Elastic-Plastic Stress Analysis In Multiply Layered For Symmetric Cross-Ply Composite Laminated Plates With A Hole

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***Abstract*—Fiber-reinforced composite materials and, in particular, polymer matrix composites have gained widespread applicability in high-performance air-craft and aerospace structures due to their high specific-stiffness and specific-strength (stiffness and strength per mass), thus, offering a considerable weight saving capability. The matrix, however, is the most significant limitations to broader and more efficient use of composite materials. It limits the overall performance of the composite system due to its inferior elastic/strength properties and large thermoelastic property mismatch with reinforcing fibers, as well as its high sensitivity to environmental conditions. In the present study, an elastic-plastic stress analysis in symmetric and antisymmetric Cross-ply laminated plastic-metal fiber composite plate is carried out by using the finite element method. The composite plate reinforced with (long galvanized wire fiber) is manufactured by using moulds under the action of 30 MPa pressure and heating up to 190°C. A laminated plate consists of four plastic matrix layers bonded symmetrically or antisymmetrically by applying pressure and heat. The first order shear deformation theory and nine-node Lagrangian finite element is used. In the numerical solution the transverse load is increased gradually.**

Keywords— Residual stresses; Thermoplastic composite; Finite element; Elastic-plastic solution

I. INTRODUCTION

The high specific-designing composite structures, which are more efficient than metallic structures.

In addition, thermoplastic composites have the unique characteristic that they may be remelted, reprocessed, and reformed thereby offering a degree of post processing freedom unavailable in thermoset composites. This characteristic makes thermoplastic easily repairable as they can be remelted locally and resolidified (repair of transverse cracking and delamination), and recyclable.

Jegley, [6] presented a manufacturing process of thermoforming and the results of a study of the effects of impact damage on compression loaded trapezoidal corrugation sandwich and semisandwich graphite-thermoplastic panels. Marissen et al. [2] İnvestigated woven fabric in $(0^o/90^o)_2$ composites with a thermoplastic matrix because they combine excellent mechanical performance with ease of processing.

Representative experimental investigations on the forming of advanced thermoplastic composites can be found in references. (Jegley, Chen, Cantwell, Shi, Wang.) [6], [1], [7], [5], [3][4] .

Elastic-plastic stress analysis was carried out in a laminated composite plate (Bahei-El-Din, Karakuzu). [9], [10]

Finite element method gives the excellent solution to the elastic-plastic stress analysis of laminated plate (Karakuzu, Sayman). [10], [8]

II. MACROMECHANICAL PROPERTIES OF A LAMINATE

2.1. Mathematical Formulation

The laminated plate of constant thickness is composed of orthotropic layers bonded symmetrically or antisymmetrically about the middle surface of the plate. In the solution of this problem, the Cartesian coordinates are used where the middle surface of the plate coincides with the x-y plane (Figure 2.1)

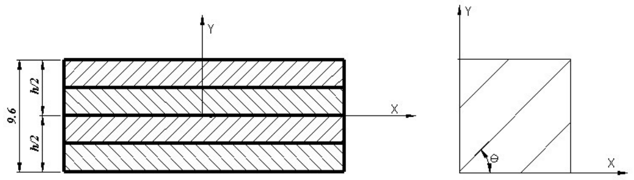


Figure 2.1

The stress-strain relations for anorthotropic layer can be written as,

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{Bmatrix}Q_{11} & Q_{12} & Q_{16}\\ Q_{12} & 22 & Q_{26}\\ Q_{16} & Q_{26} & Q_{66}\end{Bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} \quad (2.1)$$

$$\begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{44} & Q_{45}\\ Q_{54} & Q_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \quad (2.2)$$

where Q_{ij} are reduced stiffness and can be defined in terms of elastic constants D_{ij} and the orientation angle θ. Here, we use the theory of plates with transverse shear deformations theory which uses the assumption that particles of the plate originally on a line that is normal to the undeformed middle surface remain on the straight line during deformations, but this line is not necessarily normal to the deformed middle surface. By using this assumption, the displacement components of a point of coordinates x,y,z for small deformations are (Jones, Tsai, Gibson), [18], [15] , [19]

u(x,y,z) = u_0(x,y) + zψ_x(x,y)
v(x,y,z) = v_0 (x,y) - z + ψ_y(x,y)
w(x,y,z) = w(x,y) (2.3)

where , u_0, v_0, and w are displacements at any point of the middle surface and ψ_x, ψ_y are the rotations of normals to the y and x axes, respectively.

The bending strains vary linearly through the plate thickness, whereas shear Strains are assumed to be constant through the thickness as (Lin) [14]

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{vmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial y}\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{vmatrix}+z\begin{vmatrix}\frac{\partial \psi_x}{\partial x}\\ -\frac{\partial \psi_y}{\partial y}\\ \frac{\partial \psi_x}{\partial y}-\frac{\partial \psi_y}{\partial x}\end{vmatrix} \text{ or } \begin{vmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{vmatrix}=\begin{vmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{vmatrix}+z\begin{vmatrix}K_x\\ K_y\\ K_{xy}\end{vmatrix}$$

$$\begin{vmatrix}\gamma_{yz}\\ \gamma_{xz}\end{vmatrix}=\begin{vmatrix}\frac{\partial w}{\partial y}-\psi_y\\ \frac{\partial w}{\partial x}-\psi_x\end{vmatrix} \quad (2.4)$$

The total potential energy of a laminated plate under static loadings is given as:

$$\Pi = U_b + U_s + V \quad (2.5)$$

where U_b is the strain energy of bending, U_s is the strain of energy shear, and V, represents potential energy of external forces. They are as,

$$U_b=\frac{1}{2}\int_{-h/2}^{h/2}\left[\int_A\left(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\tau_{xy}\gamma_{xy}\right)dA\right]dz$$

$$U_s=\frac{1}{2}\int_{-h/2}^{h/2}\left[\int_A\left(\tau_{yz}\gamma_{yz}+\tau_{xz}\gamma_{xz}\right)dA\right]dz$$

$$V=\frac{1}{2}\int_A wpdA-\int_{\partial R}\left(N_n^b u_n^0+N_s^b u_s^0\right)ds \quad (2.6)$$

where dA = dx.dy, p is the transverse loading per unit area, and N_n^b and N_s^b are the in-plane loads applied on the boundary ∂R.

In-plane forces N_x, N_y, and N_{xy} and moments M_x, M_y, and M_{xy}, defined per unit length are given

$$\begin{Bmatrix}N_x & M_x\\ N_y & M_y\\ N_{xy} & M_{xy}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{Bmatrix}(1,z)dz$$

$$\begin{Bmatrix}Q_x\\ Q_y\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix}dz \quad (2.7)$$

Equilibrium requires that π is the stationary or π is a minimum that the second variation of Π is positive at the stationary point. It may be regarded as the principle of

$$\Pi = \Pi(u_0, v_0, w, \psi_x, \psi_y) \quad (2.8)$$

virtual displacement (Bathe). [13] By using the finite element method, the total energy is expressed as a function of displacement components as.

2.2. Finite Element Analysis

By using characteristic values found from the experiments mentioned in chapter three, we applied the finite element analysis by using computer to the laminates for variation of angles cross-ply laminate position (symmetric and antisymmetric), thickness and hole effect in the laminates and following results are found. The area of the laminates taken as 150x150 mm^2 (Figure 2.2.)

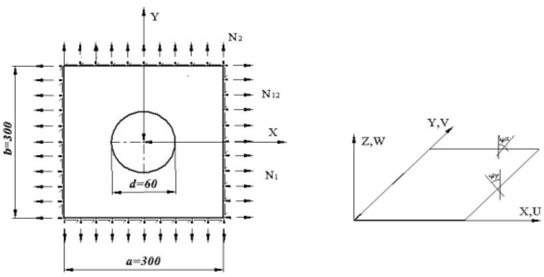


Figure 2.2. Loading laminated plate

The nine-node finite element is used in the present study. The quadratic interpolation functions for the nine-node Lagrangian finite element and mesh model.

İn the elastic-plastic solution the tangential modular matrix is used instead of the elasticity matrix (Cristfield). The yield function f is as;

$$f=\sigma_e-\sigma_0=0 \quad (2.9)$$

where σ_e is the effective stress and σ_0 is the yield stress:The tangential modular matrix is found as;

$$D_t = D\left(I-\frac{aa^T D}{a^T Da}\right) \quad (2.10)$$

Where D, I and a are the elasticity matrix, unit matrix and flow vector, respectively. The flow vector is found by using the result Prandtl-reuss equation (Mendelson and Chakrabarty) [16], [17]

$$a= \partial f - \partial \sigma \quad (2.11)$$

Once the nodal displacement are calculated, the strain components of each layer can be found and the stress components can be calculated and used to check the yield state of the material.

Since the calculated stresses do not generally coincide with the true stress in a nonlinear problem, the unbalanced nodal forces and the equivalent nodal forces must be calculated. The equivalent nodal point forces corresponding to the element stress at each iteration can be calculated as(Hu): [11]

$$\{R\}_{equivalent} = \int_{vol}[B]^T [\sigma]dA = \int_{vol}[B_b]^T [\sigma_b]dA + \int_{vol}[B_s]^T [\sigma_s]dA \quad (2.12)$$

When the equivalent nodal forces are known, the unbalanced nodal forces can be found by;

$$\{R\}_{unbalanced}= \{R\}_{applied} - \{R\}_{equivalent} \quad (2.13)$$

These unbalanced nodal forces are applied for obtaining increments in the solution and must satisfy the convergence tolerance in a nonlinear analysis. A widely used iteration procedure is the modified Newton Iteration Method as (Bathe). [2] This iterative schema can be derived from the Newton-Raphson Method Equations.

The difference between the plastic and elastic solution gives the residual stresses. The residual stresses may increase the possible failure of the laminated plates. In this solution, 169 nodes and 36 elements are used.

The stiffness matrix of the plate element is obtained by using the minimum potential energy method or the principle of virtual bending and shear stiffness matrices are:

Where

$$[K_b] = \int_A[B_b]^T [D_b][B_b]dA$$

$$[K_s] = \int_A[B_b]^T [D_s][B_s]dA$$

$$[D_b]=\begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \quad [D_s]=\begin{bmatrix} k_1^2A_{44} & 0 \\ 0 & k_2^2A_{55} \end{bmatrix} \quad (2.14)$$

$$(A_{ij,}B_{ij,}D_{ij,})=\int_{-h/2}^{h/2} Q_{ij}(1,z,z^2)dz \quad \text{(i,j=1,2,6)}$$

$$(A_{44,}A_{55,})=\int_{-h/2}^{h/2}(Q_{44,}\, Q_{55})dz \quad (2.15)$$

$|B_b|_{6x45}$, $|B_b|_{2x45}$ and D_b and D_s, are the bending and shear parts of the material matrix, respectively. $A_{55,}$ is neglected in comparison with A_{44} and A_{55} and shear correction factorsfor rectangular cross-sections are given as (Lin, Barbero), [14], [12]

$k_1^2 = k_2^2 = 5/6$.

Once the nodal displacements are calculated, the strain components of each layer can be found by partial derivation formulas and tangential modular matrix. The stress components can be calculated and used to check the yield state of the material.

III. EXPERIMENTAL PROCEDURES

3.1. Introduction

In this study, low density polyethylene (LDPE) F2.12 was used as a thermoplastic matrix. The preparation of composite laminates consisted of two important steps. The first step was that, melting granule, raw material of thermoplastics, as a flat plate. The second was combining these plates with fibers as composite plates. These two steps were applied successfully in the laboratory.

Thermoplastic plates were obtained by melting the granule in the rectangular mould. The material temperature was increased up to 160°C without pressure in five minutes. Then, the materials were waited five minutes under 2.5 MPa pressure. And the last, the temperature was decreased to 30°C under 15 MPa pressure in three minutes. Consequently, polyethylene plate was obtained.

3.2 Production of Laminates

Approximately 30 grams of granulized plastic material was inserted into an iron mould of 160 mm size and 1 mm thickness. To avoid sticking, acetate was put between the surfaces. The compression pressure and time date were reported for the low density. PE were as before chapter. Then, a single laminate was produced by cooling down to 30°C degrees under 15 MPa pressure within 3 minutes.

3.3. Production of These Composite Materials

3.3.1. Unidirectionally Galvanized Wire Fiber Reinforced Laminates

An iron square frame of 160 x 160 ⟦mm⟧^2 area was driven by 0.5 mm diameter holes along its two facing edges every other mm. Then, 0.5 mm - diameter galvanized fiber was passed through these holes producing a single directional fibered frame.

Then, this frame was sandwiched between two polyethylene laminates and inserted in a mould suitable to its thickness. Thereafter, it was put in a 2.5 MPa pressure under 160°C degrees for 5 minutes, by a press using acetate in between the faces to avoid sticking.Then, it was cooled down to 30°C degrees within 3 minutes under 15 MPa pressure. In this way, a matrix material was produced.

3.3.2. Finding V_m and V_f

Before building composite material the weight of fiber and matrix materials were measured.

	Weight (gr)	Specific weight (gr/cm^3)	Volume fraction
Fiber	13.764	7.8	0.046
PE(polyethlene)	32.680	0.9	0.953

Table 3.1 Volume fractions

After specimen preparation, the tensile tests were carried out. From these tests, both tensile strength and σ-ε diagrams were found. (k, n) plastic coefficients were calculated from this diagram by using elastic-plastic region as shown in Fig. 3.1.

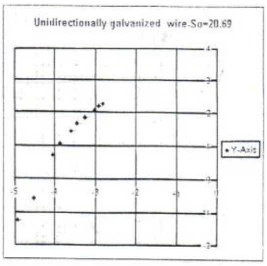


Fig. 3.1. Diagram of tensile test.

From above diagram, following values were found.

Num. of tests	$A_{0\ (mm^2)}$	X(MPa)	K(MPa)	n
20	23.700	21.010	47.183	0.713

Table 3.2. Tensile test of 1-direction.

Similarly, loading process applied to 2-direction of specimen the results were in the following table:

Num. of tests	$A_{0\ (mm^2)}$	Y(MPa)
5	23.50	5.22

Table 3.3. Tensile test of 2-direction.

Then to find shear stress in 1-2 directions was investigated. The results are given in Table 3.4.

Num. of tests	$A_{0\ (mm^2)}$	S(MPa)
5	10.75	5.85

Table 3.4. Results of shear tests

To find E1, and v12, two strain gauges bounded on the specimen one of which is parallel to loading direction and the other perpendicular to loading direction. The test results are in Table 3.5.

P(kg)	10
$A_{0\ (mm^2)}$	33.75
σ_1(MPa)	2.976
ε_1	692 E-6
ε_2	278 E-6
E_1(MPa)	4300
ν_{12}	0.4

Table 3.5. E1 and v12 test results.

Similarly, to find E2 the loading operation was applied to 2-direction and E2, was calculated then v12 was calculated the stiffness properties should satisfy the reciprocal relations The results are in Table 3.6.

P(kg)	5
$A_{0\ (mm^2)}$	27.50
σ_1(MPa)	18.18
ε_1	1899 E-6
E_1(MPa)	957
ν_{12}	0.0898

Table 3.6. E2 and v12 test results.

Shear modulus is found by applying the loading to the x-direction.

P(kg)	4
$A_{0\ (mm^2)}$	28.405
ε_x	1838.72 E-6
E_x(MPa)	765.86
G_{12}(MPa)	241.48

Table 3.7. Shear modulus test results.

3.4. Plates With a Hole

In this case elastic-plastic stress analysis is carried out in plates with a hole under the in plane loading greatest are found as shown in

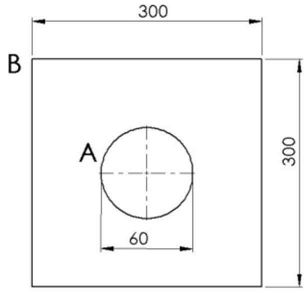


Figure 3.2.

The greatest stress components are found around the hole. Elastic-plastic, elastic and residual stress components are given in the symmetric cross-ply laminated plate at point A for 100, 200 and 300 iteration numbers, at Table 3.8.

Stresses	Iteration Number	σ_x (MPa)	σ_y (MPa)	τ_x (MPa)	τ_{yz} MPa	τ_{xz} MPa
Elatic-Plastic Stresses	100	21.205	0.533	-0.205	0	0
	200	21.435	0.470	-0.213	0	0
	300	21.616	0.437	-0.087	0	0
Elastic Stresses	100	24.169	1.804	-0.214	0	0
	200	29.307	2.187	-0.260	0	0
	300	34.446	2.571	-0.360	0	0
Residual Stresses	100	-2.964	-1.271	0.010	0	0
	200	-7.873	-1.718	0.047	0	0
	300	-12.831	-2.134	0.219	0	0

Table 3.8. Elastic-plastic, elastic and residual stress components for symmetric cross-ply laminated plates with a hole $(0^o)_2$ for 100, 200 and 300 iterations.

As seen from these Tables when we increase the iteration numbers, the intensity of residual stress become higher. The intensity of the residual stresses in the layer of 0° orientation angle is greater than in the layer of 90o orientation angle, because of the higher elastic contants.

IV. RESULTS AND DISCUSSIONS

4.1. Unidirectionally Galvanized Wire Reinforced Laminates

The mechanical properties of the laminated plate are found by using strain-gauges and Instron test machine.

E_1	E_2	G_{12}	ν_{12}
4300 MPa	957 MPa	242 MPa	0.4

Table 4.1.1. Mechanical properties and yield points of a layer.

Mechanical Properties:

Axial Strength	X	21.01 MPa
Transverse Strength	Y	5.22 MPa
Shear Strength	S	5.85 MPa
Hardening Parameter	K	47.18 MPa
Strain Hardening Parameter	n	0.71

When the laminated plates composed of four layers are loaded transversely, yield points for transverse load are given in Table 4.1.2.

	$(0^o/90^o)_2$ S and A
Yield point MPa	S 0.0412 and A 0.0384

Table 4.1.2. The yield points of the laminated plates loaded transversely.
(S. Symmetric, A. Antisymmetric)

As seen from this Table the yield point in the symmetric cross-ply laminated plate is higher than in the antisymmetric cross-ply laminated plate. The yield points in the antisymmetric angle-ply laminated plates are higher than the yield points in angle-ply symmetric laminated plates.

In symmetric cross-ply and angle-ply symmetric laminated plates the maximum stress components are at the upper and lower surfaces, and they are equal absolutely. Therefore, in symmetric laminated plates, stress components are given at the upper surface only at node 85, at the mid-point. But in antisymmetic laminated plates, stress components are given at the upper and lower surfaces, because they are different at these surfaces.

	Layer	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Elas-Plas Stresses	0^o	17.346	3.530	-0.005	- 0.01 4	- 0.03 0
Elastic Stresses	0^o	18.757	5.144	-0.007	- 0.00 7	- 0.03 2
Residual Stresses	0^o	-1.412	- 1.613	-0.007	- 0.00 7	- 0.02

Table 4.1.3. Elastic-plastic, elastic and residual stress components in $(0^o/90^o)_2$ cross-ply symmetric laminated plate for 100 iterations

	Layer	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Elastic-Plastic Stresses	0^o	18.437	3.137	-0.04	-0.02	- 0.03
Elastic Stresses	0^o	20.589	5.646	-0.04	-0.08	- 0.03
Residual Stresses	0^o	-2.152	-2.58	-0.02	-0.01	- 0.01

Table 4.1.4. Elastic-plastic, elastic and residual stress components in $(0^o/90^o)_2$ cross-ply symmetric laminated plate for 150 iterations.

	Layer	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Elastic-Plastic Stresses	0°	19.384	2.701	-0.06	-0.02	-0.04
Elastic Stresses	0°	22.421	6.148	-0.04	-0.01	-0.03
Residual Stresses	0°	-3.037	-3.447	-0.02	-0.01	-0.02

Table 4.1.5. Elastic-plastic, elastic and residual stress components in $(0^\circ/90^\circ)_2$ cross-ply symmetric laminated plate for 200 iterations.

As seen from these Tables, when we increase the iteration numbers the residual stresses become greater.

	Layer	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Elastic-Plastic Stresses	0°	15.750	3.976	-0.001	-0.053	-0.053
	90°	-3.998	-15.61	0.016	-0.056	-0.056
Elastic Stresses	0°	17.223	6.438	0.002	-0.041	-0.042
	90°	-6.392	-17.09	0.016	-0.041	-0.042
Residual Stresses	0°	-1.473	-2.462	-0.003	-0.012	-0.011
	90°	2.394	1.394	0.000	-0.015	-0.014

Table 4.1.6. Elastic-plastic, elastic and residual stress components in $(0^\circ/90^\circ)_2$ cross-ply antisymmetric laminated plate at node 85 for 100 iterations

	Layer	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Elastic-Plastic Stresses	0°	16.250	3.824	-0.01	-0.05	- 0.05
	90°	-3.298	-15.92	0.012	-0.06	- 0.06
Elastic Stresses	0°	17.823	6.838	0.008	-0.04	- 0.04
	90°	-4.526	-5.08	0.011	-0.04	- 0.03
Residual Stresses	0°	-1.127	-2.82	-0.02	-0.01	- 0.01
	90°	1.824	2.185	0.000	-0.02	- 0.02

Table 4.1.7. Elastic-plastic, elastic and residual stress components in $(0^\circ/90^\circ)_2$ cross-ply antisymmetric laminated plate at node 85 for 150 iterations

As seen from these Tables, residual stress components are different at the upper and lower surfaces in antisymmetric cross-ply laminated plates.

	Layer	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Elastic-Plastic Stresses	0°	16.907	3.666	0.000	-0.05	-0.05
	90°	-2.736	-16.22	0.006	-0.08	-0.08
Elastic Stresses	0°	18.836	7.041	0.002	-0.04	-0.04
	90°	-2.233	-13.60	-0.05	-0.04	-0.04
Residual Stresses	0°	-0.929	-3.37	-0.02	-0.01	-0.01
	90°	0.504	2.624	-0.01	-0.03	-0.03

Table 4.1.8. Elastic-plastic, elastic and residual stress components in $(0^\circ/90^\circ)_2$ cross-ply antisymmetric laminated plate at node 85 for 200 iterations

Stresses	Iteration Number	σ_x MPa	σ_y MPa	τ_x MPa	τ_{yz} MPa	τ_{xz} MPa
Elatic-Plastic Stresses	100	17.34	3.530	-0.04	-0.01	-0.030
	150	18.43	3.137	-0.02	-0.02	-0.030
	200	19.38	2.701	-0.06	-0.02	-0.042
Elastic Stresses	100	18.75	5.144	-0.05	-0.07	-0.032
	150	20.59	5.646	-0.05	-0.07	-0.035
	200	24.42	-6.148	-0.05	-0.08	-0.039
Residual Stresses	100	-1.41	-1.613	-0.01	-0.06	-0.002
	150	-2.15	-2.508	-0.02	-0.02	-0.001
	200	-3.03	-3.447	-0.03	-0.02	-0.003

Table 4.1.9. Elastic-plastic, elastic and residual stress components for symmetric cross-ply laminated plate $(0^\circ/90^\circ)_2$ at node 85 for 100, 150 and 200 iterations.

Stresses	Iteration Number	σ_x MPa	σ_y MPa	τ_x MPa	τ_{yz} MPa	τ_{xz} MPa
Elatic-Plastic Stresses	100	15.750	3.976	0.001	-0.05	-0.05
	150	15.750	3.976	0.001	-0.05	-0.05
	200	16.907	3.666	0.000	-0.05	-0.05
Elastic Stresses	100	17.223	6.438	0.002	-0.04	-0.04
	150	17.223	6.438	0.002	-0.04	-0.04
	200	18.836	7.041	0.002	-0.04	-0.04
Residual Stresses	100	-1.473	-2.46	-0.02	-0.01	-0.01
	150	-1.473	-2.46	-0.02	-0.01	-0.01
	200	-1.929	-3.37	-0.02	-0.01	-0.01

Table 4.1.10. Elastic-plastic, elastic and residual stress components for antisymmetric cross-ply laminated plate $(0^\circ/90^\circ)_2$ at node 85 for 100,150 and 200 iterations.

	Laminated Plates	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Upper surface	$(0^o/90^o)_2$	17.34	3.530	-0.04	-0.02	-0.03
Lower surface	$(0^o/90^o)_2$	-17.34	-3.53	0.005	-0.01	-0.04

Table 4.1.11. Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Upper surface	$(0^o/90^o)_2$	15.750	3.976	-0.001	-0.053	-0.053
Lower surface	$(0^o/90^o)_2$	-3.998	-15.6	0.016	-0.05	-0.05

Table 4.1.12. Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Upper surface	$(0^o/90^o)_2$	18.437	3.13	-0.02	-0.02	-0.03
Lower surface	$(0^o/90^o)_2$	-18.43	-3.13	0.02	-0.02	-0.03

Table 4.1.13. Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 150 iterations.

	Laminated Plates	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Upper surface	$(0^o/90^o)_2$	15.75	3.98	-0.02	-0.05	-0.05
Lower surface	$(0^o/90^o)_2$	-3.99	-15	0.01	-0.05	-0.05

Table 4.1.14. Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 150 iterations.

	Laminated Plates	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Upper surface	$(0^o/90^o)_2$	19.35	2.71	-0.6	-0.02	-0.04
Lower surface	$(0^o/90^o)_2$	3.495	11.2	0.03	-0.03	-0.07

Table 4.1.15. Elastic-plastic stress components at upper and lower surfaces of the symmetric laminated plates for 200 iterations.

	Laminated Plates	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Upper surface	$(0^o/90^o)_2$	16.967	3.666	-0.0	-0.057	-0.053
Lower surface	$(0^o/90^o)_2$	2.736	16.218	0.06	-0.082	-0.082

Table 4.1.16. Elastic-plastic stress components at upper and lower surfaces of the antisymmetric laminated plates for 200 iterations

	Laminated Plates	σ_x MPa	σ_y MPa	τ_{xy} MPa	τ_{yz} MPa	τ_{xz} MPa
Upper surface	$(0^o/90^o)_2$	-1.412	1.61	0.001	-0.04	-0.02
Lower surface	$(0^o/90^o)_2$	1.412	1.61	0.01	-0.04	-0.02

Table 4.1.17. Residual stress components at upper and lower surfaces of the symmetric laminated plates for 100 iterations.

The absolute value of the stress components in the symmetric laminated plates at the upper and lower surfaces are the same for σ_x , σ_y and τ_{xy} stress components, it is different at the lower and upper surfaces in the antisymmetric laminated plates.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Upper surface	$(0^o/90^o)_2$	-1.47	-2.46	0.003	-0.012	-0.011
Lower surface	$(0^o/90^o)_2$	-2.39	-1.39	0.000	-0.015	-0.014

Table 4.1.18. Residual stress components at upper and lower surfaces of the antisymmetric laminated plates for 100 iterations.

	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Upper surface	$(0^o/90^o)_2$	-3.04	-3.44	-0.02	-0.017	-0.003
Lower surface	$(0^o/90^o)_2$	3.04	3.447	-0.02	-0.017	-0.003

Table 4.1.19. Residual stress components at upper and lower surfaces of the symmetric laminated plates for 200 iterations.

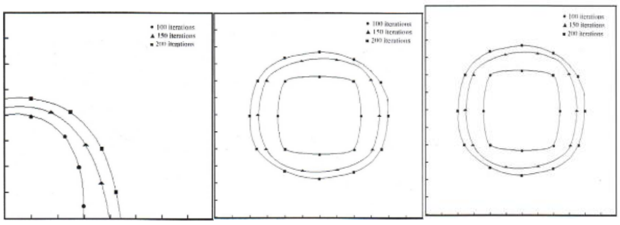
	Laminated Plates	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
Upper surface	$(0^o/90^o)_2$	-1.47	-2.46	0.003	-0.012	-0.011
Lower surface	$(0^o/90^o)_2$	-2.39	-1.39	0.000	-0.015	-0.014

Table 4.1.20. Residual stress components at upper and lower surfaces of the antisymmetric laminated plates for 200 iterations.

As seen from these Tables, the intensity of the residual stress component in cross-ply $(0^o/90^o)_2$ laminated plate is greater than in the angle-ply laminated plates.

V. FIGURES

The expansion of plastic zone in symmetric, $(0^o/90^o)_2$ laminated plates;



a) laminated plates b) upper surface c) lower surface

The expansion of plastic zone is the similar at the lower and upper surfaces for cross- ply symmetric, $(0^o/90^o)_2$,and antisymmetric, $(0^o/90^o)_2$, laminated plates.

VI. CONCLUSIONS

Elastic-plastic stress analysis has been carried out by using the first order shear deformation theory in thermoplastic composite laminated plates under transverse loading. The expansion of plastic zone and residual stresses are obtained in symmetric and antisymmetric cross-ply laminated plates. The following results of elastic-plastic stress analysis are found.

For Unidirectionally Galvanized Wire Reinforced Composite Laminated Plates:

a) The yield point in the symmetric cross-ply laminated plate is higher than in the antisymmetric cross-ply laminated plate.

b) Residual stress components in cross- ply laminated plates are great

c) Cross-ply laminated plates mechanical properties are more different than other different angle laminated plates.

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