Strength of Reinforced Concrete Members under Combined Bending and Torsion

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***Abstract*—Three concrete specimens are tested under combined bending and torsional moments. The first specimen is plain concrete, the second specimen has only longitudinal reinforcement, and the last specimen has both longitudinal and transverse reinforcement. The results are compared with theoretical predictions of three different interaction relations; an elastic interaction equation developed using maximum principal stresses, ultimate strengths interaction based on skew bending theory, and ACI interaction equations. The first two specimens fall very closely to the elastic interaction plot while the results of the third specimen are closely predicted by skew bending theory. It was found that the ACI equations underestimated the strength of the RC member.**

Keywords—Reinforced concrete, bending, torsion, steel reinforcement, yield strength, non-dimensional interaction, skew bending theory.

I. INTRODUCTION

The ultimate strength of concrete members under torsion and torsion combined with other actions can be determined using either a Truss Model or a Skew Bending Model [1-7]. In the skew bending theory, a failure mode is considered first and equations for ultimate strength are derived satisfying equilibrium conditions. In the present study, an elastic interaction equation between bending and torsion is developed using an equation for maximum principal stresses. Three concrete members are then tested and results compared with theoretical predictions.

II. EXPERIMENTAL INVESTIGATION

Three concrete members were tested under combined bending and torsional moments. Companion test cylinders having 4 in. diameter and 8 in. height were cast to determine the actual compressive strength of concrete for each specimen. A minimum of 28-day curing period was used for all cylinders. Fig. 1 shows a couple of experimental stress-strain plots along with a second degree curve approximation [8]:

$$f_c = f_c'\left[\frac{2\epsilon}{\epsilon_0} - \left(\frac{\epsilon}{\epsilon_0}\right)^2\right] \quad (1)$$

where f_c' represents the ultimate compressive strength of concrete, ϵ_o is the strain corresponding to f_c', and f_c is concrete stress at any strain level ϵ. The average compressive strength of concrete was found to be 5365 psi.

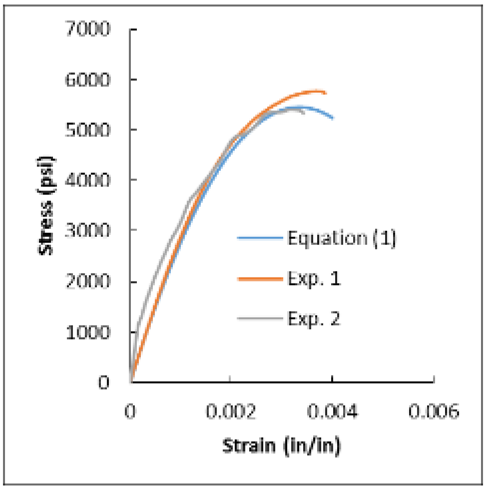

Fig. 1. *Stress-strain relation of concrete*

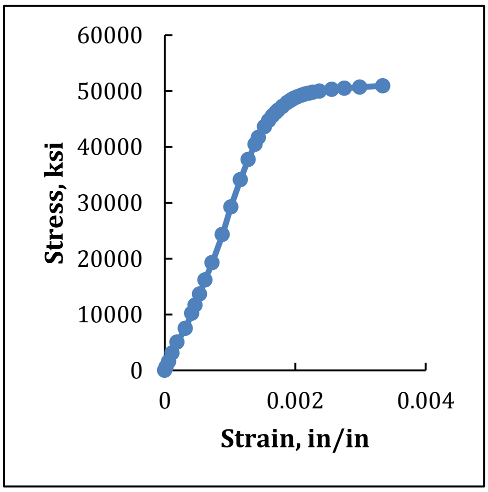

Fig. 2. *Stress-strain relation of steel rebars*

The longitudinal and transverse reinforcement is provided by #2 steel rebars (with 0.25-in. diameter). Fig. 2 shows stress-strain relation of these rebars. The average tensile yield strength was found to be 49.5 ksi and modulus of elasticity (E) was found to be 28700 ksi.

Three concrete members having a square cross section were tested under the combined action of bending and torsional moments. Each specimen was 28 in. long and had 5 in. x 5 in. cross section. The first specimen (SM01) was a plain concrete member without any steel reinforcement as shown in Fig. 3.

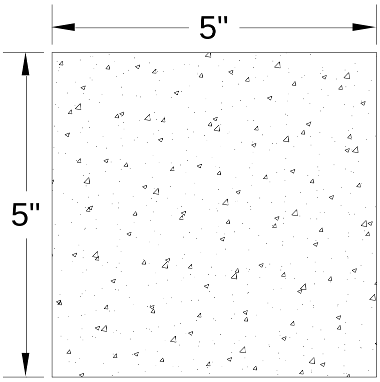

Fig. 3. *Cross section of SM01*

The second specimen (SM02) had only longitudinal reinforcement without any transverse stirrups. The reinforcement consists of 4, #2 rebars as shown in Fig. 4. The rebars were hold in place by a #2 rebar stirrup at each end.

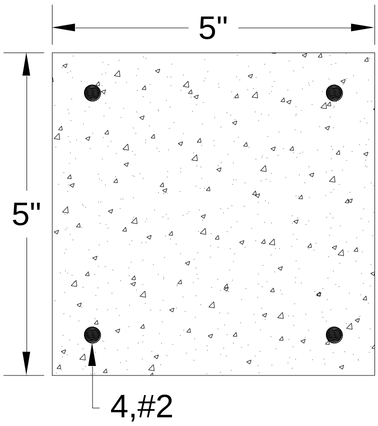

Fig. 4. *Cross section of SM02*

The third specimen (SM03) had both longitudinal reinforcement and transverse stirrups as shown in Fig. 5. The shear reinforcement was provided by #2 stirrups with a 5 in. center-to-center spacing.

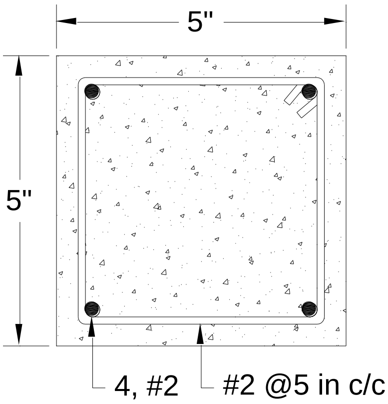

Fig. 5. *Cross section of SM03*

The test apparatus developed by Razzaq and McVinnie [9] for biaxial bending of steel beam-columns and nearly replicated at Old Dominion University (ODU) was used for testing the concrete specimens. The apparatus at ODU was modified and used by Sanders [10] and Zhao [11]. Konate [12] modified the same apparatus to apply torsion at the end for testing steel members. The bending part of the test setup along with a test specimen is shown in Fig. 6. The upper end fixture which simulates pinned-end condition is bolted to a heavy steel cross beam in an upside-down position. The cross beam is attached at its ends to steel columns which in turn are anchored to the laboratory test bed forming a large reaction frame. A solid rectangular steel moment arm having dimensions 1.0 x 2.0 x 24.0 in. is bolted to the upper to the upper fixture through which the bending moment is applied at the top end of the member. The load is applied through two 75 in. long tie rods having 0.75 in. diameter. The rods are separated at each end by 12 in. long and 0.5 in thick steel plates. The top plate sits on the steel arm with the help of a ball-and-socket arrangement. The bottom plate is attached to a 22-kip capacity compression Load Cell B through a similar arrangement. The Hydraulic Jack B is firmly bolted to a small steel reaction frame. The reaction frame is mounted to the laboratory test bed.

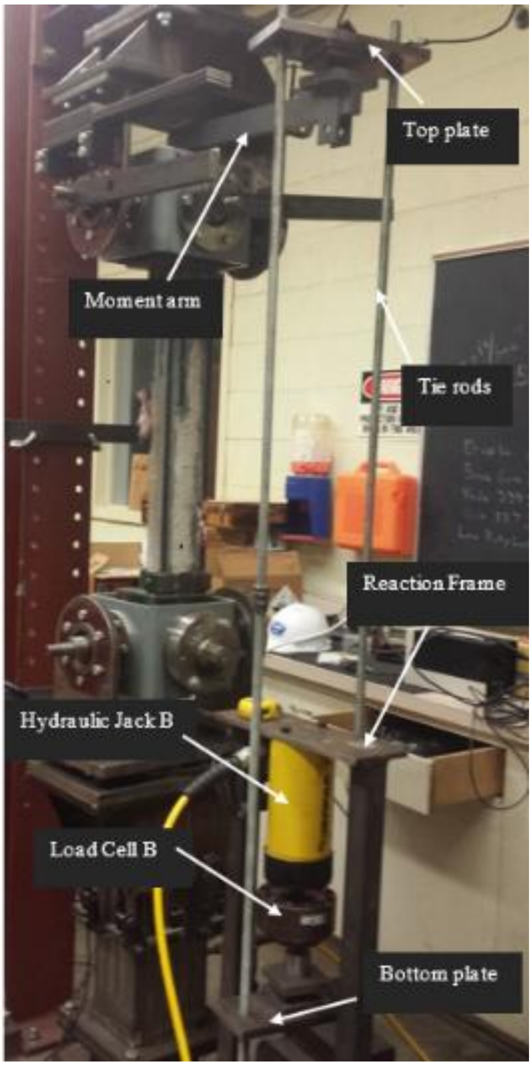

Fig. 6. *Bending test setup*

The torsion apparatus along with a test specimen is shown in the Figure 7. The eccentric force generating torsional moment at the bottom end is applied through

Hydraulic Jack C and measured by Load Cell C. The load is transmitted to the specimen by a chain as shown in Figure 5. The bottom end gimbal is attached to Steel Plate 1 which rotates on solid steel spheres. A shaft is welded to the Steel Plate 2 which is connected to the circular bearing on Steel Plate 1. When an eccentric load is applied, Steel Plate 2 rotates freely on Steel Plate 1 thus producing a torsional moment. To hold the concrete specimen ends in place, each specimen end was secured inside of a 2-inch deep end steel containment.

Fig. 7. *Torsion test setup*

The experimental load-deflection plots and final cracking pattern of the specimen are shown in Fig. 8-16. The values of the applied bending and torsional moments and corresponding maximum deflections are given in Table 1. The applied bending moment in all cases is only a fraction of the cracking moment, therefore the bending moment versus deflection plots are linear. The torsional moment versus angle of twist plot for SMO1 is almost a straight line until failure as shown in Fig. 9. The plot for SM02 has two increasing straight line regions separated by an almost horizontal region. Similarly the plot for SM03 has an initial straight portion before cracking followed by another straight portion with lesser stiffness after the cracking. The specimen failed in a brittle fashion after reaching the maximum torsion. The cracking patterns show an inclined failure surface as shown in Figure 10 for SM01.

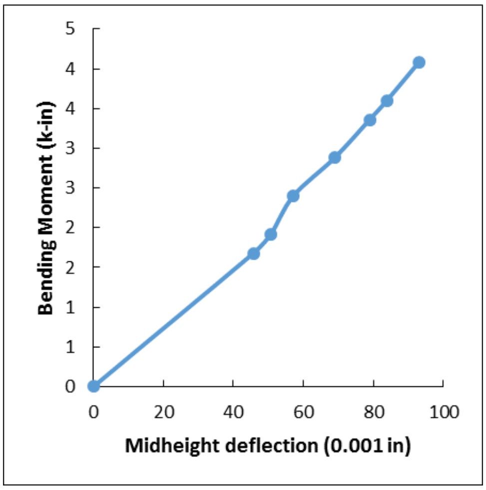

Fig. 8. *Bending versus deflection plot of SM01*

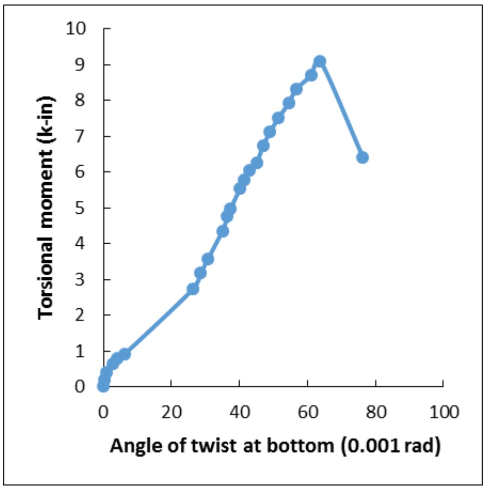

Fig. 9. *Torsion versus angle of twist plot of SM01*

Fig. 10. *Final crack pattern of SM01*

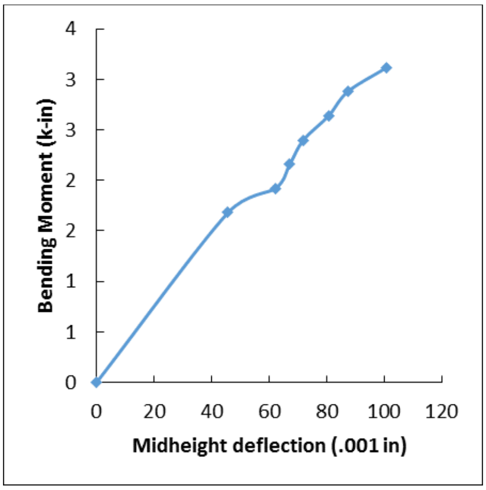

Fig. 11. *Bending versus deflection plot of SM02*

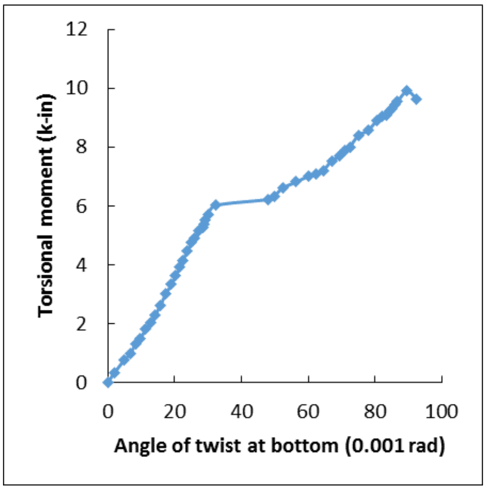

Fig. 12. *Torsion versus angle of twist plot of SM02*

Fig. 13. *Final crack pattern of SM02*

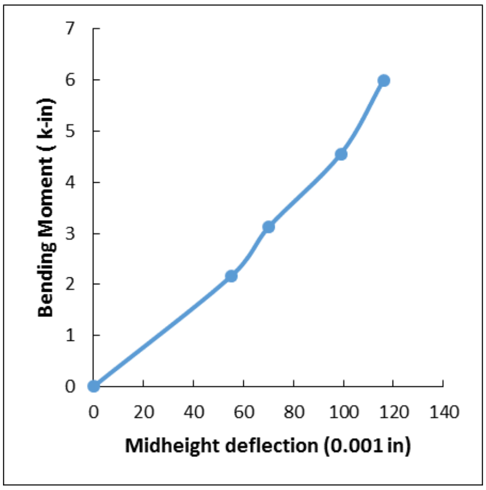

Fig. 14. *Bending versus deflection plot of SM03*

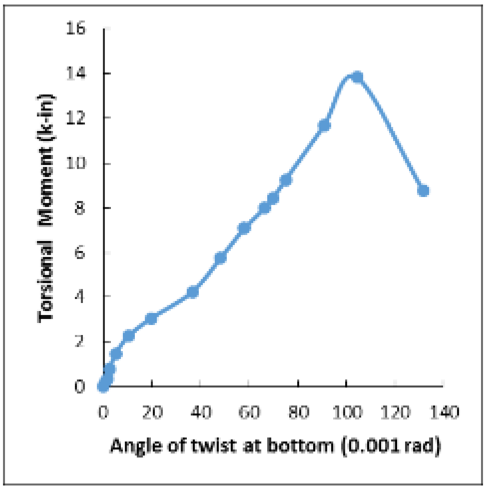

Fig. 15. *Torsion versus angle of twist plot of SM03*

Fig. 16. *Final crack pattern of SM03*

TABLE 1. SUMMARY OF TEST RESULTS

Specimen	M_{max} (k-in)	D_{max} (0.001in)	T_{max} (k-in)	Θ_{max} (0.001rad)
SM01	4.08	95.9	9.11	63.77
SM02	3.12	83.1	9.90	89.6
SM03	6.00	116.0	13.82	104.3

III. THEORETICAL PROCEDURE

The elastic flexural stress due to an applied bending moment M_x about the principal x-axis is given by [13]:.

$$\sigma_z = \frac{M_x y}{I_x} \quad (2a)$$

The elastic shear stress due to an applied torsional moment T is given by based on St. Venant theory [14]:

$$\tau = \frac{T}{\alpha x^2 y} \quad (2b)$$

A plain concrete member and a member with only longitudinal reinforcement fails when the maximum principal tensile stress reaches the tensile strength of concrete [14, 15]. Therefore, substitution of Equations 2a and 2b in the equation for principal stress results in the following interaction equation between torsion and bending:

$$T = \alpha x^2 y \sqrt{\left(f_{ct} - \frac{M_x h}{4I_x}\right)^2 - \left(\frac{M_x h}{4I_x}\right)^2} \quad (2c)$$

In the above equations, x and y are the shorter and longer sides of the cross section respectively, h is the depth of the cross section, I_x is the moment of inertia of the cross section about x-axis, α is a coefficient which depends on the ratio x/y, and f_{ct} is the splitting tensile strength of concrete. The splitting tensile strength can be approximated by ACI-318 relation [16]:

$$f_{ct} = 6.7\sqrt{f_c'}. \quad (2d)$$

For members with both longitudinal and transverse reinforcement, Elfren et al [3] developed the interaction equation given by:

$$\frac{M}{M_o} + \left(\frac{T}{T_o}\right)^2 = 1 \quad (3a)$$

In this expression, M_o and T_o are respectively, defined as follows:

$$M_o = A_s f_y h' \quad (3b)$$

$$T_o = 2h'b' \frac{A_t f_{yt}}{s} \sqrt{\frac{A_s f_y}{b'+h'} \frac{s}{A_t f_{yt}}} \quad (3c)$$

In these equations, h' and b' represent the center-to-center distances between horizontal and vertical legs of stirrups respectively, and yielding of both longitudinal and transverse reinforcement is assumed at failure.

The joint ACI-ASCE Committee 445 on torsion has adopted the same interaction equation but introduced a coefficient *r* to account for the unsymmetrical distribution of reinforcement [17]. The coefficient *r* is the ratio of the forces in top and bottom rebars at yielding. Its modified equation is:

$$r\left(\frac{T}{T_o}\right)^2 + \frac{M}{M_o} = 1 \quad (4a)$$

In which:

$$M_o = A_s f_y \left(d - \frac{a}{2}\right) \quad (4b)$$

$$T_o = \frac{2A_0 A_t f_{yt}}{s} cot\theta \quad (4c)$$

where A_o represents the area enclosed by the shear flow path and θ is the angle of the concrete compression strut with axis of the member.

IV. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

The non-dimensional interaction plots of the members are shown in Fig. 17-20 along with the points obtained from test data. The results are summarized in Table 2. The test points for the first two specimens are closely located on the interaction plot developed using elastic principal stresses. Similarly the test point of the third specimen is closely located on the interaction based on skew bending theory. However, the ACI equation significantly underestimated the strength of Specimen SMO3.

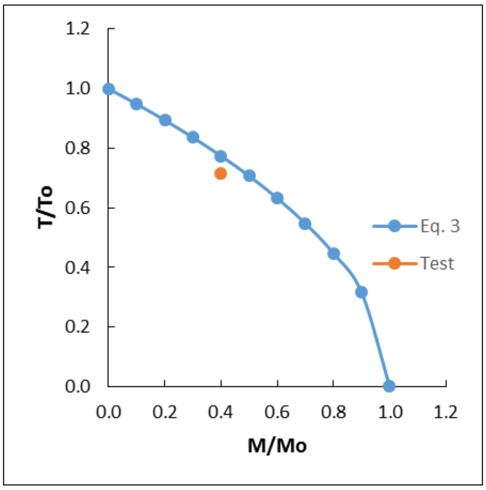

Fig. 17. *Bending-torsion interaction for SM01*

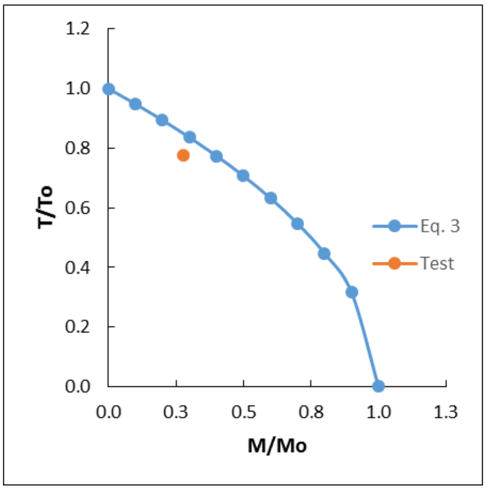

Fig. 18. *Bending-torsion interaction for SM02*

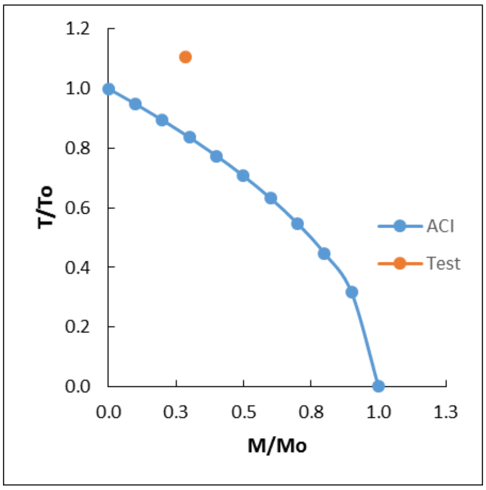

Fig. 19. *Bending-torsion interaction for SM03*

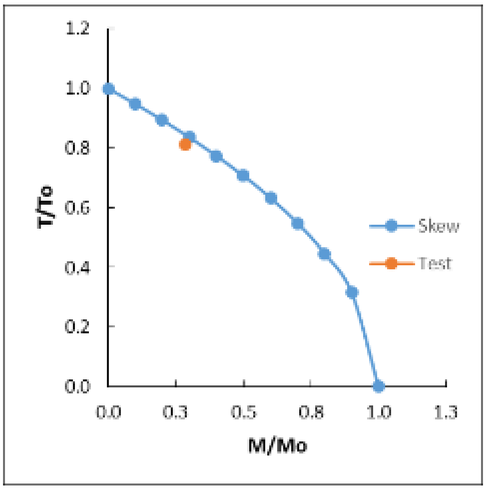

Fig. 20. *Bending-torsion interaction for SM03*

TABLE 2. COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Specimen	Applied Moment, M (k-in)	Test, T_u (k-in)	Calculated		
			Method	T_u (k-in)	$T_{u,test}/T_{u,calc}$
SM01	4.08	9.11	Elastic theory	9.89	0.92
SM02	3.12	9.90	Elastic theory	10.84	0.91
SM03	6.00	13.82	Skew Bending	14.35	0.96
	6.00	13.82	ACI	10.56	1.31

V. PRACTICAL EXAMPLES

The following examples demonstrate the use of interaction equations to check the capacity of members under combined bending and torsion.

A. *Example 1*

A reinforced concrete member having cross section 18in x 12in is reinforced with four #8 longitudinal rebars. The #3 transverse stirrups are provided at 5in center-to-center. Check if the member will fail under a combined loading of M = 1000 k-in and T = 200 k-in. Given $f_c' = 5$ ksi, $f_y = 60$ ksi.

SOLUTION

Using (3b) and (3c) one gets M_o = 1386 k-in and T_o = 534 k-in. Substituting these values and the given values of M and T in (3a) results in 0.86 which is less than 1.00. Therefore, the member is safe under the given applied loads.

B. Example 2

If the torsion in Example 1 in increased to T = 300 k-in, find spacing of the transverse stirrups that will make the member safe.

SOLUTION

Using (3a) results in T_o = 568 k-in which is used in (3c) to find s = 5.30 in. Therefore transverse stirrups should be provided at a distance less than 5.30 in. in order for the member to be on the safe side of the interaction diagram.

VI. CONCLUSIONS

The interaction equation developed based on elastic principal stresses closely predicts the response of plain concrete members and members with only longitudinal reinforcement. The interaction equation based on skew bending theory predicted the strength of RC member very closely. The ACI equation underestimates the strength of RC members, resulting in a safe but uneconomical design.

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REFERENCES

1. Thomas, T.C.H., *Torsion of Structural Concrete-Plain Cocnrete Rectangular Sections.* ACI Special Publication 18.
2. Thomas, T.C.H., *Torsion of Structural Concrete-Behavior of Reinforced Concrete Rectangular Members.* ACI Special Publication 18.
3. Elfren, L., I. Karlsson, and A. Losberg, *Torsion-Bending-Shear Interaction for Concrete Beams.* 1974. 100(ST8): p. 1657-1676.
4. Denis, M. and P.C. Michael, *Diagonal Compression Field theory-A Rational Model For Structural Concrete in Pure Torsion.* Journal Proceedings. 71(8).
5. Lessig, N.N., *Determination of load carrying capacity of rectangular reinforced concrete elements subjected to flexure and torsion.* 1959.
6. M.P. Collins, P.F.W.F.E.A. and A.S. Hall, *Ultimate Strength of Reinforced Concrete Beams Subjected to Combined Torsion and Bending.* Special Publication. 18.
7. Thomas, T.C.H. and M.O. Y. L, *Softening of Concrete in Torsional Members - Theroy and Tests.* Journal Proceedings. 82(3).
8. Lin, T.Y. and N.H. Burns, *Design of prestressed concrete structures / T. Y. Lin, Ned H. Burns*. 1981: New York : Wiley, c1981. 3d ed.
9. Razzaq, Z. and W.W. McVinnie, *Theoretical and Experimental Behavior of Biaxially Loaded Columns.* Journal of Structural Mechanics, 1986. 14(3).
10. Sanders, E.C., *Experimental Investigation of Steel Angle Column Buckling and Comparison to Seven International Design Specifications*. 2009, Old Dominion University.
11. Zhao, Y., *Thermo-elasto-plastic behavior of biaxially loaded steel beam-columns inducing those from World Trade Center towers*. 2013, Old Dominion University: Ann Arbor. p. 222.
12. Konate, M., *Inelastic Behavior and Strength of Steel Beam-Columns with Applied Torsion*, in *Civil and Environmental Engineering*. 2015, Old Dominion University.
13. Popov, E.P., *Engineering mechanics of solids*. 2nd ed.. ed, ed. T.A. Balan. 1998, Upper Saddle River, N.J.: Upper Saddle River, N.J. : Prentice Hall.
14. Hsu, T.T.C., *Torsion of reinforced concrete / Thomas T.C. Hsu*. 1984: New York : Van Nostrand Reinhold, c1984.
15. Thomas, T.C.H., *Torsion of Structural Concrete-Interaction Surface for Combined Torsion, Shear, and Bending in Beams Without Stirrups.* Journal Proceedings. 65(1).
16. ACI committee 318, *318-11: Building Code Requirements for Structural Concrete and Commentary.* 2014.
17. Joint, A.C.I.A.C., *445.1R-12: Report on Torsion in Structural Concrete.* Technical Documents.