Controlling of Operation Height in a Coupled Tanks System with Model-Reality Differences

[1] **Sy Yi Sim**
Department of Electrical Engineering Technology
Universiti Tun Hussein Onn Malaysia
Batu Pahat, Malaysia
sysim@uthm.edu.my

[2] **Sie Long Kek**
Department of Mathematics and Statistics
Universiti Tun Hussein Onn Malaysia
Batu Pahat, Malaysia
slkek@uthm.edu.my

[3] **Ta Wee Seow**
Department of Construction Management
Universiti Tun Hussein Onn Malaysia
Batu Pahat, Malaysia
tawee@uthm.edu.my

[4] **Kim Gaik Tay**
Department of Communication Engineering
Universiti Tun Hussein Onn Malaysia
Batu Pahat, Malaysia
tay@uthm.edu.my

***Abstract*—In this paper, an efficient computational approach is proposed to control the normal operating height in a coupled tanks system. Since the dynamics of the coupled tanks system is in a nonlinear manner, determination of the optimal water level in the tanks is challenging but useful for operation decision. For simplicity, the linear model of the coupled tanks system dynamics is suggested to give the true operating height of the coupled tanks. In our approach, the adjustable parameter is added into the model used. The aim is to measure the differences between the real plant and the model used repeatedly during the computation procedure. In this way, the optimal solution of the model used can be updated iteratively. On this basis, system optimization and parameter estimation are integrated. At the end of the iteration, the converged solution approximates to the correct optimal solution of the original optimal control problem, in spite of model-reality differences. For illustration, the numerical parameters of the coupled tank system are studied and the applicable of the approach proposed is shown. In conclusion, the efficiency of the approach proposed in achieving the desired water level of the coupled tanks is highly presented.**

Keywords—optimal control, iterative solution, coupled tanks system, model-reality differences, adjusted parameter

I. Introduction

Modelling of a coupled tanks system, which is joined by two or more tanks, in engineering process is a significant experimental task. The applications of the coupled tanks system have been well-defined and widely studied in the engineering community, especially, the use of control and optimization techniques [1]–[4] in determining the operation height in the coupled tanks system. In fact, the practical aspects of the coupled tanks system are ranged from chemical process to mechanical system [5]–[9]. Additionally, the basic concept of the coupled tanks system is based on the mass balance equation, where the inflow and the outflow are balance in the system. Mathematically, the coupled tanks system is formulated as a set of the ordinary differential equations (ODEs), where the solution of the set of ODEs, both for the analytic and the numeric, are fruitful results [10]–[12].

In this paper, a simple coupled tanks system is considered [1]–[2]. This coupled tanks system, which has two joint tanks, two inflows and two outflows, is formulated by two nonlinear ODEs. Then, the determination of the water level in the tanks is defined as an optimal control problem so that the water level could be controlled at the normal operating height. In our approach, the linear model, which is simplified from the original optimal control problem, is proposed [13]–[14]. After that the adjusted parameter is added into the model used. The aim is to measure the differences between the real plant and the model used repeatedly during the computation procedure. In this way, the optimal solution of the model used can be updated iteratively. Therefore, system optimization and parameter estimation are integrated. At the end of the iteration, the converged solution approximates to the correct optimal solution of the original optimal control problem, in spite of model-reality differences [15]–[18], and the desired water level in the tanks would be achieved.

The rest of the paper is organized as follows. In Section 2, the optimal control problem of the coupled tanks system and the simplified linear optimal control model are described. In Section 3, an expanded optimal control problem, which integrates system optimization and parameter estimation, is introduced. Then, an iterative algorithm is derived for solving the optimal control problem of the coupled tanks system. In Section 4, the numerical study of the coupled tanks system is illustrated to achieve the desired water level in the tanks by using the algorithm proposed. Finally, some concluding remarks are made.

II. Problem Statement

Consider the coupled tanks system with three valves as shown in Fig. 1. The dynamic equation for each tank is given by

Tank 1: $$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{o1} - Q_3 \quad (1)$$

Tank 2: $$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{o2} + Q_3 \quad (2)$$

where H_1 and H_2 are the height of water in Tanks 1 and 2, respectively, A_1 and A_2 are cross sectional area of Tanks 1 and 2, respectively, Q_{i1} and Q_{i2} are pump flow rate into Tanks 1 and 2, respectively, Q_{o1} and Q_{o2} are flow rate of water out of Tanks 1 and 2, respectively, and Q_3 is flow rate of water between Tanks 1 and 2.

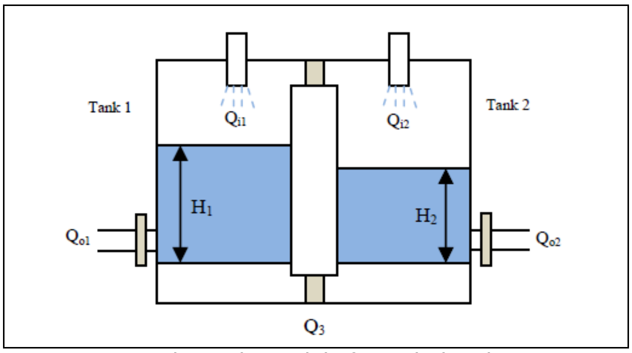

Fig. 1. *Schematic model of coupled tanks system*

Each outlet drain is modelled as a simple orifice. According to Bernoulli's equation for steady, non-viscous, incompressible fluid, the outlet flow in each tank is proportional to the square root of the head of water in the tank, and the flow between the tanks is proportional to the square root of the head of differential [5]. That is,

$$Q_{o1} = a_1 \sqrt{H_1} \quad (3)$$

$$Q_{o2} = a_2 \sqrt{H_2} \quad (4)$$

$$Q_3 = a_3 \sqrt{H_1 - H_2} \quad (5)$$

where a_1, a_2 and a_3 are proportionality constants, which their values depend on the coefficients of discharge, the cross sectional area of each orifice and the gravitational constant. Therefore, the system dynamics of the coupled tanks system in (1) and (2) becomes

$$A_1 \frac{dH_1}{dt} = Q_{i1} - a_1\sqrt{H_1} - a_3\sqrt{H_1 - H_2} \quad (6)$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - a_2\sqrt{H_2} + a_3\sqrt{H_1 - H_2}\,. \quad (7)$$

Suppose that for the set of inflows Q_{i1} and Q_{i2}, the water level in the tanks is at some steady state levels H_1 and H_2. Consider small variations in each inflow, which are represented by q_1 in Q_{i1}, q_2 in Q_{i2}, h_1 in H_1 and h_2 in H_2. Equations (6) and (7) are rewritten by

$$A_1 \frac{d(H_1 + h_1)}{dt} = (Q_{i1} + q_1) - a_1\sqrt{H_1 + h_1} - a_3\sqrt{H_1 - H_2 + h_1 - h_2} \quad (8)$$

$$A_2 \frac{d(H_2 + h_2)}{dt} = (Q_{i2} + q_2) - a_2\sqrt{H_2 + h_2} + a_3\sqrt{H_1 - H_2 + h_1 - h_2} \quad (9)$$

Subtracting (6) and (7) from (8) and (9), the system model of the coupled tanks becomes

$$\frac{dh_1}{dt} = \frac{q_1}{A_1} - \frac{a_1}{A_1}(\sqrt{H_1 + h_1} - \sqrt{H_1}) - \frac{a_3}{A_1}(\sqrt{H_1 - H_2 + h_1 - h_2} - \sqrt{H_1 - H_2}) \quad (10)$$

$$\frac{dh_2}{dt} = \frac{q_2}{A_2} - \frac{a_2}{A_2}(\sqrt{H_2 + h_2} - \sqrt{H_2}) - \frac{a_3}{A_2}(\sqrt{H_1 - H_2 + h_1 - h_2} - \sqrt{H_1 - H_2}) \quad (11)$$

Then, this problem of controlling the water level in Tank 2, which is defined as an optimal control problem and is referred to as Problem (P), is described by

Problem (P):

Find the optimal small variations of the flow rate $q = (q_1 \quad q_2)^{\mathrm{T}}$ in order to minimize the cost function

$$J_0 = \tfrac{1}{2}\int_{t_0}^{t_p} ((h_2 - h_2^s)^2 + (q_1 - q_1^s)^2 + (q_2 - q_2^s)^2)\ dt \quad (12)$$

subject to the system dynamics (10) and (11) with the output measurement $y = h_2$, where h_2^s, q_1^s and q_2^s are the steady states of the value, t_0 is the initial time and t_p is the fixed terminal time.

It is noticed that Problem (P) is a complex problem and solving this kind of problem is computational demanding. However, the optimal solution of Problem (P) could be obtained by solving the simplified problem, which is referred to as Problem (M), given by

$$\min_q J_1 = \tfrac{1}{2}\int_{t_0}^{t_p} ((h_2 - h_2^s)^2 + (q_1 - q_1^s)^2 + (q_2 - q_2^s)^2)\ dt$$

subject to

(13)

$$\frac{dh_1}{dt} = \frac{q_1}{A_1} - \frac{a_1}{2A_1\sqrt{H_1}}h_1 - \frac{a_3}{2A_1\sqrt{H_1 - H_2}}(h_1 - h_2) + \alpha_1$$

$$\frac{dh_2}{dt} = \frac{q_2}{A_2} - \frac{a_2}{2A_2\sqrt{H_2}}h_2 + \frac{a_3}{2A_2\sqrt{H_1 - H_2}}(h_1 - h_2) + \alpha_2$$

$$y = h_2$$

where $\alpha = (\alpha_1 \quad \alpha_2)^{\mathrm{T}}$ is the adjustable parameter. From (13), the state equation can be written by

$$\begin{pmatrix} \dot{h}_1 \\ \dot{h}_2 \end{pmatrix} = \begin{pmatrix} -k_1 - k_3 & k_3 \\ k_4 & -k_2 - k_4 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + \begin{pmatrix} A_1^{-1} & 0 \\ 0 & A_2^{-1} \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} + \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \quad (14)$$

and the output measurement is rewritten as

$$y = (0 \quad 1) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad (15)$$

with

$$k_1 = \frac{a_1}{2A_1\sqrt{H_1}}, \quad k_2 = \frac{a_2}{2A_2\sqrt{H_2}},$$

$$k_3 = \frac{a_3}{2A_1\sqrt{H_1 - H_2}}, \quad k_4 = \frac{a_3}{2A_2\sqrt{H_1 - H_2}}.$$

Here, it is highlighted that adding the adjustable parameter into the state equation in Problem (M) is to measure the differences between the system model and the linear model used repeatedly. By virtue of this, the optimal solution of the model used could be updated, in turn, approximates to the correct optimal solution of Problem (P), in spite of model-reality differences.

III. System Optimization and Parameter Estimation

Now, denote $x = (h_1 \quad h_2)^{\mathrm{T}}$ and $u = (q_1 \quad q_2)^{\mathrm{T}}$ as the state variable and the control input, respectively, and $f : \Re^2 \times \Re^2 \times \Re \to \Re^2$ represents the set of state equation in (10) and (11), which describes the system dynamics of the coupled tanks system with state variable $x(t)$ and control input $u(t)$. The weighting matrices are given as

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The terms of $x^s = (h_1^s \quad h_2^s)^{\mathrm{T}}$ and $u^s = (q_1^s \quad q_2^s)^{\mathrm{T}}$ are the steady states of the state variable and the control input.

Let us introduce an expanded optimal control problem, which is referred to as Problem (E), given by

$$\min_u J_2 = \tfrac{1}{2}\int_{t_0}^{t_p} (x(t)-x^s)^{\mathrm{T}} Q (x(t)-x^s) + (u(t)-u^s)^{\mathrm{T}} R (u(t)-u^s)) + \tfrac{1}{2} r_1 \| u(t) - v(t) \|^2 + \tfrac{1}{2} r_2 \| x(t) - z(t) \|^2 \, dt$$

subject to (16)

$$\dot{x}(t) = Ax(t) + Bu(t) + \alpha(t)$$
$$y(t) = Cx(t)$$
$$Az(t) + Bv(t) + \alpha(t) = f(z(t), v(t), t)$$
$$u(t) = v(t)$$
$$x(t) = z(t)$$

where $v(t) \in \Re^2$ and $z(t) \in \Re^2$ are introduced to separate the control variable and the state variable in the optimization problem from the respective signals in the parameter estimation problem, and $\| \cdot \|$ denotes the usual Euclidean norm. The terms $\tfrac{1}{2} r_1 \| u(t) - v(t) \|^2$ and $\tfrac{1}{2} r_2 \| x(t) - z(t) \|^2$ with $r_1 \in \Re$ and $r_2 \in \Re$ are introduced to improve the convexity and to facilitate the convergence of the resulting iterative algorithm. It is important to note that the algorithm is designed in such a way that the constraints $u(t) = v(t)$ and $x(t) = z(t)$ are satisfied due on the termination of iterations, assuming convergence is achieved. The state constraint $z(t)$ and the control constraint $v(t)$ are used for the computation of the parameter estimation and the matching scheme, while the corresponding state constraint $x(t)$ and control constraint $u(t)$ are reserved for optimizing the linear model-based optimal control problem. Hence, system optimization and parameter estimation are mutually interactive.

A. Necessary Conditions

Define the Hamiltonian function by

$$\begin{aligned} H(t) = {} & \tfrac{1}{2}((x(t)-x^s)^{\mathrm{T}} Q (x(t)-x^s) \\ & + (u(t)-u^s)^{\mathrm{T}} R (u(t)-u^s)) \\ & + \tfrac{1}{2} r_1 \| u(t) - v(t) \|^2 + \tfrac{1}{2} r_2 \| x(t) - z(t) \|^2 \\ & - \lambda(t)^{\mathrm{T}} u(t) - \beta(t)^{\mathrm{T}} x(t) \\ & + p(t)^{\mathrm{T}} (Ax(t) + Bu(t) + \alpha(t)) . \end{aligned} \quad (17)$$

where $p(t) \in \Re^2$ is the Lagrange multiplier. Then, the augmented cost function becomes

$$\begin{aligned} J_a = \tfrac{1}{2}\int_{t_0}^{t_p} & H(t) - p(t)^{\mathrm{T}} \dot{x}(t) + \theta(t)^{\mathrm{T}} (y(t) - Cx(t)) \\ & + \mu(t)^{\mathrm{T}} (f(z(t), v(t), t) - Az(t) - Bv(t) - \alpha(t)) \\ & + \lambda(t)^{\mathrm{T}} v(t) + \beta(t)^{\mathrm{T}} z(t)) \; dt \end{aligned} \quad (18)$$

where $p(t)$, $\theta(t)$, $\mu(t)$, $\lambda(t)$ and $\beta(t)$ are the appropriate multipliers to be determined later.

Applying the calculus of variation [13]–[14], the following necessary conditions for optimality are obtained:

(a) Stationarity:

$$0 = \nabla_u H = R(u(t) - u^s) + B^{\mathrm{T}} p(t) + r_1(u(t) - v(t)) - \lambda(t) . \quad (19)$$

(b) Costate equation:

$$-\dot{p}(t) = \nabla_x H = Q(x(t) - x^s) + A^{\mathrm{T}} p(t)$$

$$+r_2(x(t)-z(t))-\beta(t)\,. \tag{20}$$

(c) State equation:

$$\dot{x}(t)=\nabla_p H = Ax(t)+Bu(t)+\alpha(t)\,. \tag{21}$$

(d) Output equation:

$$y(t)=Cx(t)\,. \tag{22}$$

(e) Boundary condition:

$x(0)$ and $p(t_p)$ are given.

(f) Adjustable parameter equation:

$$f(z(t),v(t),t)=Az(t)+Bv(t)+\alpha(t)\,. \tag{23}$$

(g) Multiplier equations:

$$\lambda(t)=-\left(\frac{\partial f}{\partial v}-B\right)^{\mathrm{T}}\hat{p}(t) \tag{24}$$

$$\beta(t)=-\left(\frac{\partial f}{\partial z}-A\right)^{\mathrm{T}}\hat{p}(t) \tag{25}$$

where $\mu(t)=\hat{p}(t)$ and $\theta(t)=0$.

(h) Separable variables:

$$z(t)=x(t)\,,\ v(t)=u(t)\,,\ \hat{p}(t)=p(t)$$

B. Modified Optimal Control Problem

Refer to the necessary conditions (19)–(22), a modified optimal control problem, which is referred to as Problem (MM), is defined by

$$\min_u J_3=\tfrac{1}{2}\int_{t_0}^{t_p}(x(t)-x^s)^{\mathrm{T}}Q(x(t)-x^s)$$
$$+(u(t)-u^s)^{\mathrm{T}}R(u(t)-u^s))$$
$$-\lambda(t)^{\mathrm{T}}u(t)-\beta(t)^{\mathrm{T}}x(t)$$
$$+\tfrac{1}{2}r_1\|u(t)-v(t)\|^2+\tfrac{1}{2}r_2\|x(t)-z(t)\|^2\,dt$$

subject to

$$\dot{x}(t)=Ax(t)+Bu(t)+\alpha(t)$$
$$y(t)=Cx(t) \tag{26}$$

with the specified $\alpha(t)$, $\lambda(t)$, $\beta(t)$, $v(t)$ and $z(t)$, where the boundary conditions $x(0)$ and $p(t_p)$ are given.

C. Optimal Control Law

The optimal control law for Problem (MM), which is the expanded optimal control policy, is a feedback control. This control law is explained in the following theorem.

Theorem 1 (Expanded optimal control policy): Assume that the expanded optimal control policy exists. Then, this optimal control law is the feedback control law for Problem (MM), given by

$$u(t)=-K(t)x(t)+u_{ff}(t)+R_a^{-1}Ru^s \tag{27}$$

where

$$u_{ff}(t)=-R_a^{-1}B^{\mathrm{T}}s(t)+R_a^{-1}\lambda_a(t) \tag{28}$$

$$K(t)=R_a^{-1}B^{\mathrm{T}}S(t) \tag{29}$$

$$\dot{S}(t)=-S(t)A-A^{\mathrm{T}}S(t)-Q_a+S(t)BR_a^{-1}B^{\mathrm{T}}S(t) \tag{30}$$

$$\dot{s}(t)=-(A-BK(t))^{\mathrm{T}}s(t)-K(t)^{\mathrm{T}}\lambda_a(t)+\beta_a(t)$$
$$-S(t)\alpha(t)-S(t)BR_a^{-1}Ru^s+Qx^s \tag{31}$$

with the boundary conditions $S(t_p)=0$ and $s(t_p)=0$, and $R_a=R+r_1I_2$, $Q_a=Q+r_2I_2$, $\lambda_a(t)=\lambda(t)+r_1v(t)$ and $\beta_a(t)=\beta(t)+r_2z(t)$.

Proof: From the necessary condition (19), we obtain

$$u(t)=-R_a^{-1}B^T p(t)+R_a^{-1}\lambda_a(t)+R_a^{-1}Ru^s\,. \tag{32}$$

Applying the sweep method [13]–[15],

$$p(t)=S(t)x(t)+s(t) \tag{33}$$

into (32), after some algebraic manipulations, the feedback control law (27) is obtained, where (28) and (29) are satisfied.

Also, substitute (33) into the costate equation (20) to yield

$$-\dot{p}(t)=Q_ax(t)+A^T(S(t)x(t)+s(t))-\beta_a(t)-Qx^s \tag{34}$$

Differentiating both sides (33) with respect to *t* gives

$$\dot{p}(t)=\dot{S}(t)x(t)+S(t)\dot{x}(t)+\dot{s}(t) \tag{35}$$

Notice that (34) and (35) are equivalent. That is,

$$-\dot{S}(t)x(t)-S(t)\dot{x}(t)-\dot{s}(t)$$
$$=Q_ax(t)+A^T(S(t)x(t)+s(t))-\beta_a(t)-Qx^s \tag{36}$$

Then, consider the state equation (21) and the feedback control (27) in (36). After doing some algebraic manipulations by taking into account (28) and (29), then (30) and (31) are satisfied. This completes the proof.

Now, taking (27) into (21), the state equation becomes

$$\dot{x}(t)=(A-BK(t))x(t)+Bu_{ff}(t)+BR_a^{-1}Ru^s+\alpha(t) \tag{37}$$

with the output measurement

$$y(t)=Cx(t)\,. \tag{38}$$

D. Iteratve Procedure

From the discussion above, the calculation procedure is summarized as an iterative algorithm given below:

Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 2458-9403
Vol. 4 Issue 2, February - 2017

Iterative Algorithm 1

Data $A,\ B,\ C,\ Q,\ R,\ x_0,\ t_0,\ t_p,\ r_1,\ r_2,\ k_v,\ k_z,\ k_p$ and f.

Step 0 Compute a nominal solution. Assuming that $\alpha(t)=0,\ r_1=0,\ r_2=0$, solve Problem (M) to obtain $u(t)^0,\ x(t)^0,\ p(t)^0$, where (29) and (30) are computed for $K(t)$ and $S(t)$, respectively. Set iteration number $i=0$, $v(t)^0=u(t)^0$, $z(t)^0=x(t)^0,\ \hat{p}(t)^0=p(t)^0,\ t\in[t_0,\ t_p]$.

Step 1 Compute the parameter $\alpha(t)^i$ from (23). This is called the parameter estimation step.

Step 2 Compute the multipliers $\lambda(t)^i$ and $\beta(t)^i$ from (24) and (25), respectively.

Step 3 Using $\alpha(t)^i,\ \lambda(t)^i,\ \beta(t)^i,\ v(t)^i$ and $z(t)^i$, solve Problem (MM) using the result that is presented in Theorem 1. This is called the system optimization step.

3.1 Solve (31) backward to obtain $s(t)^i$ and solve (28), either backward or forward to obtain $u_{ff}(t)^i$.

3.2 Use (27) to obtain the new control $u(t)^i$.

3.3 Use (37) to obtain the new state $x(t)^i$.

3.4 Use (33) to obtain the new costate $p(t)^i$.

3.5 Use (38) to obtain the new output $y(t)^i$.

Step 4 Test the convergence and update the optimal solution of Problem (P). In order to provide a mechanism for regulating convergence, a simple relaxation method is employed:

$$\begin{aligned} v(t)^{i+1} &= v(t)^i + k_v(u(t)^i - v(t)^i) \\ z(t)^{i+1} &= z(t)^i + k_z(x(t)^i - z(t)^i) \\ \hat{p}(t)^{i+1} &= \hat{p}(t)^i + k_p(p(t)^i - \hat{p}(t)^i) \end{aligned} \quad (39)$$

where $k_v, k_z, k_p \in (0,\ 1]$ are scalar gains. If $v(t)^{i+1}=v(t)^i$ within a given tolerance, stop; else set $i=i+1$, and repeat the procedure staring with Step 1.

Remarks:

(a) The nominal solution can be the optimal solution that is obtained from the standard linear quadratic regulator (LQR) optimal control problem.

(b) The off-line computation for solving (29) and (30) is done at Step 0 before the iteration begin with assuming $\alpha(t)=0,\ \beta(t)=0$ and $\lambda(t)=0$.

(c) The numerical scheme for solving the ordinary differential equations $S(t)$ and $s(t)$ can be used.

(d) The relaxation method given in (39) establishes a matching scheme for the updating of the iterative solution.

IV. Result and Discussion

For illustration, the numerical simulation on the coupled tanks system is studied here. Table 1 shows the value of each parameter of the coupled tanks system [1]. The steady state values are set at $x^s=(0.0\quad 0.6037)^{\mathrm{T}}$ and $u^s=(1.0\quad 0.7)^{\mathrm{T}}$. The initial state is $x(0)=(0.0\quad 0.0)^{\mathrm{T}}$, and the time interval is $t\in[0.0,10.0]$. The algorithm proposed is implemented in MATLAB 12 in order to obtain the results.

TABLE I. Parameters of Coupled Tanks System

Parameter	Value	Unit
H_1	17.00	cm
H_2	15.00	cm
a_1	10.78	cm$^{3/2}$/sec
a_2 , a_3	11.03	cm$^{3/2}$/sec
A_1 , A_2	32.00	cm^2

The simulation results show that the algorithm proposed spent the elapsed time 0.163674 seconds to obtain the converged solution with two number of iterations. The value of the final cost function is 10.59 units. Moreover, the graphical solutions for the final control input, the final state variable and the final output measurement are, respectively, shown in Fig. 2(a), 2(b) and 2(c). The trajectory of the control input diverges at the beginning and then turn towards to the steady state value at $u^s=(1.0\quad 0.7)^{\mathrm{T}}$ after 0.5 second. It stays at the steady state after 2 seconds. With this control input, the water level at Tank 2 is increasing linearly from the empty situation to stay at the normal operating height at $y=0.6037$ cm after 4.5 seconds.

From these simulation results, it is noticed that the water level for both tanks should be non-negativity. By virtue of this, the constraint of $x\geq 0$ can be added for this purpose. However, the algorithm proposed shows the efficiency in controlling the water level at the normal operating height in Tank 2 is achieved. Hence, the applicable of the algorithm proposed is certainly demonstrated.

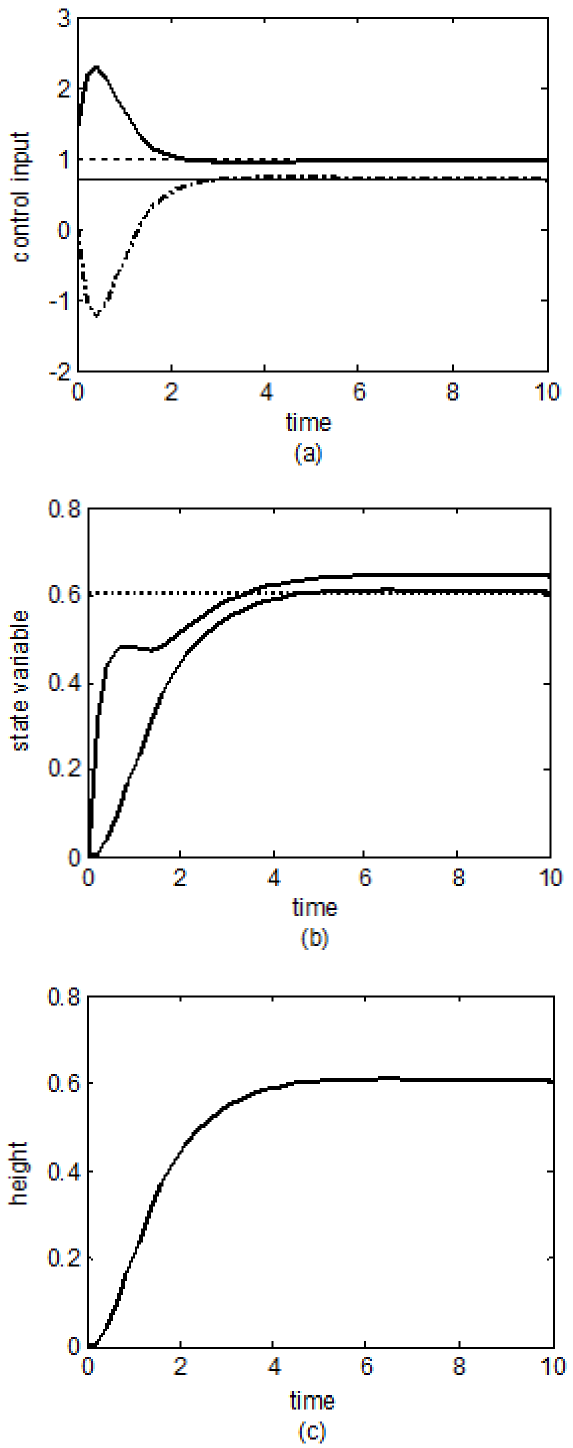

Fig. 2. (a) *Final control trajectory and the steady values*, (b) *Final state trajectory and the steady value and* (c) *Final output trajectory*

V. Concluding Remarks

In this paper, the optimal control policy for the coupled tanks system with three valves was discussed. The mathematical formulation of the coupled tanks system was made according to the mass balance equation. The flow rate for both inflows was used in controlling the water level of the second tank. This problem was defined as the nonlinear optimal control problem. In our approach, the simplified linear model of the original optimal control problem, which is added with the adjusted parameter, was solved repeatedly. Because of the different structure, the adjusted parameter could capture the differences between the real plant and the model used, in turn, updating the optimal solution of the model used. After the convergence was achieved, the iterative solution approximated to the correct optimal solution of the original optimal control problem, in spite of model-reality differences. For illustration, the parameters of the coupled tanks system were considered in which the simulation result was obtained. From the result obtained, the water level of the second tank was able to be controlled at the desired steady state value by using the algorithm proposed. Hence, the efficiency of the algorithm was shown. In conclusion, it is emphasized that the usefulness of the algorithm proposed in controlling the water level in the coupled tanks system is highly presented.

Acknowledgment

The authors would like to thank the Universiti Tun Hussein Onn Malaysia (UTHM) for financial supporting to this study under the Fundamental Research Grant Scheme (FRGS) VOT 1561.

References

[1] H.I. Jaafar, S.Y.S. Hussein, N.A. Selamat, M.N.M. Nasir and M.H. Jali, Analysis of Transient Response for Coupled Tank System via Conventional and Particle Swarm Optimization (PSO) Techniques, *International Journal of Engineering and Technology*, vol. 6, no. 5, pp. 2002-2007, 2014.

[2] S.A. Jagnade, R.A. Pandit and A.R. Badge, Modeling, Simulation and Control of Flow Tank System, *International Journal of Science and Research* (IJSR), vol. 4, Iss. 2, pp. 657–669, 2015.

[3] A. Sharma, O.P. Verma and M. Singh, Mathematical Modeling and Intelligent Control of Two Coupled Tank System, *Imperial Journal of Interdisciplinary Research* (IJIR), vol. 2, iss. 10, pp. 1589–1593, 2016.

[4] K. Ashokkumar, A Nonlinear Controller for Liquid Level Control System, *International Journal of Scientific & Engineering Research* (IJSER), vol. 4, iss. 6, pp. 987–992, 2013.

[5] N. Tompkins, "Synchronization Dynamics of Coupled Chemical Oscillators", PhD Thesis, Brandeis University, 2015.

[6] R.M, Soumya, S. Bidyadhar, G. Subhojit, PI Controller Design for a Coupled Tank System Using LMI Approach: An Experimental Study, *Journal of Chemical Engineering & Process Technology*, pp. 1–8, 2016.

[7] M. Saad, A. Albagul and Y. Abueejela, Performance Comparison between PI and MRAC for Coupled-Tank System, *Journal of Automation and Control Engineering*, vol. 2, no. 3, 316-321, 2014.

[8] N.B. Almutairi and M. Zribi, Sliding Mode Control of Coupled Tanks, *Mechatronics*, vol. 16, iss. 7, pp. 427–441, 2006.

[9] T.G. Hicks, "Handbook of Mechanical Engineering Calculations", 2nd Ed, McGraw-Hill Education: New York, 2006.

[10] J.A. John and R.M. Francis, Modelling and Nonlinear Control Design for Coupled Twin Tank Level Process, *International Journal of Science and Research* (IJSR), vol. 4, iss. 8, pp. 618–622, 2015.

[11] J. Seung, A.F. Atiya, A.G. Parlos and K. Chong, Parameter Estimation for Coupled Tank Using Estimate Filtering, *International Journal of Control and Automation*, vol. 6, no. 5, pp. 91–102, 2013.

[12] A. Srinivasan and P. Nivethitha, Pneumatic Control Valve Stiction Detection and Quantification, *International Journal of Advance Engineering and Research Development*, vol. 2, iss. 4, pp. 121–127, 2015.

[13] F.L. Lewis, "Applied Optimal Control and Estimation: Digital Design and Implementation," Prentice Hall, Inc, 1992.

[14] A.E. Bryson and Y.C. Ho, "Applied Optimal Control," Hemisphere Publishing Company, New York, 1975.

[15] S.L. Kek, K.L. Teo and M.I.A. Aziz, An Integrated Optimal Control Algorithm for Discrete–Time Nonlinear Stochastic System, *International Journal of Control*, vol. 83, pp. 2536–2545, 2010.

[16] P.D. Roberts and T.W.C. Williams, On an Algorithm for Combined System Optimization and Parameter Estimation, *Automatica*, vol. 17, pp. 199–209, 1981.

[17] P.D. Roberts and V.M. Becerra, Optimal Control of a Class of Discrete-Continuous Nonlinear Systems Decomposition and Hierarchical structure, *Automatica*, vol. 37, pp. 1757–1769, 2001.

[18] V.M. Becerra and P.D. Roberts, Dynamic Integrated System Optimization and Parameter Estimation for Discrete Time Optimal Control of Nonlinear Systems, *International Journal of Control*, Vol. 63, pp. 257–281, 1996.