science. Using $\varkappa_{SI}$-hypothesis, development teams can orchestrate and optimize the activities of physical phenomena and engineering systems.

The comparative uncertainty provides us with a universal metric of length, energy and information quantity as a tool for reasoning, as an aid for constructing pictures and models of the world.

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