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Dynamic vibration characteristics of non-homogenous beam-model MEMS

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***Abstract*—In this paper a Timoshenko micro-beam model based on the modified couple stress theory is established to capture the size effects on the vibrational and dynamical behavior of the system. Mechanical properties of the micro-beam are supposed to vary through thickness based on the power law. Governing equations are derived using the Hamilton's principle. For free vibration analysis, a closed-form solution approach is presented. The verification of the model is accomplished by comparing the obtained results with benchmark results existing in the literature. A detailed consideration for the effects of material length scale parameter, power index and ratio of beam length to beam thickness upon the vibrational behavior of the model are reported. It is observed that these parameters have substantial role in the dynamic behavior of micro-structures.**

Keywords—Micro-beam; Modified couple stress theory; Vibration Analysis; Timoshenko Beam Theory, Nondimensional Frequency

I. INTRODUCTION

With the advance of Micro/Nano-technology, devices with thickness dimensions at the order of microns and sub microns are of high interest. They have some benefits in comparison to macro devices such like: they are light, often inexpensive and more durable. The most used geometrical shapes for the simulations of such devises are beams and plates. Since the controlled experiments in micro structures are difficult and cost a lot, affording the decent mathematical models is vital [1], [2].

Functionally graded materials (FGMs) belong to a class of graded composites. These non-homogenous materials are a mixture of two components with diverse mechanical properties which are being designed to exploit the benefits of the both constituents. An intentionally smooth variation of the materials in one specific direction is used to make graded materials capable of possessing different properties. Functionally graded materials usually consist of metal and ceramic phases. Ceramic part of the material delivers high temperature resistance and metal part impedes fracture due to thermal stresses in few cycles of loading. Material gradation causes smaller stress impositions. Moreover these stresses can be located in desired positions of the structure. Ceramic parts, because of their low thermal conduction coefficients can bear high temperature gradients. The ductile metal phase also increases the potential to withstand under rough dynamic loadings. Facing high shear stresses at the location of jointing two different components of composites is a serious problem which can be obviated by using material variant rules, existing in functionally graded materials. Thus ameliorated stress distribution, enhanced thermal gradient resistance and increased toughness, are all salient characters of functionally graded materials that make such mechanical devices applicable in biomechanics, optoelectronics, Micro/Nano-technologies. With simultaneous growth of micro structures and material technologies, functionally graded materials are extensively used in micro-scaled devices such as thin films in the form of shape memory alloys, micro-switches, micro-actuators, Micro/Nano-electro-mechanical systems (MEMS and NEMS) [3] through [8].

Some researchers have published different works regarding non-homogenous micro-structures. Studies in [9] through [12] are experimentally validated. They show that when the dimensions of a structure are scaled down, it is crucial to capture the size-dependent response of the system which is not afforded by classical continuum mechanics theories; as a result, researches tried to present non-classical theories in order to consider the small scale effects in micro and Nano-structures. Strain gradient theory, couple stress theory and modified couple stress theory are of the most prevalent theories used. Strain gradient theory proposed in [13] accounts for three intrinsic material length scale parameters. The mentioned parameters above, relate curvature tensor, deviator tensor and dilatation tensor to the stress tensor. Couple stress theory presented in [14], [15] and [16] uses four material parameters, including two classical and two additional parameters. Based on this theory, a group of researchers in [17] proposed that three equilibrium equations should be considered for the material element; classical equilibrium equations of forces and moments of forces and an additional equation for the equilibrium of moments of couples. They made an inference that this additional equation implies the symmetry of the couple stress tensor. So they improved the constitutive equations and presented the modified couple stress theory. Besides the continuum theories, beam theories play significant role in the prediction of static and dynamic response of the structures. Euler-Bernoulli beam theory is simple in comparison to the other theories since the shear characteristics and deformations are neglected. This is a decent theory when the ratio of length to the thickness is larger than 20. However, to prognosticate more accurate behavior, especially when the beam is thicker, some restrictive assumptions should be omitted and shear characteristics should be accounted. In this case Timoshenko and other high order beam theories are suitable.

Utilizing modified couple stress theory, the mechanical properties of an Euler–Bernoulli beam in the static condition in [18] is studied. In [19] size-dependent free vibration behavior of carbon reinforced polymer micro-cantilevers based on the modified couple stress theory is carried out, in which used Euler-Bernoulli beam model and the axially-graded material in in consideration. In another research, free vibration behavior of micro-scaled structures using the modified couple stress theory and three different beam theories: Euler-Bernoulli, Timoshenko and third order shear deformation theories is accomplished [20]. In [21] functionally graded Timoshenko Nano-beam model for free vibration analysis is modeled, and non-local elasticity theory and principle of minimum potential energy for obtaining the governing equations are utilized. In [22], researchers have studied dynamic response of a functionally graded micro beam based on the strain gradient theory. They modeled thick beams using Timoshenko beam theory. Another group of researchers in [23] presented Timoshenko size-dependent model, using the nonlocal elasticity theory. Free vibration analysis of Euler-Bernoulli micro beams using an approximate method is carried out in [24], which is based on the modified couple stress theory. In another research paper, vibration analysis of a temperature-dependent micro-beam is reported. This study is based on the modified couple stress theory and the thermo-mechanical properties of the system are varying according to temperature shifts and thermal stresses play a major role in the characteristic determination [25].

In this paper, free lateral vibration response of functionally graded Timoshenko micro beam is presented. The beam is graded in the direction of the thickness. The modified couple stress theory in addition to the Hamilton's principle is used to obtain the governing equations, boundary and initial conditions. The effects of material length scale parameter upon dimensionless natural frequencies of a simply-supported micro beam are reported. Also, for first time, higher order modes of vibration influenced by gradient index and wide range of slenderness ratios are obtained. The other noble case of this paper is to use different distribution rule for functionally graded materials and inertia-based process for calculating the dimensionless frequencies.

II. MATHEMATICAL MODELING

a. *Modified couple stress theory*

With the modified couple stress theory, strain energy function depends on both strain and rotation gradients. In the Cartesian coordinates, u_x, u_y and u_z are defined as the displacement field vectors in the directions of the beam's length, width and thickness. The displacement gradient tensor $u_{i,j}$ can be decomposed into symmetric and anti-symmetric parts as the tensors of strain and rotation, respectively:

$$u_{i,j} = \varepsilon_{ij} + \omega_{ij} \tag{1}$$

Where

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

$$\omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}) \tag{3}$$

Using the alternator ϵ rotation vector dual to the rotation tensor can be defined as:

$$\theta_i = \frac{1}{2}\omega_{jk}\epsilon_{ijk} \tag{4}$$

Now gradient of rotation is defined as:

$$\kappa_{ij} = \theta_{i,j} = \frac{1}{2}\omega_{kl,j}\epsilon_{ikl} \tag{5}$$

Where κ denotes the curvature tensor.

Constitutive equations of the modified couple stress theory are defined by the strain energy density function e_s [23]:

$$e_s = \frac{1}{2}\lambda\varepsilon_{ii}\varepsilon_{jj} + G\,\varepsilon_{ij}\varepsilon_{ij} + 2Gl^2\kappa_{ij}\kappa_{ij} \tag{6}$$

That λ and G are two classical Lame constants. l Is Lame-type non-classical material parameter which introduces the couple stress effects.

Eq. (1) leads to the constitutive equations as:

$$\sigma_{ij} = \frac{\partial e_s}{\partial \varepsilon_{ij}} = 2G\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} \tag{7}$$

$$\varrho_{ij} = \frac{\partial e_s}{\partial \kappa_{ij}} = 4Gl^2\kappa_{ij} \tag{8}$$

In which σ_{ij} and ϱ_{ij} denote the classical stress and couple stress tensors.

The strain energy of the deformed linear elastic body based on the classical strain and curvature tensors is associated to the symmetric classical stress and deviatoric couple stress tensor. This energy for a body occupying volume V is defined as:

$$U_s = \frac{1}{2}\int_V \left(\sigma_{ij}:\varepsilon_{ij} + \varrho_{ij}:\kappa_{ij}\right) dV \tag{9}$$

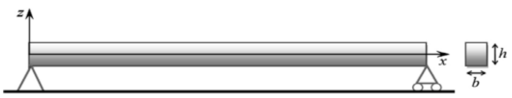

Figure 1. Schematic of the micro FG beam.

b. Fuctionally graded materials

In Figure 1. a functionally graded micro beam of length L, width b and thickness h is shown. We supposed that the micro beam is made up of two dissimilar materials and the effective mechanical properties of the beam vary through the thickness direction.

Using decent medley rule for micro structures, one can describe the effective mechanical properties P based on the rule of mixture as:

$$P = P_a V_a + P_s V_s \tag{10}$$

Where P_a and P_s are the effective mechanical properties of the constituents, V_a and V_s are the volume fractions, restricted by the following equation:

$$V_a + V_s = 1 \tag{11}$$

We used power-law form to define the mechanical properties of the micro structure. The volume fraction of the second material is defined by:

$$V_a = (\frac{z}{h} + \frac{1}{2})^k \tag{12}$$

In which k denotes the power-law exponent that specifies the material variation contour through the thickness direction. The three requisite properties for delineation are Young's modulus, shear modulus and density which can be replaced with P as follows:

$$E(z) = (E_a - E_s)\left(\frac{z}{h} + \frac{1}{2}\right)^k + E_s \tag{13}$$

$$G(z) = (G_a - G_s)\left(\frac{z}{h} + \frac{1}{2}\right)^k + G_s \tag{14}$$

$$\rho(z) = (\rho_a - \rho_s)\left(\frac{z}{h} + \frac{1}{2}\right)^k + \rho_s \tag{15}$$

It can be easily depicted that the micro functionally graded beam reaches pure material properties at the top and bottom surfaces.

c. Timoshenko beam theory

In the Timoshenko beams, shear strain due to distortion is not being neglected; hence, rotation angle is not equal to derivation of the lateral displacement. According to this theory, axial displacement, u, lateral displacement, w, of any point on the neutral axis is expressed as follows [24, 25]:

$$u_x(x, y, z) = u(x, t) - z\theta(x, t) \tag{16}$$

$$u_y(x, y, z) = 0 \tag{17}$$

$$u_x(x, y, z) = w(x, t) \tag{18}$$

Where θ, shows the total angle of rotation of the cross section.

Nonzero strains are obtained as:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial \theta}{\partial x} \tag{19}$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} - \theta\right) \tag{20}$$

$$\kappa_{xy} = \kappa_{yx} = \frac{-1}{4}\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \theta}{\partial x}\right) \tag{21}$$

d. Governing equations of motion

Governing equations expressing the vibrational motions are obtained using the Hamilton's principle.

$$\delta\left[\int_{t_1}^{t_2} (T - Us)dt\right] = 0 \tag{22}$$

In which T is kinetic energy and U_s is potential energy.

Exerting the Eqs. (13)-(21) into Eq. (24) gives the potential energy. Also kinetic energy for a Timoshenko beam is obtained using Eq. (19):

$$U_s = \frac{1}{2}\int_V \{(2G(z) + \lambda(z))\varepsilon_{xx}{}^2 + 4G(z)\varepsilon_{xz}{}^2 + 4G(z)l^2\kappa_{xy}{}^2\} dV \tag{23}$$

$$T = \frac{1}{2}\int_V \rho(z)\{\left(\frac{\partial u_x}{\partial t}\right)^2 + (\frac{\partial u_z}{\partial t})^2\} dV \tag{24}$$

Using variation calculus and the detail by detail method, final form of the energy terms can be expressed as following:

$$\int_{t_1}^{t_2} \delta U_s = \int_{t_1}^{t_2}\int_0^L \left\{(S_1\frac{\partial^3 u}{\partial x^3} + S_2\frac{\partial^4 w}{\partial x^4} + S_3\frac{\partial^3 \theta}{\partial x^3} - S_7\left(\frac{\partial^2 w}{\partial x^2} - \frac{\partial \theta}{\partial x}\right))\delta w - (S_4\frac{\partial^2 u}{\partial x^2} + S_5\frac{\partial^3 w}{\partial x^3} + S_6\frac{\partial^2 \theta}{\partial x^2} - S_7\left(\frac{\partial w}{\partial x} - \theta\right))\delta\theta - (D_1\frac{\partial^2 u}{\partial x^2} + D_2\frac{\partial^3 w}{\partial x^3} + D_3\frac{\partial^2 \theta}{\partial x^2})\delta u\right\} dxdt \tag{25}$$

$$\int_{t_1}^{t_2} \delta T = \int_{t_1}^{t_2}\int_0^L \left\{\left(D_4\frac{\partial^3 u}{\partial x\partial t^2} + D_5\frac{\partial^4 w}{\partial x^2\partial t^2} + D_6\frac{\partial^3 \theta}{\partial x\partial t^2} - F_4\frac{\partial^2 w}{\partial t^2}\right)\delta w - \left(D_7\frac{\partial^2 u}{\partial t^2} + D_8\frac{\partial^3 w}{\partial x\partial t^2} + D_9\frac{\partial^2 \theta}{\partial t^2}\right)\delta\theta - (F_1\frac{\partial^2 u}{\partial t^2} + F_2\frac{\partial^3 w}{\partial x\partial t^2} + F_3\frac{\partial^2 \theta}{\partial t^2})\delta u\right\} dxdt \tag{26}$$

Coefficients used in Eqs. (20) and (21) are defined in the following:

$$S_2 = \frac{1}{4}l^2\int_A G(z)dA \tag{27}$$

Neglecting the shear effects, some of the coefficients are obtained based on the modulus of elasticity.

$$S_4 = \int_A E(z)(-z)dA \tag{28}$$

$$S_6 = \int_A E(z)(z^2)dA + \frac{1}{4}l^2\int_A G(z)dA \tag{29}$$

$$S_7 = k_s\int_A G(z)dA \tag{30}$$

$$D_1 = \int_A E(z)dA \tag{31}$$

$$D_7 = \int_A \rho(z)(-z)dA \tag{32}$$

$$F_1 = \int_A \rho(z)dA \tag{33}$$

$$S_1, D_2, D_4, D_5, D_6, D_8, F_2 = 0, \tag{34}$$

$$D_3 = S_4, F_3 = D_7 \tag{35}$$

Using Eqs. (27)-(35) into the Eq. (22), gives a system of coupled partial differential equations, known as the governing equations.

III. Solution procedure

For the governing equations, related to free vibration of a simply-supported FG micro beam, an analytical solution based on the Navier method is presented. Navier method as a kind of discretizing approach which expands the displacement and rotation functions as products of undetermined coefficients and specific trigonometric functions which satisfy the boundary conditions, at $=0, L$. The functions are defined in the following form:

$$w(x,t)=\sum_{n=1}^{\infty} a_n \sin(\alpha x) e^{i\omega_n t}, \alpha=(n\pi)/L \quad (36)$$

$$\theta(x,t)=\sum_{n=1}^{\infty} b_n \cos \alpha x\, e^{i\omega_n t} \quad (37)$$

$$u(x,t)=\sum_{n=1}^{\infty} c_n \cos \alpha x\, e^{i\omega_n t} \quad (38)$$

In which a_n, b_n, c_n are the coefficients to be determined for each n. Boundary conditions of a simply-supported beam are as follows:

$$w|_{x=0,L}, \qquad \left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0,L} \quad (39)$$

Substituting Eqs. (36)-(38) into Eqs. (25)-(26) leads to the following system of equations:

$$(F_4 {\omega_n}^2 - S_2 {\alpha_n}^4 - S_7 {\alpha_n}^2) a_n + (-S_3 {\alpha_n}^3 + S_7 \alpha_n) b_n = 0 \quad (40)$$

$$(-S_5 {\alpha_n}^3 + S_7 \alpha_n) a_n + (D_9 {\omega_n}^2 - S_6 {\alpha_n}^2 - S_7) b_n + (D_7 {\omega_n}^2 - S_4 {\alpha_n}^2) c_n = 0 \quad (41)$$

$$(F_3 {\omega_n}^2 - D_3 {\alpha_n}^2) b_n + (F_1 {\omega_n}^2 - D_1 {\alpha_n}^2) c_n = 0 \quad (42)$$

Determinant of the coefficient matrix of the above equations, gives frequency equation in the form of polynomial for each n. Setting this polynomial equal to zero gives the frequency of each mode.

IV. Results and discussion

Steel and alumina (Al_2O_3) are the two constituents of the simply-supported functionally graded micro beam investigated in this study. The mechanical properties vary through the thickness direction based on a power-law, Table 1 shows these mechanical properties of pure steel and alumina. The beam length and width are equal to $L=10000, b=1000$ micro meters. Natural frequencies are non-dimensionalized using the following equation, $\widehat{\omega}=\omega L^2\sqrt{\rho_a A/E_a I}$, where $I=bh^3/12$ is the second moment of inertia of the beam cross section and A is area of the cross section. The dimensionless frequencies are accompanied with slenderness ratio, material distribution for three modes of vibration. According to experimental tests reported by Lame, non-classical parameter (l) is taken 15 micro meters. k_s represents the shear correction factor which for rectangular cross sections is equal to 5/6. For verification and check validity of the present analysis, the results are compared with those of Euler-Bernoulli FG Nano beam ([21]) in Tables 2-3 and Timoshenko FG Nano beam ([23]) in Tables 4-5.

Putting the material length scale parameter equal to zero gives the equations which the corresponding solution procedure culminates in dimensionless frequencies of classical theory shown in Tables 2-5. Tables 4-5 show that, generally the modified couple stress theory overestimates classical frequencies rather than those of the non-local elasticity theory. Dimensionless frequencies based on the modified couple stress theory for the first three modes of vibration are shown in Tables 6-7; results are shown in the division of different slenderness ratios and diverse power indexes. According to Tables 6-7, as the power index increases the frequency decreases and as the slenderness ratio increases the frequency increases.

Figs. 1-3 demonstrate the variation of dimensionless frequencies with varying material distribution (power index) and specific slenderness ratios for the first, second and three mode of vibration. It can be observed that there is a sharp gradient when the power index is in small range $0<k<2$; rate of changes reduce at $2<k<5$ and finally, the frequency has the least variation with power index for $5<k<10$. To make an inference for so large power index values, we expect an independent behavior of the micro structure. Figs. 1-3 also substantiate the reduction of frequencies by increasing power index and decreasing slenderness ratio concluded from Tables 6-7.

Fig. 4 demonstrates the gradation of dimensionless frequency with slenderness ratios. As it can be seen, for all power indexes, frequency increases as the slenderness ratio takes larger quantities. Also it seems that the rates of changes with slenderness ratio are similar to each other for different material distributions.

V. Conclusion

The free vibration analysis of functionally graded micro beam based on the modified couple stress theory is presented in this study. The simply-supported micro beam is modeled according to the Timoshenko beam theories. The non-classical constitutive equations are formed due to assumptions of the modified couple stress theory. The Hamilton's principle is used to derive the governing equations, initial and boundary conditions. Based on the Navier method, an analytical solution is proposed. Numerical results express the effect of the material length scale parameter upon the vibration behavior of the FG micro beam. As a result it seems crucial to exert the non-classical parameter, mentioned above in the vibration analysis of micro structures. The other incisive case playing a tangible role in the analysis is the power index. Due to the results, it is feasible to reach the specific frequency by selecting the appropriate power quantity. Effects of the slenderness ratios and comparative dimensions of the micro beam are characterized such that smooth gradation of frequency is obtained by different beam length to thickness ration. Consequently, a fastidious design with respect to the material length scale parameter, decent power index value and slenderness ratio results in the desired and predictable dynamic behavior of the FG micro beams.

TABLE 1. Mechanical Properties of FGM Constituents

Properties	Steel	Alumina
ρ	7800 $(\mathrm{Kg/m^3})$	3960 $(\mathrm{Kg/m^3})$
E	210 (GPa)	390 (GPa)
ν	0.3	0.24

TABLE 2. Comparison of Non-dimensional Natural Frequencies With [21] (2012). (k=0,0.1,0.2,0.5)

L/h	Mode of vibration	$k = 0$	[21] $k = 0$	$k = 0.1$	[21] $k = 0.1$	$k = 0.2$	[21] $k = 0.2$	$k = 0.5$	[21] $k = 0.5$
20	$n = 1$	9.8509	9.8797	9.1813	9.2129	8.6792	8.7200	7.7322	7.8061
	$n = 2$	38.9365	39.6419	36.2903	36.9488	34.3057	34.9276	30.5615	31.1114
	$n = 3$	84.5731	89.6599	78.8629	83.5840	74.5713	79.0261	66.4399	70.4278
50	$n = 1$	9.9965	9.8724	9.3181	9.2045	8.8092	8.7115	7.8489	7.7998
	$n = 2$	39.9079	38.8778	37.1997	36.8214	35.1683	34.8045	31.3341	31.0042
	$n = 3$	88.0944	88.6492	82.1110	82.9145	77.6230	78.3897	69.1537	69.8720
100	$n = 1$	10.3913	9.8700	9.6900	9.2038	9.1634	8.7111	8.1670	7.7981
	$n = 2$	41.5447	39.4849	38.7409	36.8000	36.6358	34.7868	32.6519	30.9909
	$n = 3$	88.6414	88.8594	82.6154	82.8224	78.0967	78.3028	69.5750	69.7968

TABLE 3. Comparison of Non-dimensional Natural Frequencies With [21] (2012). (k=1,2,5,10)

L/h	Mode of vibration	$k = 1$	[21] $k = 1$	$k = 2$	[21] $k = 2$	$k = 5$	[21] $k = 5$	$k = 10$	[21] $k = 10$
20	$n = 1$	6.9830	7.0904	6.4099	6.5244	5.9289	6.0025	5.6631	5.7058
	$n = 2$	27.5964	28.0910	25.3240	25.7847	23.4137	23.8575	22.3632	22.7937
	$n = 3$	59.9393	63.6216	54.8715	58.4009	50.5741	53.9949	48.3104	51.5621
50	$n = 1$	7.0876	7.0852	6.5033	6.5189	6.0118	5.9990	5.7417	5.7001
	$n = 2$	28.2943	28.0048	25.9604	25.7083	23.9968	23.7856	22.9186	22.7194
	$n = 3$	62.4458	63.1454	57.3038	57.9879	52.9822	53.6120	50.6098	51.1902
100	$n = 1$	7.3713	7.0833	6.7527	6.5182	6.2286	5.9970	5.9464	5.7005
	$n = 2$	29.4706	27.9902	26.9970	25.6984	24.9011	23.7762	23.7730	22.7115
	$n = 3$	62.8355	63.0799	57.6835	57.9299	53.3600	53.5616	50.9705	51.1345

TABLE 4. Comparison of Non-dimensional Natural Frequencies With [23] (2013). (k=0,0.1,0.2,0.5)

L/h	$k = 0$	[23] $k = 0$	$k = 0.2$	[23] $k = 0.2$	$k = 0.5$	[23] $k = 0.5$
20	9.8509	9.8296	8.6792	8.6600	7.7322	7.7149
50	9.9965	9.8631	8.8092	8.6895	7.8489	7.7413
100	10.3913	9.8680	9.1634	8.6938	8.1670	7.7451

TABLE 5. Comparison of Non-dimensional Natural Frequencies With [23] (2013). (k=1,2,5,10)

L/h	$k = 1$	[23] $k = 1$	$k = 5$	[23] $k = 5$	$k = 10$	[23] $k = 10$
20	6.9830	6.9676	5.9289	5.9172	6.2286	5.6521
50	7.0876	6.9917	6.0118	5.9389	5.7417	5.6730
100	7.3713	6.9952	6.2286	5.9421	5.9464	5.6760

TABLE 6. Non-dimensional Natural Frequencies Based On The Modified Couple Stress Theory. (k=0,0.1,0.2,0.5)

L/h	Mode of vibration	$k = 0$	$k = 0.1$	$k = 0.2$	$k = 0.5$
10	$n = 1$	9.7183	9.0576	8.5622	7.6276
	$n = 2$	37.1922	34.6661	32.7707	29.1903
	$n = 3$	78.5071	73.1835	69.1854	61.6187
30	$n = 1$	9.8999	9.2272	8.7227	7.7712
	$n = 2$	39.3869	36.7108	34.7039	30.9176
	$n = 3$	87.8443	81.8767	77.4009	68.9546
90	$n = 1$	10.2935	9.5979	9.0757	8.0882
	$n = 2$	41.1491	38.3683	36.2810	32.3332
	$n = 3$	92.4923	86.2419	81.5502	72.6765

Table 7. Non-dimensional natural frequencies based on the modified couple stress theory. (k=1,2,5,10)

L/h	Mode of vibration	$k = 1$	$k = 2$	$k = 5$	$k = 10$
10	$n = 1$	6.8877	6.3210	5.8448	5.5826
	$n = 2$	26.3458	24.1516	22.2994	21.2955
	$n = 3$	55.5778	50.8746	46.8814	44.7601
30	$n = 1$	7.0182	6.4418	5.9579	5.6907
	$n = 2$	27.9199	25.6235	23.6940	22.6310
	$n = 3$	62.2624	57.1284	52.8103	50.4393
90	$n = 1$	7.3011	6.6909	6.1749	5.8957
	$n = 2$	29.1864	26.7470	24.6835	23.5675
	$n = 3$	65.6023	60.1177	55.4778	52.9693

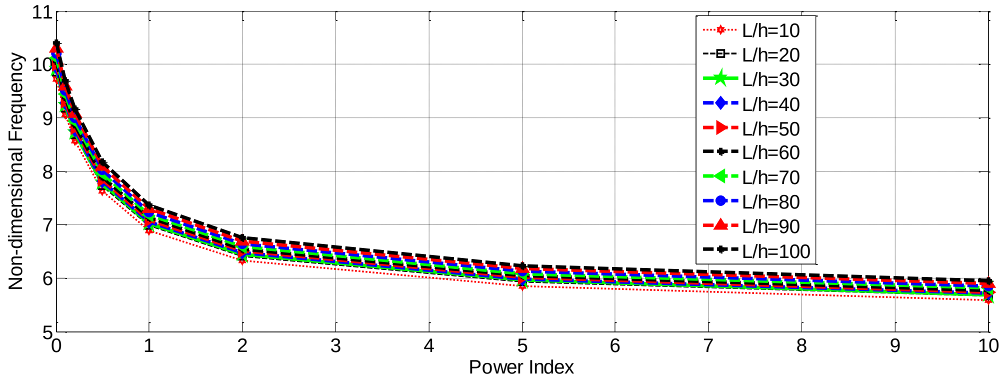

Fig 1. Variation of non-dimensional natural frequencies of the first mode with power index.

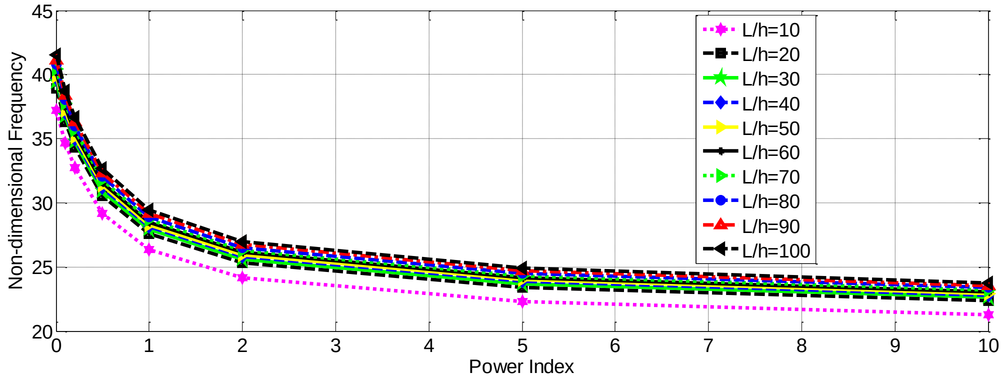

Fig 2. Variation of non-dimensional natural frequencies of the second mode with power index.

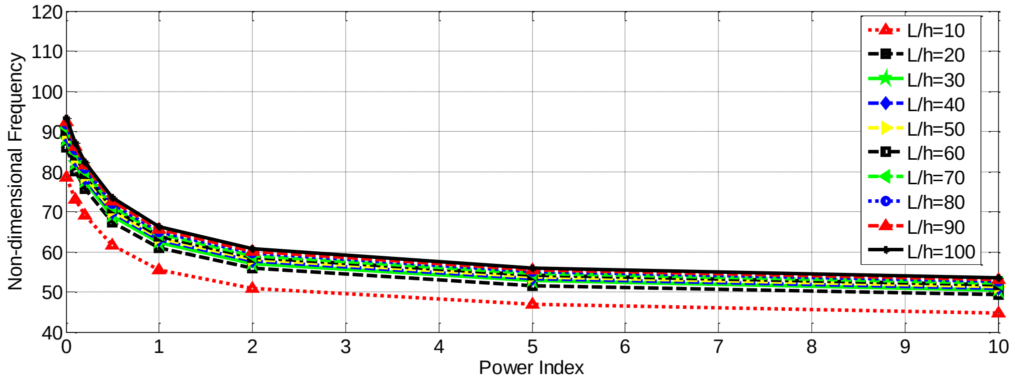

Fig 3. Variation of non-dimensional natural frequencies of the third mode with power index.

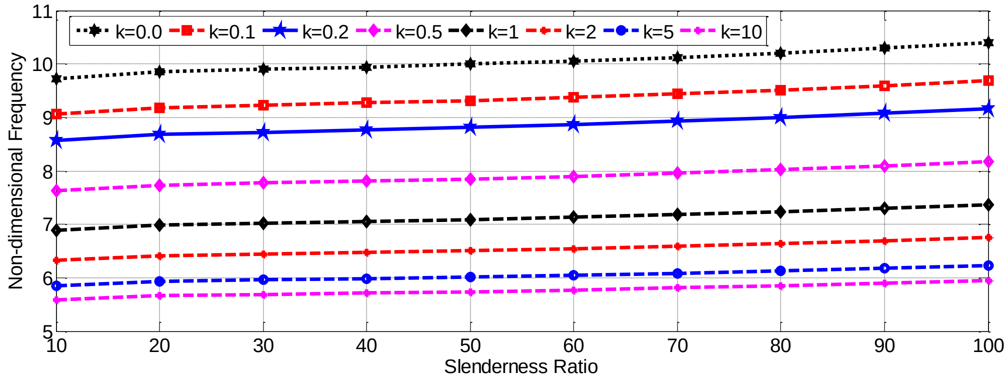

Fig 4. Gradation of non-dimensional natural frequencies with slenderness ratios.

VI. References

[1] WT Koiter, Couple-stresses in the theory of elasticity, I and II. Proc K Ned Akad Wet B 1964; 67: 17–44.

[2] RD Mindlin, HF Tierstenm, Effects of couple-stresses in linear elasticity. Arch Ration Mech Anal 1962; 11-1: 415–48

[3] A Ghanbari, A Babaei, The new boundary condition effect on the free vibration analysis of micro-beams based on the modified couple stress theory. International Research Journal of Applied and Basic Science 2015; 9(3): 274-279

[4] A Babaei, A Ghanbari, F Vakili-Tahami, Size-dependent behavior of functionally graded micro-beams, based on the modified couple stress theory. Thechnology 2015; 3(5): 364-372

[5] FACM Yang, A.C.M. Chong, D.C.C. Lam, P. Tong, Couple stress based strain gradient theory for elasticity. International Journal of Solids and Structures 2002; 39-10: 2731-2743.

[6] WT Koiter, Couple-stresses in the theory of elasticity, I and II. Proc K Ned Akad Wet B 1964; 67: 17–44.

[7] SK Park, XL Gao, Bernoulli–Euler beam model based on a modified couple stress theory. Journal of Micromechanics and Microengineering 2006; 16-11: 2355.

[8] HM Ma, XL Gao, JN Reddy, A microstructure-dependent Timoshenko beam model based on a modified couple stress theory. Journal of the Mechanics and Physics of Solids 2008; 56-12: 3379-3391.

[9] M Asghari, MT Ahmadian, MH Kahrobaiyan, M Rahaeifard, On the size-dependent behavior of functionally graded micro-beams. Materials & Design 2010; 31-5: 2324-2329.

[10] A Ghanbari, A Babaei, F Vakili-Tahami. Free Vibration Analysis of Micro Beams Based on the Modified Couple Stress Theory, Using Approximate Methods. Technology 2015; 3-02: 136-143

[11] RD Mindlin, Micro-structure in linear elasticity. Arch. Ration. Mech. Anal 1964; 16: 51-78.

[12] HG Georgiadis, EG Velgaki, High-frequency Rayleigh waves in materials with micro-structure and couple-stress effects. International Journal of Solids and Structures 2003; 40-10: 2501-2520.

[13] S Sahmani, M Bahrami, R Ansari, Nonlinear free vibration analysis of functionally graded third-order shear deformable microbeams based on the modified strain gradient elasticity theory. Composite Structures 2014; 11: 219-230.

[14] M Fathalilou, M Sadeghi, G Rezazadeh, Micro-inertia effects on the dynamic characteristics of micro-beams considering the couple stress theory. Mechanics Research Communications 2014; 60: 74-80.

[15] JS Stolken, AG Evans, Microbend test method for measuring the plasticity length scale. Acta Mater 1998; 46-14: 5109–15

[16] ACM Chong, DCC Lam, Strain gradient plasticity effect in indentation hardness of polymers. Mater Res 1999; 14-10: 4103–10

[17] AW McFarland, JS Colton, Role of material microstructure in plate stiffness with relevance to microcantilever sensors. Micromech Microeng 2005; 1060–7

[18] R. Aghazadeh, E. Cigeroglu, S. Dag. Static and free vibration analyses of small-scale functionally graded beams possessing a variable length scale parameter using different beam theories. European Journal of Mechanics-A/Solids 2014; 46: 1-11. 2014.01.002

[19] H Rokni, A Milani, AS, RJ Seethaler, Size-dependent vibration behaviour of functionally graded CNT-Reinforced polymer microcantilevers: Modeling and optimization. European Journal of Mechanics-A/Solids 2015; 49: 26-34

[20] M Simsek. Nonlinear static and free vibration analysis of microbeams based on the nonlinear elastic foundation using modified couple stress theory and He's variational method. Composite Structures 2014; 112: 264-272

[21] MA Eltaher, SA Emam, FF Mahmoud. Free vibration analysis of functionally graded size-dependent nanobeams, Applied Mathematics and Computation 2012; 218-14: 7406-7420

[22] R Ansari, R Gholami, S Sahmani. Free vibration analysis of size-dependent functionally graded microbeams based on the strain gradient Timoshenko beam theory, Composite Structures 2011; 94-1: 221-228.

[23] O Rahmani, O Pedram. Analysis and modeling the size effect on vibration of functionally graded nanobeams based on nonlocal Timoshenko beam theory, International Journal of Engineering Science 2013; 77: 55-70

[24] A Ghanbari, A Babaei, F Vakili-Tahami. Free Vibration Analysis of Micro Beams Based on the Modified Couple Stress Theory, Using Approximate Methods. Technology 2015; 3-02: 136-143

[25] A Babaei, MRS Noorani, A Ghanbari, Temperature-dependent free vibration analysis of functionally graded micro-beams based on the modified couple stress theory. Microsystem Technologies 2017. 1-12