Chaotic behaviour in a Resource-Economy-Pollution dynamic system

Xinghua Fan, Huihui Xu, Jiuli Yin*
Faculty of Science
Jiangsu University, Jiangsu 212013
Zhenjiang,China
*Corresponding author. Yjl@ujs.edu.cn

Cheng Ning
Xieqiao High School
Xieqiao Town, Jingjiang
Jiangsu 214513, China

***Abstract*—It is a new attempt to study the relationship between economic growth and environmental quality from nonlinear dynamical system perspective. A Resource-Economy-Pollution dynamical system (REP) is proposed by considering the close relationship between resource, pollution and economic growth. Pollutants in air,water and land are integrated to make the environmental variable. Different relationships between environment and economy are resulted from the REP system. On one hand, chaotic relationship is obtained by dynamical analysis of the system. This relationship indicts that a small change in economy growth may cause catastrophic environmental damage. On the other hand, fold relationship is discovered by identifying system parameters using empirical data. This relationship is relative stable in that an economy status corresponds to two or three different environmental quality levels.**

Keywords—Resource-Economy-Pollution system; chaotic attractor; dynamic system; economic growth; environmental quality; environmental Kuznets curve

I. INTRODUCTION

The most famous theory about the relationship between economic growth and environmental quality is the Environmental Kuznets Curve (EKC)[1-4]. The EKC refers to an inverted U-shaped relationship between economic output per capita and some measures of environmental quality. It argues that environmental pollution increases with income at low levels of income and decreases with income at high levels of income [3,4].The EKC was initially observed for some elements of air pollution (suspended particles and NO_x). There is some evidence of an EKC relationship existing for certain countries and for a larger set of environmental variables including urban air quality (SO2 , NO_x, CO, smoke and particulates) and river quality [5-9].

EKC hypothesis has been facing some challenges. It has been shown that countries with similar levels of wealth perform differently, without any clear or systematic signs of convergence [10]. Furthermore, the environmental Kuznets curve hypothesis was rejected by a few empirical studies [11,12].

We apply the theory of nonlinear dynamical system to describe the relationship between economic growth and environmental quality. Dynamical method has been widely used in economics, biology and engineering field [13-17]. The relationship between economic growth and the environment is complex. Natural resources are, vital for securing economic growth and development, not just today but for future generations. Solely focussing on economic growth to deliver environmental outcomes could be counter-productive. We integrate natural resources in the relation between the environment and economy then set up a Resource-Economy-Pollution (REP) dynamical system. Pollutants in air,water and land are composite to make the environmental variable. By using dynamic system method, different relationships, such as chaotic and stable relationships between environment and economy can be shown.

The remainder of this paper is organized as follows. Section II sets up the dynamical system of REP. Section III focuses on dynamical analysis of the REP system. Chaotic relationship between environment and economy is discussed. Section IV shows the stable relationships between environment and economy by empirical study. Conclusions are presented in Section V.

II. RESOURCE-ECONOMY-POLLUTION DYNAMICAL SYSTEM

Let x(t) represent the total resource consumed in a region during a given period, y(t) represent the economy scale and z(t) represent the pollution variable. We first make the composite pollution variable then set up the Resource-Economy-Pollution dynamical system.

A. Composite pollution variable

The composite pollution variable is consolidated from three pollutants, namely, waste-water, exhaust and solid waste We let the composite pollution variable be the linear combination of the three standardized emissions. Analytic Hierarchy Process (AHP) is applied to find the combination coefficients [18]. AHP is a theory of measurement through pairwise comparisons and relies on the judgements of experts to derive priority scales. We construct the pairwise comparison matrix in this study as the following

$$A = (a_{ij}) = \begin{pmatrix} 1 & 3/2 & 5 \\ 3/2 & 1 & 6 \\ 1/5 & 1/6 & 1 \end{pmatrix}. \quad (1)$$

The element a_{ij} indicates element i is a_{ij} times more important or dominant over element j with respect to the overall environmental quality. Followed by the process of AHP[19], we obtain the composite pollution variable as

$$z = 0.5788z_1 + 0.8059z_2 + 0.1247z_3 \quad (2)$$

where z_1, z_2 and z_3 represent respectively the amount of waste water emissions, waste gas emissions and industrial solid emissions.

B. *REP dynamic system*

Quantifying the relationship between resources, the environment and economy we can get the Resource-Economy-Pollution dynamical system. The basic idea is that the environment, economic and social issues are interlinked. Fig. 1 shows the relationship among the three variables.

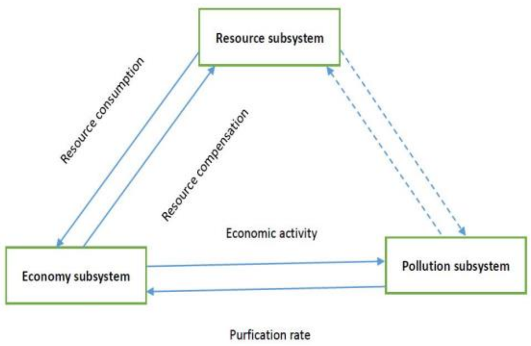

Fig.1 Relation between resource, economy and pollution

In Fig. 1, the dot arrow means an indirect influence. Economic growth requires natural resources and thus gives rise to environmental pollution. Each viable evolves in its own way and we call it a subsystem. Resource subsystem provides natural resource inputs and environmental services for economic production. Growth is generated by industries where the output is a clean and healthy natural environment. Environmental degradation causes economic recession.

We propose the following Resource-Economy-Pollution dynamic system:

$$\begin{cases} \frac{dx}{dt} = a_1 x + a_2 y - a_3 yz, \\ \frac{dy}{dt} = b_1 x(1 - \frac{x}{M}) - b_2 y - b_3 z, \\ \frac{dz}{dt} = c_1 xy - c_2 z, \end{cases} \quad (3)$$

where a_i,b_i, c_i,$(i = 1,\cdots,3)$ are positive system parameters and M represents the maximum value of resource consumption.

Now we give some explanation to Eq. (3). First, the more resource consumed, the faster the rate. Because economic development contributes to resource consumption, the rate of change of resource consumption is positively proportional to the economy scale. Thus $dx(t)/dt$ has the second term $a_2 y(t)$ at the right hand side. When part of waste can be recycled, then resources are saved and the rate of consumption is slow down. Thus we have the third term $-a_3 yz$ in the first equation of system (3).

The second formula in Eq.(3) expresses the complicated relationship between the rate of change of economy scale $dy(t)/dt$, resource consumption and pollution. The rate of change of economy scale $dy(t)/dt$ is associated with resource consumed $x(t)$ and the potential share of resource $(1 - x/M)$ simultaneously, in a positive proportion to their product. When there are plenty of resource, i.e., $x < M$ and $1 - x/M > 0$, the economy develops fast. However, the economy slows down when there are not enough resource i.e.,$x > M$ and $1 - x/M < 0$. The accession of economy counteracts its rate of change. Thus $dy(t)/dt$ is inversely proportional to e $y(t)$. The more serious pollution is, the slower economic development. Thus $dy(t)/dt$ is inversely proportional to pollution $z(t)$.

The third formula in Eq.(3) shows the rate of change of pollution. Resource consumption and economic scale promote pollution. Then we have the term $c_1 xy$. The environment can slow down the pollution by its functions such as pollution filtering,waste sink and waste decomposition. Thus we get $-c_2 z$.

III. CHAOTIC RELATIONSHIP BETWEEN THE ENVIRONMENT AND ECONOMY

A. *Equilibrium analysis*

We analyze the dynamics of the system governed by Eq.(3). The Jacobian matrix of the Eq.(3) at an equilibrium $S(x, y, z)$ is:

$$J = \begin{pmatrix} a_1 & a_2 - a_3 z & -a_3 y (4) \\ b_1 - \frac{2b_1 x}{M} & -b_2 & -b_3 (5) \\ c_1 y & c_1 x & -c_2 (6) \end{pmatrix}. \quad (4)$$

The characteristic polynomial of J at the equilibrium $S(x, y, z)$ is:

$$f(\lambda) = |\lambda I - J| = \lambda^3 + p\lambda^2 + q\lambda + r, \quad (5)$$

where $p = a_2 + c_2 - b_1, q = a_3 c_1 x + b_3 c_1 y^2 + a_1 b_3 z + a_2 c_2 - c_2 b_1 - a_1 b_2 - a_2 b_1 + \frac{2a_1 x(b_2 - b_3 z)}{M}$, $r = (b_2 - b_3 z)(c_1 a_3 y - a_1 c_2) + c_1 b_3 y(a_1 x - a_2 y) - b_1(a_2 c_2 - c_1 a_3 x) + \frac{2a_1 x(b_2 c_2 - c_2 b_3 z - c_1 b_3 xy)}{M}$, I is third-order unit matrix. By the Routh-Hurwitz criterion, all real eigenvalues and all real parts of complex conjugate eigenvalues of Eq.(5) are negative if and only if the following conditions hold: $p > 0, r > 0, pq - r > 0$. For certain parameters, Eq.(5) has unstable saddle-focus points.

We determine the type of the equilibrium points for the following parameter conditions:

$$a_1 = 0.065; a_2 = 0.035; a_3 = 0.065;$$
$$b_1 = 0.5; b_2 = 0.088; b_3 = 0.06; M = 10; \quad (6)$$
$$c_1 = 0.468; c_2 = 0.06$$

The values of the coordinates of the equilibrium points and their corresponding eigenvalues are list in Tab. 1. There are stable and unstable equilibrium points, especially unstable saddle-focus points. Thus the system has different dynamical behaviors.

The divergence satisfies

$$\nabla V = b_1 - a_2 - c_2. \quad (7)$$

If $b_1 - a_2 - c_2 < 0$, then the dynamic system presented in Eq.(3) is a dissipative system. Thus there may be chaotic phenomenon.

B. *Numerical simulation results*

We study the chaotic phenomenon from Lyapunov exponents and bifurcation analysis. c_1 is taken as the bifurcation parameter.

Vary c_1 in the interval [0.2,0.9], and fix parameters as in Eq.(8) :

$$a_1 = 0.065; a_2 = 0.035; a_3 = 0.065; b_1 = 0.5;$$
$$b_2 = 0.088; b_3 = 0.06; M = 10; c_2 = 0.06 \qquad (8)$$

TABLE I. EQUILIBRIUM POINTS AND THEIR CORRESPONDING EIGENVALUES

Equilibrium point	λ_1	λ_2, λ_3	*Type*
$S_1(0,0,0)$	-0.06	-0.1643, 0.1413	unstable saddle
$S_2(0.2129, 0.5552, 0.9219)$	-0.0906	$0.0038 \pm 0.1537i$	unstable saddle-focus
$S_3(6.5001, 0.3634, 18.425$	0.3232	-0.2031 ± 0.312	unstable saddle-focus
$S_4(13.3741, -0.3555, -37.0836)$	-0.0806	$-0.0012 \pm 1.5565i$	asymptotic stable
$S_5(-0.0871, -0.9303, 0.632$	-0.1299	$0.0234 \pm 0.1679i$	unstable saddle-focus

The Lyapunov exponent spectrum is shown in Fig.2 with the initial condition [0.196,0.36,0.88]. The corresponding bifurcation diagram with respect to changing parameter c_1 is shown in Fig.3 with the initial condition [0.82,0.29,0.48].

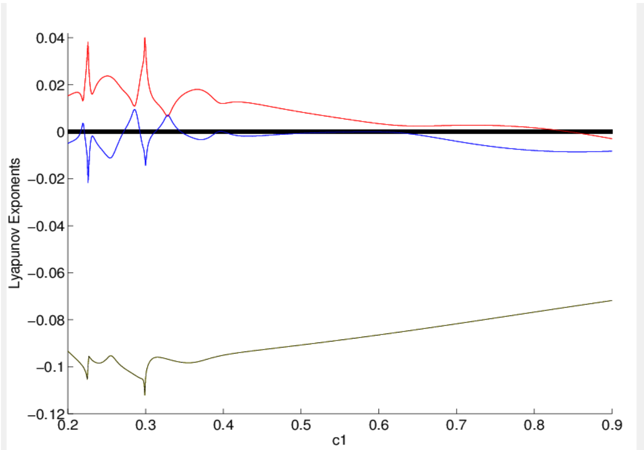

Fig. 2 Lyapunov exponents spectrum

According to Figs.2 and 3, the maximum Lyapunov exponent is positive for a large range of parameters, there exist period-doubling windows. Thus there is chaos for some parameter $\boldsymbol{c_1}$.

A chaotic attractor is observed for parameters as in Eq.(6). Fig.4 shows the three dimensional chaotic attractors. The attractor is different from the previous chaotic attractor,

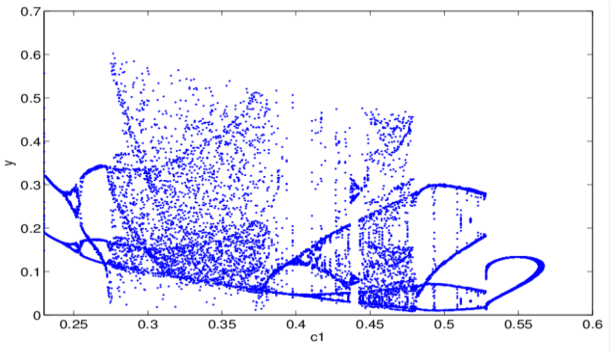

Fig. 3 Bifurcation diagram of $\boldsymbol{y}$

Lü attractor [20], Energy resources attractor [21] and Energy-saving and emission-reduction attractor [15]. Fig. 5 shows the chaotic relationship between the pollution and the economy. It is different from the EKC in that it not a simple curve.

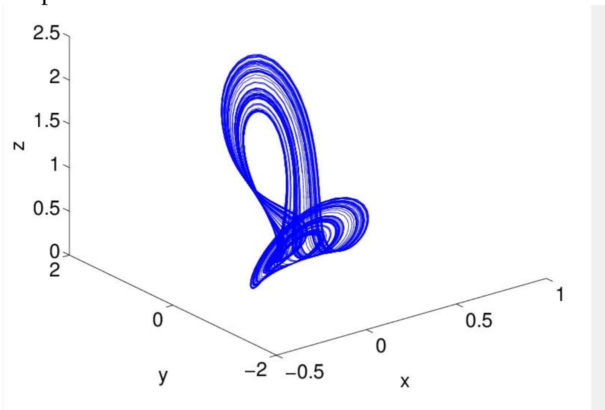

Fig 4. A chaotic attractor of PRE

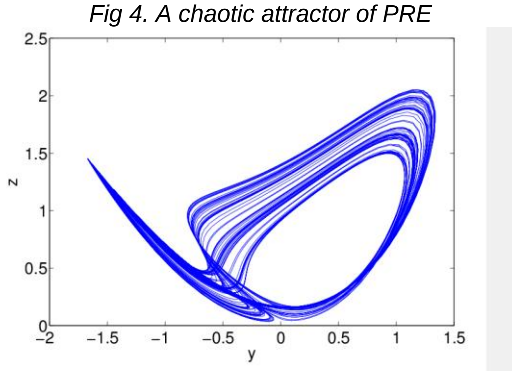

Fig. 5 Chaotic relationship

IV. FOLD RELATIONSHIP

We carried out an empirical study to show that there exists stable relationship between the environment and economy. The first step is to identify system parameters by using statistical data. The artificial neural network (ANN) method is applied.

A. Data

Empirical data are collected from China Statistical Yearbook (2004~2013). We select data of coal, oil, natural gas and water resource as resource data, Gross domestic product (GDP) as economic data and the amount of waste water emissions, waste gas emissions and industrial solid waste emissions as environmental data indicators. Because waste water and exhaust are difficult to be stored, emissions

Year	coal	oil	natural gas	total water	GDP	sewage	waste gas	solid waste
2004	161657.26	45825.92	5296.46	24129.56	142763.2	4824094	237696	1762
2005	189231.16	46523.68	6272.86	28053.1	158963.9	5245089	268988	1654.7
2006	207402.11	50131.73	7734.61	25330.14	179129.94	5144802	330990	1302.1
2007	225795.45	52945.14	9343.26	25255.16	204565.42	5568494.16	388169	1196.7
2008	229236.87	53542.04	10900.77	27434.3	224246.18	5716801	403866	781.8
2009	240666.22	55124.66	11764.41	24180.2	244958.22	5890877.25	436064	710.5
2010	249568.42	62752.75	14425.92	30906.41	270995.45	6172562	519168	498.2
2011	271704.19	65023.22	17803.98	23256.7	296705.3	6591922.44	674509	433.3
2012	275464.53	68363.46	19302.62	29526.88	319692.69	6847612.14	635519	144.2
2013	280999.36	71292.12	22096.39	27957.86	344262.17	6954432.7	669361	129.3

are adopted. Since tremendous amount of solid waste is stored instead of being discharged, production is employed. Considering China is a big developing economy, we use industrial waste water emissions, industrial exhaust emissions and industrial solid waste production as indictors that influence the comprehensive evaluation variable of environmental quality. They are measured at different units of spatial and temporal aggregation. All data are normalized as $\overline{x_i} = (x_i - x_{min})(x_{max} - x_{min})$, where x_{max} and x_{min} are the maximum and minimum of x_i.

B. Parameter identification

Eq.(3) is discredited to the following difference equation(see [22]):

$$\begin{cases} x_{k+1} = x_k + T(a_1 x_k + a_2 y_k - a_3 y_k z_k), \\ y_{k+1} = y_k + T[b_1 x_k(1 - \frac{x_k}{M}) - b_2 y_k - b_3 z_k], \\ z_{k+1} = z_k + T(c_1 x_k y_k - c_2 z_k). \end{cases} \quad (9)$$

Parameters are identified with the help of ANN method. Fig.6 shows the structure of an ANN with three layers (input layer, hidden layer and output layer). In this study, a 3-4-3 (3-4-4,3-4-2) structure is used for the first (second, third) sub-equation of Eq. 9.

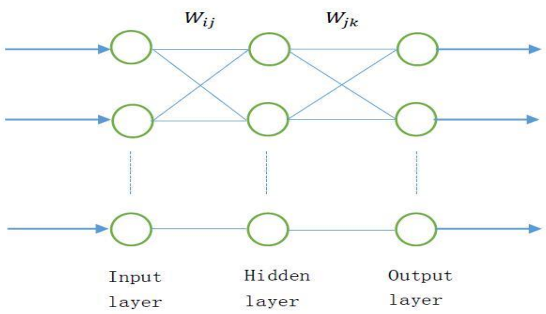

Fig. 6 The ANN structure of system (9)

The BP algorithm [23,24] of artificial neural network is an effective method to identify the coefficients of a new system, which has the virtue of tiny error. The ANN toolboxes of MATLAB (2012a) are used to run the models. Select the beginning $n-1$ sets of data in Tab. 2 as the input, and the latter $n-1$ sets as the output. Select proper multilayered feed-forward neural network, and let all the adjustable parameters be random numbers. Substitute the difference Eq. (9) with the output results. Compare the data with output target and the error e is obtained. Stop the debugging when e reaches 10^{-3}, the parameters of actual system are obtained.

We set the learning rate as 0.01, minimum error as 10^{-3}, simulation time as 3×10^4. The activation functions used in the hidden layer and out layer are taken as hyperbolic tangent sigmoid function and logarithmic sigmoid function respectively. The weight factors W_{jk} and W_{ij} are selected as random number.

TABLE II. STATISTIC DATA OF YEAR 2004-2013.

Note: Units for coal, oil and natural gas are 10^4 standard coal, total water, waste gas and solid waste are 100 million cubic meters, sewage is 10^4 tons and GDP is 10^8 yuan.

Through the above procedure, we obtain the identified parameters as shown in Tab. 3.

TABLE III IDENTIFIED PARAMETERS

a_1	a_2	a_3	b_1	b_2	b_3	M	c_1	c_2
0.005	0.024	0.0257	0.0019	0.0143	0.0113	0.0117	0.0076	0.0314

Fixing parameters as in Tab.3, we obtain the phase diagram of the actual system as shown in Fig. 7.

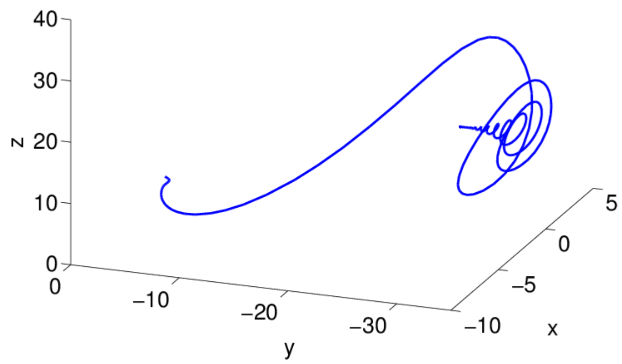

Fig. 7 Stable revolution of REP

It shows that a stable evolution trend. Fig. 8. shows a stable relation between the environment and economy. We call it fold relationship because the graph looks like a parabola and can be folded at its vertex. Corresponding to an economy status, there are two or three different environmental quality levels. In policy term, counties in the same developing level may have different environment quality. This coincides with the observation that the EKC relationship cannot be generalized to across all counties and income levels.

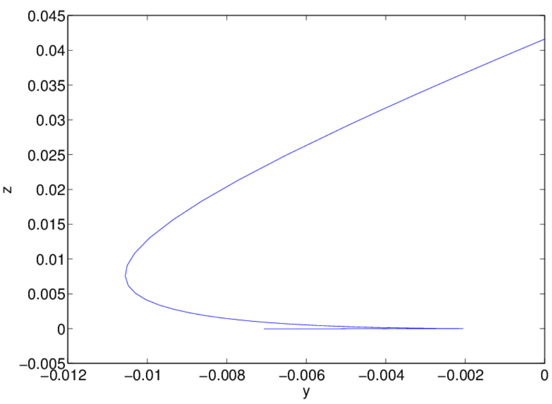

Fig. 8 Fold relationship

V. Conclusions

A new three-dimension non-linear dynamics model for the REP system was established. Chaotic dynamics characteristics of the system was researched by using Lyapunov exponents and bifurcation diagrams, which all proved the system as a chaotic system and also indicated parameters of the system have sensitivity. This system displayed a special chaotic attractor named REP attractor. Coefficients of the actual system were obtained based on the BP algorithm of artificial neural network. The result of empirical analysis displayed that the actual system was stable.

We presented two different possible relationships between the environment and economy: chaotic and fold one. The chaotic relationship agreed with the opinion of Arrow, et al. [25], suggesting that the risk of small changes causing catastrophic damage.

This system could be used as a predictor of environmental performance as countries develop. The definition of environmental quality used in this study was based on a combination of pollutants in air, water and land. As such, the conclusions reached by this analysis could be applicable to more types of environmental damage.

Acknowledgment

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