Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 2458-9403
Vol. 4 Issue 2, February - 2017

Analysis Of Epidemic Model By Differential Transform Method*

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Abstract— **The aim of this paper is to apply the differential transformation method (DTM) to solve epidemic model SAEIQRS (Susceptible-Antidotal-Exposed-Infected-Quarantined-Recovered-Susceptible) for a given constant population. This mathematical model is described by nonlinear first order ordinary differential equations. First, we find the solution of this model by using the differential transformation method (DTM). In order to show the efficiency of the method, we compute the solution by using fourth-order Runge-Kutta method (RK4) and then compare the solutions obtained by DTM and RK4. We illustrated the profiles of the solutions, from which we speculate that the DTM and RK4 solutions agreed very well.**

Keywords— Differential Equation, Differential transform method; Epidemic model; Runge-Kutta method

I. INTRODUCTION

Mathematical modeling is commonly used in the application of biological infectious diseases. These models describe the pattern and controlling approach of the infection of disease properly and the behavior and relationship between different sub-populations of a certain region. Ordinary differential initial value problems are frequently arises in these modeling. Simple formation of mathematical model is not enough for disease control. To detect and cure these diseases properly, we need an effective method to solve these models [1]. For the solution of the systems of linear and nonlinear differential equations, there are many methods like exact, approximate and purely numerical are available. Most of these methods are computationally intensive or need complicated symbolic computations [2]. Generally, the exact solutions of these models are unavailable and usually are very complex.

The differential transform method (DTM) is a computational method that can be used to solve linear (or nonlinear) ordinary (or partial) differential equations with their corresponding initial conditions. Pukhov [3], proposed the concept of differential transform, where the image of a transformed function is computed by differential operations. This method becomes a numerical-analytical technique that formalizes the Taylor series in a different manner. A pioneer in using this method to solve initial value problems is Zhou [4], who introduced it in a study of electrical circuits. Additionally, this method has been used to solve differential algebraic equation [5], Schrödinger equations [6], fractional differential equation [7], Lane-Emden type equation, free vibration analysis of rotating beams, unsteady rolling motion of sphere equation in inclined tubes. The main advantage of this method is that it can be applied directly to linear and nonlinear ODEs without requiring linearization, discretization or perturbation. Another important advantage is that, this method is capable of reducing the size of computational work, and still accurately provides the series solution with fast convergence rate [8].

The purpose of this paper is to employ the differential transformation method (DTM) to systems of differential equations, which describes the SAEIQRS epidemic model and approximating the solutions in a sequence of time intervals. In order to illustrate the accuracy of the DTM, the obtained results are compared with the fourth-order Runge-Kutta method.

The organization of this paper is as follows: In Section 2, the formations of SAEIQRS model are presented. Some basic definitions and the operation properties of differential transformation method are introduced in Section 3. Section 4 is devoted to present the numerical results of the application of the method to SAEIQRS models. Comparisons between the differential transform method (DTM) and the fourth-order Runge-Kutta (RK4) solutions are presented in section 5. Finally, Section 6 summarizes the work.

II. FORMULATION OF SAEIQRS MODEL

Compartmental mathematical model SAEIQRS (Susceptible-Antivirus-Exposed-Infected-Quarantine-Recovered-Susceptiblie) has been developed for understanding the transmission of computer virus. In this model a total number of populations n(t) at a time t, are divided into the following six compartments:

s(t): The number of susceptible computers at a time t, which are uninfected, and having no immunity.
a(t): The number of antidotal computers at a time t that may be recent or old updated.

e(t): The number of exposed computers at a time t that are susceptible to infection.
i(t): The number of infected computers at a time t that have to be cured.
q(t): The number of infected computers at a time t that are quarantined.
r(t): Uninfected computers at a time t having temporary immunity.

To characterized the model, we consider, B is the birth rate (new computers attached to the network), μ is the natural death rate (crashing of the computers due to other reason other than the attack of virus), k_1 is the crashing rate of computer due to the attack of virus, β is the rate of transmission of virus attack when susceptible computers contact with infected ones (s to e), α is the rate at which the susceptible computers begin the antidotal process (s to a), point to be noted that $\alpha = 0$ bears the meaning of no vaccination, ϕ_1 is the rate of virus attack when antidotal computers contact infected computers before obtaining recent update (a to e), ϕ_2 is the rate of recovery by antidotal computers (a to r), γ is the rate coefficient of exposed class (e to i), σ_1 and σ_2 are the rate of coefficients of infectious class (i to r) and (i to q), δ is the rate coefficient of quarantine class (q to r), η is the rate coefficient of recovery class (r to s).

The system of nonlinear ordinary differential equations representing this model is given as follows:

$$\begin{aligned}
\frac{ds(t)}{dt} &= B - \mu s(t) - \beta s(t)i(t) - \alpha s(t) + \eta r(t)\\
\frac{da(t)}{dt} &= \alpha s(t) - \mu a(t) - \phi_2 a(t) - \phi_1 a(t)i(t)\\
\frac{de(t)}{dt} &= \beta s(t)i(t) - \mu e(t) - \gamma e(t) + \phi_1 a(t)i(t)\\
\frac{di(t)}{dt} &= \gamma e(t) - (\mu + k_1)i(t) - \sigma_1 i(t) - \sigma_2 i(t)\\
\frac{dq(t)}{dt} &= \sigma_2 i(t) - (\mu + k_1)q(t) - \delta q(t)\\
\frac{dr(t)}{dt} &= \sigma_1 i(t) + \delta q(t) - \mu r(t) + \phi_2 a(t) - \eta r(t)
\end{aligned} \tag{2.1}$$

With the initial conditions

$$\begin{aligned}
&s(0) = s_0,\quad a(0) = a_0,\quad e(0) = e_0,\\
&i(0) = i_0,\quad q(0) = q_0,\quad r(0) = r_0
\end{aligned} \tag{2.2}$$

Summing the equations of system (2.1) we obtain,

$$\frac{d}{dt}[s(t)+a(t)+e(t)+i(t)+q(t)+r(t)] =$$
$$B - \mu[s(t)+a(t)+e(t)+i(t)+q(t)+r(t)] - k_1[i(t)+q(t)]$$

Therefore the total population may vary with time t.

III. Basic definitions and the operation properties of differential transformation method (DTM)

In this section, we discussed about the basic definitions and operation properties of differential transform method. To understand the method properly, we repeat the definitions and operation properties from [1,2,5,9,10,11]. The method consists of a given system of differential equations and related initial conditions. These are transformed into a system of recurrence equations that finally leads to a system of algebraic equations whose solutions are the coefficients of a power series solution [12].

The differential transformation $F(k)$ of a function $f(x)$ is defined as follows:

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=0} \tag{3.1}$$

In equation (3.1), $f(x)$ is the original function and $F(k)$ is the transformed function, which is called T-function. Differential inverse transform of $F(k)$ is defined as

$$f(x) = \sum_{k=0}^{\infty} x^k F(k) \tag{3.2}$$

From equation (3.1) and (3.2), we obtain

$$f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}\left[\frac{d^k f(x)}{dx^k}\right]_{x=0} \tag{3.3}$$

Equation (3.3) implies that the concept of differential transform is derived from the Taylor series expansion, but the method does not evaluate the derivatives symbolically. However, relative derivatives are calculated by an iterative way which is described by the transformed equations of the original functions.

Using equations (3.1) and (3.2), the following mathematical operations can be obtained:

i. If $f(x) = g(x) \pm h(x)$, then $F(k) = G(k) \pm H(k)$.

ii. If $f(x) = cg(x)$, then $F(k) = cG(k)$, where c is a constant.

iii. If $f(x) = \frac{dg(x)}{dx}$, then $F(k) = (k+1)G(k+1)$.

iv. If $f(x) = \frac{d^m g(x)}{dx^m}$, then $F(k) = (k+1)(k+2)......(k+m)G(k+m)$.

v. If $f(x) = 1$, then $F(k) = \delta(k)$.

vi. If $f(x)=x$, then $F(k)=\delta(k-1)$.

vii. If $f(x)=x^m$, then

$$F(k)=\delta(k-m)=\begin{cases}1, & if \quad k=m\\ 0, & if \quad k\neq m\end{cases},$$

δ is the Kronecker delta.

viii. If $f(x)=g(x)h(x)$, then

$$F(k)=\sum_{m=0}^{k}H(m)G(k-m).$$

ix. If $f(x)=e^{mx}$, then $F(k)=\dfrac{m^k}{k!}$.

x. If $f(x)=(1+x)^m$, then

$$F(k)=\frac{m(m-1)(m-2).......(m-k+1)}{k!}.$$

xi. If $f(x)=\sin(\omega x+\alpha)$, then

$$F(k)=\frac{\omega^k}{k!}\sin(\frac{\pi k}{2}+\alpha).$$

xii. If $f(x)=\cos(\omega x+\alpha)$, then

$$F(k)=\frac{\omega^k}{k!}\cos(\frac{\pi k}{2}+\alpha).$$

xiii. If $F(k)=D[f(x)]$, $G(k)=D[g(x)]$, and c_1, c_2 are independent of x and k, then

$$D[c_1f(x)+c_2g(x)]=c_1F(k)+c_2G(k)$$

(Symbol D denoting the differential transform process).

xiv. If $f(x)=g(x)h(x)$, $g(x)=D^{-1}[G(k)]$, $h(x)=D^{-1}[H(k)]$ and $\otimes$ denote the convolution, then

$$D[f(x)]=D[g(x)h(x)]=G(k)\otimes H(k)$$
$$=\sum_{r=0}^{k}H(r)G(k-r)$$

xv. If $f(x)=f_1(x)f_2(x)...f_{n-1}(x)f_n(x)$, then

$$F(k)=\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}...\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2}F_1(k_1)F_2(k_2-k_1)$$
$$\cdots F_{n-1}(k_{n-1}-k_{n-2})F_n(k-k_{n-1})$$

IV. APPLICATION TO SAEIQRS MODEL

In this section, the differential transformation technique is applied to solve nonlinear differential equation system that arises from SAEIQRS epidemiological model.

Let S(k), A(k), E(k), I(k), Q(k) and R(k) denote the differential transformation of s(t), a(t), e(t), i(t), q(t) and r(t) respectively, then by using the fundamental operations of differential transformation method, discussed in Section 3, we obtained the following recurrence relation to each equation of the system (2.1) :

$$S(k+1)=\frac{1}{k+1}[B\delta(k)+\eta R(k)-(\mu+\alpha)S(k)-\beta\sum_{m=0}^{k}S(m)I(k-m)] \quad (4.1)$$

$$A(k+1)=\frac{1}{k+1}[\alpha S(k)-(\mu+\phi_2)A(k)-\phi_1\sum_{m=0}^{k}A(m)I(k-m)] \quad (4.2)$$

$$E(k+1)=\frac{1}{k+1}[-(\mu+\gamma)E(k)+\beta\sum_{m=0}^{k}S(m)I(k-m)+\phi_1\sum_{m=0}^{k}A(m)I(k-m)] \quad (4.3)$$

$$I(k+1)=\frac{1}{k+1}[\gamma E(k)-(\mu+k_1+\sigma_1+\sigma_2)I(k)] \quad (4.4)$$

$$Q(k+1)=\frac{1}{k+1}[\sigma_2 I(k)-(\mu+k_1+\delta)Q(k)] \quad (4.5)$$

$$R(k+1)=\frac{1}{k+1}[\sigma_1 I(k)+\delta Q(k)+\phi_2 A(k)-(\mu+\eta)R(k)] \quad (4.6)$$

Now, consider the initial conditions S(0)=30, A(0)=5, E(0)=2, I(0)=0, Q(0)=0, R(0)=3 and parameter value $B=0.01,\ \beta=0.09,\ \gamma=0.45,\ \sigma_1=0.35,$ $\sigma_2=0.3,\ \delta=0.65,\ \eta=0.01,\ \mu=0.05,$ $k_1=0.035,\ \alpha=0.65,\ \phi_1=0.2,\ \phi_2=0.3$

Applying the initial conditions and parameter values in (4.1)-(4.6), we get

S(1)= -20.96, S(2)= 6.1276,
S(3)=-0.354410333333334,
S(4)= -0.612774749166667,
S(5)= 0.385621333823333,
S(6)= -0.124970337952831,
S(7)= 0.011038050444019, …

A(1)= 17.75, A(2)= -10.36825,
A(3)= 1.657525833333334,
A(4)= 0.651321997916667,
A(5)= -0.689096313183333,
A(6)= 0.317635394362128,
A(7)= -0.066789015984164,…

E(1)= -1, E(2)= 1.915,
E(3)= -0.505511666666667,
E(4)= -0.118781722916667,
E(5)= 0.276216114060417,
E(6)= -0.178642739337144,
E(7)= 0.053470746172690,…

I(1)= 0.9, I(2)= -0.55575 ,
I(3)= 0.42340875,
I(4)= -0.134671420312500,
I(5)= 0.009106343723437,
I(6)= 0.019600681448410,
I(7)= -0.013542247652328,…

Q(1)= 0, Q(2)= 0.135, Q(3)= -0.08865,
Q(4)= 0.04804509375, Q(5)= -0.015142914,
Q(6)= 0.002310324151172,
Q(7)= 0.000597445169059,…

R(1)= 1.32, R(2)= 2.7804, R(3)= -1.1280205,
R(4)= 0.163877385625,
R(5)=0.033931654013125,
R(6)= -0.035903411165431,
R(7)= 0.015115253169110,…

Then, the closed form of the solution, where k=7, can be written as

$$s(t)=\sum_{k=0}^{\infty}t^k S(k)=30-20.96t+6.1276t^2-$$
$$0.354410333333334t^3-0.612774749166667t^4+$$
$$0.385621333823333t^5-0.124970337952831t^6+$$
$$0.011038050444019t^7+\ldots$$

$$a(t)=\sum_{k=0}^{\infty}t^k A(k)=5+17.75t-10.36825t^2+$$
$$1.657525833333334t^3+0.651321997916667t^4-$$
$$0.689096313183333t^5+0.317635394362128t^6-$$
$$0.066789015984164t^7+\ldots$$

$$e(t)=\sum_{k=0}^{\infty}t^k E(k)=2-t+1.915t^2-0.505511666666667t^3-$$
$$0.118781722916667t^4+0.276216114060417t^5-$$
$$0.178642739337144t^6+0.053470746172690t^7+\ldots$$

$$i(t)=\sum_{k=0}^{\infty}t^k I(k)=0-0.9t-0.55575t^2+0.42340875t^3-$$
$$0.134671420312500t^4+0.009106343723437t^5+$$
$$0.019600681448410t^6-0.013542247652328t^7+\ldots$$

$$q(t)=\sum_{k=0}^{\infty}t^k Q(k)=0-0t+0.135t^2-0.08865t^3+$$
$$0.04804509375t^4-0.015142914t^5+0.002310324151172t^6+$$
$$0.000597445169059t^7+\ldots$$

$$r(t)=\sum_{k=0}^{\infty}t^k R(k)=3+1.32t+2.7804t^2-1.1280205t^3+$$
$$0.163877385625t^4+0.033931654013125t^5-$$
$$0.035903411165431t^6+0.015115253169110t^7+\ldots$$

V. COMPARISON BETWEEN DTM AND RK4

In this section, we compared the numerical results obtained by fourth order Runge-Kutta method (RK4) with the results obtained by differential transformation method (DTM). To obtain the solution by RK4, we coded the RK4 algorithm in a computer using MATLAB and the variables are in long format in all the calculations.

Table 1, 2, 3, 4, 5 and 6 shows the solution for s(t), a(t), e(t), i(t), q(t) and r(t) respectively obtained by differential transformation method (DTM) and fourth order Runge-Kutta (RK4) method and the absolute differences between DTM and RK4.

We depict the solution obtained by differential transform method (DTM) and fourth order Runge-Kutta (RK4) method of s(t), a(t), e(t), i(t) and r(t) in Fig. 1, 2, 3, 4, 5 and 6 respectively.

Table I. THE ABSOLUTE ERROR INVOLVED THE DIFFERENTIAL TRANSFORMATION METHOD ALONG WITH THE RESULT OBTAINED BY THE RUNGE–KUTTA FOURTH-ORDER METHOD FOR s(t).

t	s(t) by DTM (8 iterate)	s(t) by RK4	|DTM-RK4|
0.1	27.964864044538555	27.964863379278441	0.000000665260114
0.2	26.049403819746907	26.049402914228487	0.000000905518419
0.3	24.249799816018207	24.249799604498925	0.000000211519282
0.4	22.561501673783944	22.561507674377030	0.000006000593086
0.5	20.979484526430973	20.979521665091642	0.000037138660669
0.6	19.498432054107489	19.498582893819560	0.000150839712070
0.7	18.112851811595306	18.113335877499377	0.000484065904072
0.8	16.817128393426067	16.818444340021557	0.001315946595490
0.9	15.605519999418515	15.608675143397299	0.003155143978784
1.0	14.472103963814519	14.478957142777162	0.006853178962643

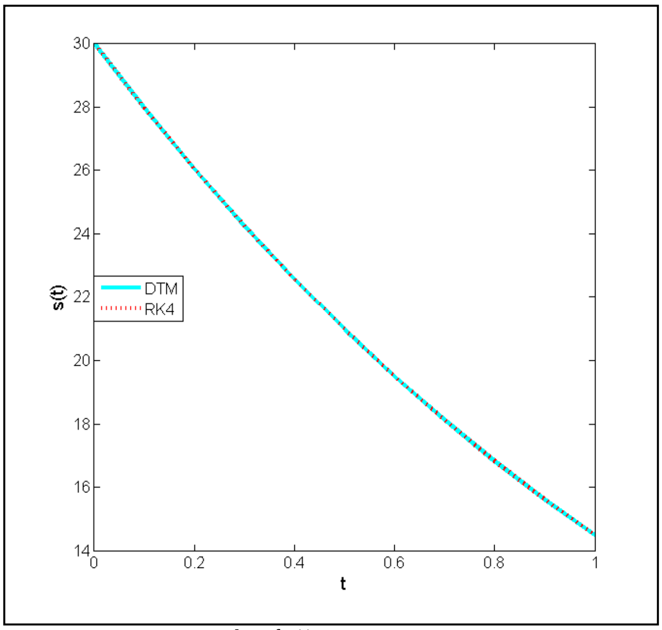

Fig. 1. Plot of s(t) versus time t

Table II. THE ABSOLUTE ERROR INVOLVED THE DIFFERENTIAL TRANSFORMATION METHOD ALONG WITH THE RESULT OBTAINED BY THE RUNGE–KUTTA FOURTH-ORDER METHOD FOR a(t).

t	a(t) by DTM (8 iterate)	a(t) by RK4	|DTM-RK4|
0.1	6.673033578026486	6.673031418838586	0.000002159187900
0.2	8.149371284808950	8.149367193791498	0.000004091017452
0.3	9.440428851086784	9.440422381704202	0.000006469382582
0.4	10.557970757684522	10.557956765102256	0.000013992582266
0.5	11.513742858099011	11.513699030069448	0.000043828029563
0.6	12.319232713578367	12.319093589980174	0.000139123598194
0.7	12.985523979028644	12.985140939701834	0.000383039326810
0.8	13.523211178084161	13.522307487340928	0.000903690743232
0.9	13.942341205677426	13.940485675098957	0.001855530578469
1.0	14.252347896444633	14.248989476202851	0.003358420241781

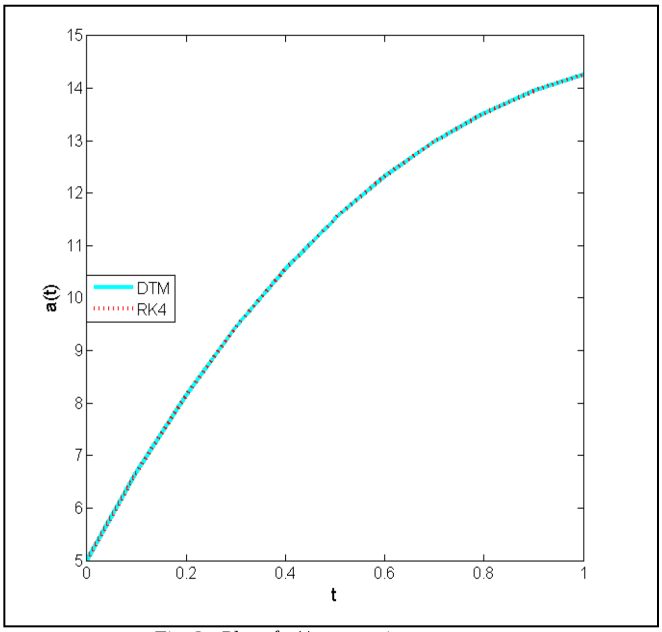

Fig. 2. Plot of a(t) versus time t

Table III. THE ABSOLUTE ERROR INVOLVED THE DIFFERENTIAL TRANSFORMATION METHOD ALONG WITH THE RESULT OBTAINED BY THE RUNGE–KUTTA FOURTH-ORDER METHOD FOR e(t).

t	e(t) by DTM (8 iterate)	e(t) by RK4	|DTM-RK4|
0.1	1.918635199026518	1.918638157727330	0.000002958700812
0.2	1.872443496356733	1.872448637415107	0.000005141058374
0.3	1.858291721696753	1.858298447405271	0.000006725708518
0.4	1.873190780044850	1.873198488339097	0.000007708294247
0.5	1.914395384951094	1.914400670806213	0.000005285855119
0.6	1.979456016772884	1.979439365999276	0.000016650773608
0.7	2.066250055182429	2.066134539387964	0.000115515794465
0.8	2.173019035182276	2.172569950688433	0.000449084493843
0.9	2.298438975884924	2.297056349229353	0.001382626655571
1.0	2.441750731312629	2.438086793421569	0.003663937891060

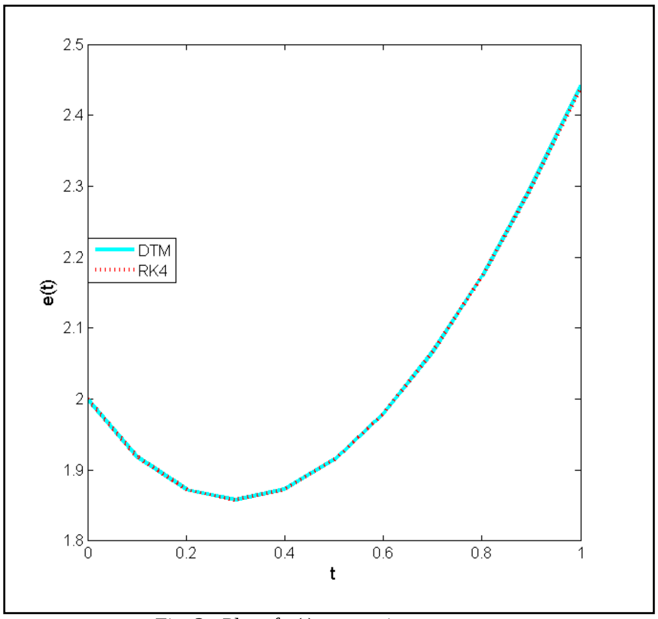

Fig. 3. Plot of e(t) versus time t

Table IV. THE ABSOLUTE ERROR INVOLVED THE DIFFERENTIAL TRANSFORMATION METHOD ALONG WITH THE RESULT OBTAINED BY THE RUNGE–KUTTA FOURTH-ORDER METHOD FOR i(t).

t	i(t) by DTM (8 iterate)	i(t) by RK4	|DTM-RK4|
0.1	0.084852550917863	0.084851851974760	0.000000698943102
0.2	0.160945790860334	0.160944632806613	0.000001158053721
0.3	0.230357153367931	0.230355975626266	0.000001177741665
0.4	0.294881917372387	0.294883036604181	0.000001119231794
0.5	0.356056665059674	0.356070984932862	0.000014319873188
0.6	0.415176376345211	0.415242394608936	0.000066018263725
0.7	0.473298334668457	0.473525454810874	0.000227120142417
0.8	0.531226018814050	0.531879727638228	0.000653708824178
0.9	0.589466155466705	0.591118774648552	0.001652619181848
1.0	0.648152107207019	0.651929388590734	0.003777281383715

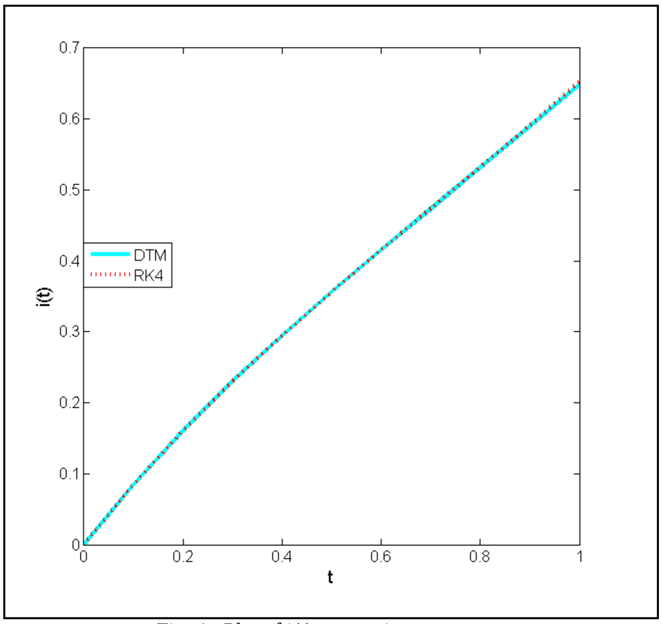

Fig. 4. Plot of i(t) versus time t

Table V. THE ABSOLUTE ERROR INVOLVED THE DIFFERENTIAL TRANSFORMATION METHOD ALONG WITH THE RESULT OBTAINED BY THE RUNGE–KUTTA FOURTH-ORDER METHOD FOR q(t).

t	q(t) by DTM (8 iterate)	q(t) by RK4	|DTM-RK4|
0.1	0.001266005450304	0.001266196619250	0.000000191168946
0.2	0.004762981925564	0.004763305366925	0.000000323441361
0.3	0.010110632865920	0.010111012329850	0.000000379463931
0.4	0.017011732902528	0.017011876638298	0.000000143735770
0.5	0.025239118652120	0.025237756617398	0.000001362034722
0.6	0.034625246282042	0.034618041342507	0.000007204939535
0.7	0.045054616958142	0.045029552985168	0.000025063972975
0.8	0.056459371287883	0.056387947081045	0.000071424206838
0.9	0.068818353871019	0.068640422489163	0.000177931381856
1.0	0.082159949070231	0.081759552977624	0.000400396092607

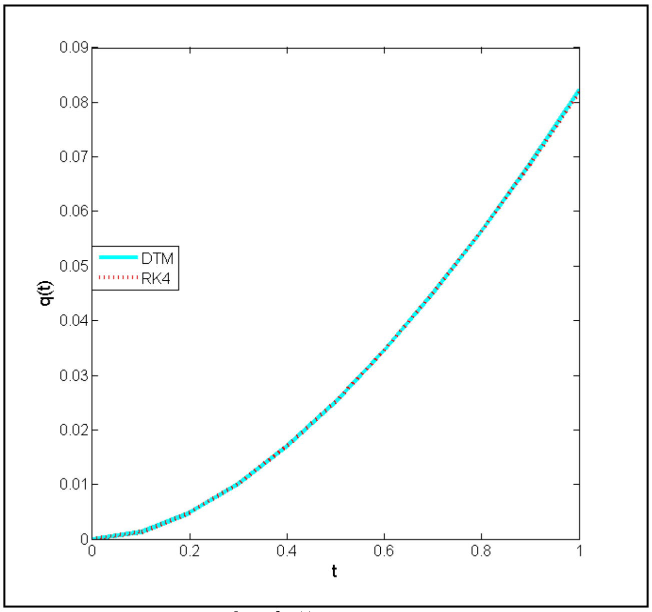

Fig. 5. Plot of q(t) versus time t

Table VI. THE ABSOLUTE ERROR INVOLVED THE DIFFERENTIAL TRANSFORMATION METHOD ALONG WITH THE RESULT OBTAINED BY THE RUNGE–KUTTA FOURTH-ORDER METHOD FOR r(t).

t	r(t) by DTM (8 iterate)	r(t) by RK4	|DTM-RK4|
0.1	3.158692672163217	3.158693034390637	0.000000362227420
0.2	3.366462793603211	3.366463464220000	0.000000670616789
0.3	3.617166439361943	3.617167170762139	0.000000731400196
0.4	3.905091113667753	3.905090138538556	0.000000975129197
0.5	4.224957235405397	4.224945844617992	0.000011390787406
0.6	4.571916627392842	4.571863534102826	0.000053093290015
0.7	4.941555627552412	4.941370761870564	0.000184865681848
0.8	5.329910440063882	5.329372697094120	0.000537742969762
0.9	5.733502344587106	5.732129981379218	0.001372363207889
1.0	6.149400381641805	6.146236384188384	0.003163997453421

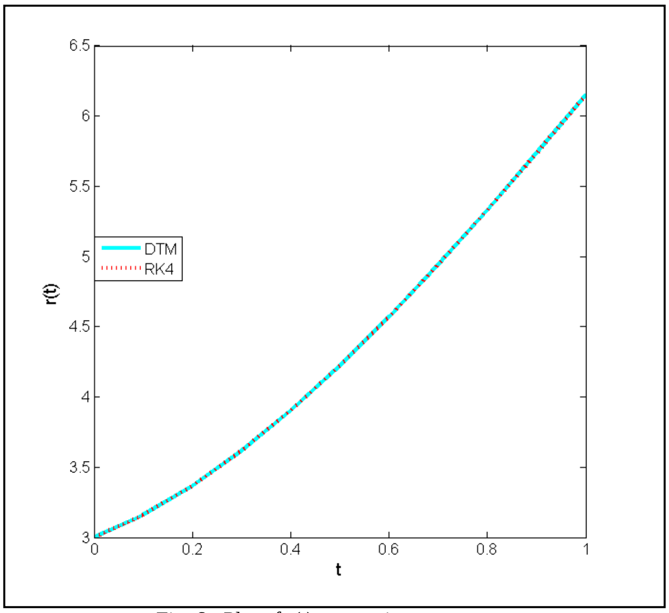

Fig. 6. Plot of r(t) versus time t

VI. CONCLUSION

Differential transform method (DTM) has been successfully applied to solve the SAEIQRS model with given initial conditions. This method provides an explicit solution which is very useful for understanding and analyzing an epidemic model. Without any linearization, discretization or perturbation, this method is applied directly to the system of nonlinear ordinary differential equations. The comparison of the solutions obtained by this method with the fourth-order Runge-Kutta method shows the efficiency and accuracy of the method. Based on the numerical results it can be concluded that the DTM is a mathematical tool, which enables one to find approximate accurate analytical solutions for epidemiological models represented by systems of nonlinear ordinary differential equations.

APPENDIX

A. *STEPS OF DIFFERENTIAL TRANSFORM METHOD*

With the initial conditions S(0)=30, A(0)=5, E(0)=2, I(0)=0, Q(0)=0, R(0)=3 and parameter
$B=0.01,\ \beta=0.09,\ \gamma=0.45,\ \sigma_1=0.35, \sigma_2=0.3,$
$\delta=0.65,\ \eta=0.01,\ \mu=0.05, k_1=0.035,\ \alpha=0.65,$
$\phi_1=0.2,\ \phi_2=0.3$

We have from (4.1)-(4.6),
For k=0,

$$S(1)=\frac{1}{0+1}[B\delta(0)+\eta R(0)-(\mu+\alpha)S(0)-\beta\sum_{m=0}^{0}S(m)I(0-m)]$$
$$=\frac{1}{1}[B\delta(0)+\eta R(0)-(\mu+\alpha)S(0)-\beta\{S(0)I(0)\}]$$
$$=-20.96$$

$$A(1)=\frac{1}{0+1}[\alpha S(0)-(\mu+\phi_2)A(0)-\phi_1\sum_{m=0}^{0}A(m)I(0-m)]$$
$$=\frac{1}{1}[\alpha S(0)-(\mu+\phi_2)A(0)-\phi_1\{A(0)I(0)\}]$$
$$=17.75$$

$$E(1)=\frac{1}{0+1}[-(\mu+\gamma)E(0)+\beta\sum_{m=0}^{0}S(m)I(0-m)+\phi_1\sum_{m=0}^{0}A(m)I(0-m)]$$
$$=\frac{1}{1}[-(\mu+\gamma)E(0)+\beta\{S(0)I(0)\}+\phi_1\{A(0)I(0)\}]$$
$$=-1$$

$$I(1)=\frac{1}{0+1}[\gamma E(0)-(\mu+k_1+\sigma_1+\sigma_2)I(0)]$$
$$=0.9$$

$$Q(1)=\frac{1}{0+1}[\sigma_2 I(0)-(\mu+k_1+\delta)Q(0)]$$
$$=0$$

$$R(1)=\frac{1}{0+1}[\sigma_1 I(0)+\delta Q(0)+\phi_2 A(0)-(\mu+\eta)R(0)]$$
$$=1.32$$

For k=1,

$$S(2)=\frac{1}{1+1}[B\delta(1)+\eta R(1)-(\mu+\alpha)S(1)-\beta\sum_{m=0}^{1}S(m)I(1-m)]$$
$$=\frac{1}{2}[B\delta(1)+\eta R(1)-(\mu+\alpha)S(1)-\beta\{S(0)I(1)+S(1)I(0)\}]$$
$$=6.1276$$

$$A(2)=\frac{1}{1+1}[\alpha S(1)-(\mu+\phi_2)A(1)-\phi_1\sum_{m=0}^{1}A(m)I(1-m)]$$
$$=\frac{1}{2}[\alpha S(1)-(\mu+\phi_2)A(1)-\phi_1\{A(0)I(1)+A(1)I(0)\}]$$
$$=-10.36825$$

$$E(2)=\frac{1}{1+1}[-(\mu+\gamma)E(1)+\beta\sum_{m=0}^{1}S(m)I(1-m)+\phi_1\sum_{m=0}^{1}A(m)I(1-m)]$$
$$=\frac{1}{2}[-(\mu+\gamma)E(1)+\beta\{S(1)I(0)+S(0)I(1)\}+\phi_1\{A(1)I(0)+A(0)I(1)\}]$$
$$=1.915$$

$$I(2)=\frac{1}{1+1}[\gamma E(1)-(\mu+k_1+\sigma_1+\sigma_2)I(1)]$$
$$=-0.55575$$

$$Q(2)=\frac{1}{1+1}[\sigma_2 I(1)-(\mu+k_1+\delta)Q(1)]$$
$$=0.135$$

$$R(2)=\frac{1}{1+1}[\sigma_1 I(1)+\delta Q(1)+\phi_2 A(1)-(\mu+\eta)R(1)]$$
$$=2.7804$$

For k=2,

$$S(3)=\frac{1}{2+1}[B\delta(2)+\eta R(2)-(\mu+\alpha)S(2)-\beta\sum_{m=0}^{2}S(m)I(2-m)]$$
$$=\frac{1}{3}[B\delta(2)+\eta R(2)-(\mu+\alpha)S(2)-\beta\{S(0)I(2)+S(1)I(1)+S(2)I(0)\}]$$
$$=-0.354410333333334$$

$$A(3)=\frac{1}{2+1}[\alpha S(2)-(\mu+\phi_2)A(2)-\phi_1\sum_{m=0}^{2}A(m)I(2-m)]$$
$$=\frac{1}{3}[\alpha S(2)-(\mu+\phi_2)A(2)-\phi_1\{A(0)I(2)+A(1)I(1)+A(2)I(0)\}]$$
$$=1.657525833333334$$

$$E(3)=\frac{1}{2+1}[-(\mu+\gamma)E(2)+\beta\sum_{m=0}^{2}S(m)I(2-m)+\phi_1\sum_{m=0}^{2}A(m)I(2-m)]$$
$$=\frac{1}{3}[-(\mu+\gamma)E(2)+\beta\{S(0)I(2)+S(1)I(1)+S(2)I(0)\}+\phi_1\{A(0)I(2)+A(1)I(1)+A(2)I(0)\}]$$
$$=-0.505511666666667$$

$$I(3)=\frac{1}{2+1}[\gamma E(2)-(\mu+k_1+\sigma_1+\sigma_2)I(2)]$$
$$=0.42340875$$

$$Q(3)=\frac{1}{2+1}[\sigma_2 I(2)-(\mu+k_1+\delta)Q(2)]$$
$$=-0.08865$$

$$R(3)=\frac{1}{2+1}[\sigma_1 I(2)+\delta Q(2)+\phi_2 A(2)-(\mu+\eta)R(2)]$$
$$=-1.1280205$$

For k=3,

$$S(4)=\frac{1}{3+1}[B\delta(3)+\eta R(3)-(\mu+\alpha)S(3)-\beta\sum_{m=0}^{3}S(m)I(3-m)]$$
$$=\frac{1}{4}[B\delta(3)+\eta R(3)-(\mu+\alpha)S(3)-\beta\{S(0)I(3)+S(1)I(2)+S(2)I(1)+S(3)I(0)\}]$$
$$=-0.612774749166667$$

$$A(4)=\frac{1}{3+1}[\alpha S(3)-(\mu+\phi_2)A(3)-\phi_1\sum_{m=0}^{3}A(m)I(3-m)]$$
$$=\frac{1}{4}[\alpha S(3)-(\mu+\phi_2)A(3)-\phi_1\{A(0)I(3)+A(1)I(2)+A(2)I(1)+A(3)I(0)\}]$$
$$=0.651321997916667$$

$$E(4)=\frac{1}{3+1}[-(\mu+\gamma)E(3)+\beta\sum_{m=0}^{3}S(m)I(3-m)+\phi_1\sum_{m=0}^{3}A(m)I(3-m)]$$

$$=\frac{1}{4}[-(\mu+\gamma)E(3)+\beta\{S(0)I(3)+S(1)I(2)+S(2)I(1)+S(3)I(0)\}+\phi_1\{A(0)I(3)+A(1)I(2)+A(2)I(1)+A(3)I(0)\}]$$

$$=-0.118781722916667$$

$$I(4)=\frac{1}{3+1}[\gamma E(3)-(\mu+k_1+\sigma_1+\sigma_2)I(3)]$$

$$=-0.1346714203125$$

$$Q(4)=\frac{1}{3+1}[\sigma_2 I(3)-(\mu+k_1+\delta)Q(3)]$$

$$=0.04804509375$$

$$R(4)=\frac{1}{3+1}[\sigma_1 I(3)+\delta Q(3)+\phi_2 A(3)-(\mu+\eta)R(3)]$$

$$=0.163877385625$$

For k=4,

$$S(5)=\frac{1}{4+1}[B\delta(4)+\eta R(4)-(\mu+\alpha)S(4)-\beta\sum_{m=0}^{4}S(m)I(4-m)]$$

$$=\frac{1}{5}[B\delta(4)+\eta R(4)-(\mu+\alpha)S(4)-\beta\{S(0)I(4)+S(1)I(3)+S(2)I(2)+S(3)I(1)+S(4)I(0)\}]$$

$$=0.385621333823333$$

$$A(5)=\frac{1}{4+1}[\alpha S(4)-(\mu+\phi_2)A(4)-\phi_1\sum_{m=0}^{4}A(m)I(4-m)]$$

$$=\frac{1}{5}[\alpha S(4)-(\mu+\phi_2)A(4)-\phi_1\{A(0)I(4)+A(1)I(3)+A(2)I(2)+A(3)I(1)+A(4)I(0)\}]$$

$$=-0.689096313183333$$

$$E(5)=\frac{1}{4+1}[-(\mu+\gamma)E(4)+\beta\sum_{m=0}^{4}S(m)I(4-m)+\phi_1\sum_{m=0}^{4}A(m)I(4-m)]$$

$$=\frac{1}{5}[-(\mu+\gamma)E(4)+\beta\{S(0)I(4)+S(1)I(3)+S(2)I(2)+S(3)I(1)+S(4)I(0)\}+\phi_1\{A(0)I(4)+A(1)I(3)+A(2)I(2)+A(3)I(1)+A(4)I(0)\}]$$

$$=0.276216114060417$$

$$I(5)=\frac{1}{4+1}[\gamma E(4)-(\mu+k_1+\sigma_1+\sigma_2)I(4)]$$

$$=0.009106343723437$$

$$Q(5)=\frac{1}{4+1}[\sigma_2 I(4)-(\mu+k_1+\delta)Q(4)]$$

$$=-0.015142914$$

$$R(5)=\frac{1}{4+1}[\sigma_1 I(4)+\delta Q(4)+\phi_2 A(4)-(\mu+\eta)R(4)]$$

$$=0.033931654013125$$

Similarly for more values of k, we can calculate the corresponding values of S, A, E, I, Q and R.

References

[1] P. Kumari, “Analysis of Analytical and Numerical Methods of Epidemic Models”, International Journal of Engineering Research and General Science, Volume 3, Issue 6, November-December 2015.

[2] I.H.A.H. Hassan, “Application to Differential Transformation Method for Solving Systems of Differential Equations”, Applied Mathematical Modeling, 2008, Volume 32, Number 12, PP. 2552–2559.

[3] G.E. Pukhov, “Differential Transformations of Functions and Equations”, Naukova Dumka Publication, Kiev, 1980.

[4] J.K. Zhou, “Differential Transformation and its Applications for Electrical Circuits”, Huarjung University Press, Wuuhahn, China, 1986.

[5] F. Ayaz, “Application of Differential Transform Method to Differential Algebraic Equations”, Applied Mathematics and Computation, 2004, Volume 152, Number 3, PP. 649-657.

[6] S.V. Kanth and K. Aruna, “Two-dimensional Differential Transform Method for Solving Linear and Nonlinear Schrodinger Equations”, Chaos Solitons and Fractals, 2009, Volume 41, Number 5, PP. 2277-2281.

[7] A. Arikoglu and I. Ozkol, “Solution of Fractional Differential Equations by using Differential Transform Method”, Chaos Solitons and Fractals, 2007, Volume 34, Number 5, PP. 1473-1481.

[8] A. Yazdani, J. Vahidi and S. Ghasempour, “Comparison Between Differential Transform Method and Taylor Series Method for Solving Linear and Nonlinear Ordinary Differential Equations”, International Journal of Mechatronics, Electrical and Computer Technology, Volume 6, Number 20, April 2016, PP. 2872-2877.

[9] F.S. Akinboro, S. Alao and F.O. Akinpelu, “Numerical Solution of SIR Model using Differential Transformation Method and Variational Iteration Method”, General Mathematics Notes, Volume 22, Number 2, June 2014, PP. 82-92.

[10] A.J. Arenas, G.G. Parra and B.M.C. Charpentier, “Dynamical Analysis of the Transmission of Seasonal Diseases using the Differential Transformation Method”, Mathematical and Computer Modeling, 2009, Volume 50, Number 5, PP. 765-776.

[11] A. M. Batiha and B. Batiha, “Differential Transformation Method for a Reliable Treatment of the Nonlinear Biochemical Reaction Model”, Advanced Studies in Biology, 2011, Volume 3, No. 8, PP. 355-360.

[12] S.F.M. Ibrahim and S.M. Ismail, “Differential Transformation Approach to A SIR Epidemic Model with Constant Vaccination”, Journal of American Science, 2012, Volume 8, Number 7, PP. 764-769.