

# A Multi-Channel GLR Detector for High-Frequency Surface Wave Radar

M. R. Moniri, M. Nezamabadi

Yadegar-e-Imam Khomeini (RAH) Shahre Rey Branch, Islamic Azad University  
Tehran, Iran

**Abstract**—This article explores the M-AR-GLR detector with regard to the Gaussian spectrum on the actual data of the sea clutter that is collected by multi-channel high frequency surface wave radar (HFSWR). We will show a large number of elements have no effect on the covariance matrix of interference and we can only consider two adjacent elements. Also, whatever spatial dependency of received samples is less, detector performance will improve. Findings show that the performance of detection will improve by fewer numbers of the interference samples. Also in the small signal to noise ratios, this detector can be better than Kelly's GLR detector [1].

**Keywords**—Multi-channel; HFSWR; Auto-Regressive process; Gaussian Spectrum;

## I. INTRODUCTION

In the real world, clutter leads to false detection of small targets and it prevent the development of detection, tracking and searching systems. In the sea environment, targets such as missiles and ships could be placed in the sky, horizon and the sea which may have different clutters such as clouds, horizon line and shining sea. False detection caused by the cloud clutter will be removed by classification based on location feature and clutters that have horizon line source can be solved by using time stable filter [2]. HFSWR, is a monitoring instrument that due to having detecting capability of stealth targets as well as targets are flying at low altitude, ARM missiles attack prevention and having a large surveillance area, plays an important role in the modern air defense system [3]. These radars that operate in the 3-15 MHz frequency, in addition to the line of sight propagation, in the propagation mode of surface wave are also able to reveal the targets that are located in over the horizon and at a long distance [4]. However, doing this, despite the clutter and interference sources present in the environment is limited. Sea clutter and especially its Bragg element is the dominant source of clutter in the operating environment of HFSWR. The sea clutter can be modeled by non-zero Doppler effect which is spread near to zero Doppler frequency. Low speed targets take a Doppler frequency nearby sea Bragg Doppler frequency and thus the conventional Doppler processing are not just sufficient to detect them [5,6].

In S bands sea radar (operating frequency 2-4 GHz), range cell is a few meters in a few meters and given that targets such as ships and vessels, occupies several cells with these dimensions, as a result, SCR in these radars, is a good value. However, in HFSWR (given that they have longer range than the other sea radars), we have to use high portion. For this purpose, according to the radar equation, there are ways that can be pointed to cases such as using large number of pulses and a large pulse width which increase the SNR. However, when using a large pulse width,

SCR is reduced. Also, for have a long range, narrow bandwidth is used that causes to range cell is increased. Similarly, since this radar works in the HF band, its beam width is wide. As a result, SCR is reduced and detection process will be difficult. Typically, range cell in HFSWR is 1.5 km in 4-5 km.

Unfortunately, conventional array processing are only able to remove interference on the side lobe [7]. However, adaptive beam forming that updates own space filtering weigh vector in CPI, can remove HF interference that HFSWR behaviors as time changeable or as non-homogeneous in terms of space structure [4]. Space-Time Adaptive Processing (STAP) is a processing scheme in the fields of space and time that can remove adaptively broad band jamming signal [8]. Based on the STAP processing, Zhang, Yang, Deng [9] presented an algorithm where based on angle – Doppler map, targets within heterogeneous clutter of surface wave radar will be detected. Fabrizio and Farina [10] presented Time-Varying STAP (TV-STAP) algorithm that is based on Stochastic Constraint STAP(SC-STAP) algorithm, but in terms of computational burden is more efficient than the SC-STAP algorithms, this algorithm was proposed by Abramovich, Spencer, Anderson, Gorokhov [11,12] for HFSWR. Other adaptive detection techniques based on STAP processing are Joint Domain Localized (JDL) process [13], Direct Data Domain ( $D^3$ ) [14] (was evaluated by Cristallini, Bürger [15]), Parametric Adaptive Matched Filter (PAMF) [16] and Multistage Wiener Filter (MWF) [17]. Saleh [18] used these algorithms on real data obtained from HFSWR. Another approach in this field is detection theory and most popular theory about that is GLR adaptive detector proposed by Kelly [1]. In this test, a decision about the presence or absence of the target is based on a comparison of maximum likelihood function with a threshold level. The likelihood function is achieved based on Maximum Likelihood estimation of unknown parameters at the detection problem. Based on asymmetric structures, we consider an algorithm for adaptive detection problem in Gaussian noise with asymmetric covariance matrix for Multi-Input Multi-Output (MIMO) radars with space distributed antenna [19] and Fabrizio and Farina [20] use this detector for surface wave radar. One of the ways to implement adaptive detector is using Auto-regressive (AR) model for Gaussian clutter [21]. On this basis, Moniri and Nayebi [22] provided M-AR-GLR multi-channel detector for Airborne Radar. In this detector, according to the GLR test definition, the maximum conditional probability density functions under the premise of the absence of the target and the presence of the target should be calculated on the unknown parameter (Here, coefficient matrix of vector processing of M-order AR model that is used to model interference and also its signal power). However, in multi-channel mode, its estimating in this way is not possible because process factor elements are independent. Hence, the amplitude of the input signal was considered clear and constant. In Single-channel mode (AR-GLR detector) because the process coefficients is formed by

an element, Simplifying the relations and imaging the primary data on signal empty space to eliminate the target signal effect is possible and then bleach can be established. This detector is called AR-GC-GLR detector that was presented by Moniri, Nayeibi [23]. In this detector, to improve detector efficiency and to reduce its need to secondary data, the interference covariance matrix form was considered as Gaussian.

In this study, according to the suitability of the AR process to model the sea clutter[10], we're going to investigate the M-AR-GLR detector for sea clutter that is collected by multi-channel surface wave radar and also by considering the Gaussian form for interference covariance function form. In section II, the respective formulas will be introduced. Next, in section III the results of applying it on real data of HFSWR are presented and finally conclusions will be discussed in IV.

## II. M-AR-GC-GLR

In this section, we discuss the multichannel detection problem addressed in this paper. All notations used are the same [22].

The multichannel binary detection problem is expressed as:

$$H_0 : \begin{cases} y_0(n) = \underline{n}(n) \\ y_k(n) = \underline{n}_k(n) \end{cases} \quad n = 1, \dots, N \quad (1)$$

$$H_1 : \begin{cases} y_0(n) = \alpha \underline{s}(n) + \underline{n}(n) \\ y_k(n) = \underline{n}_k(n) \end{cases} \quad k = 1, \dots, K$$

where,  $y_0(n)$  is  $J \times 1$  received observation vector and  $y_k(n)$  are  $J \times 1$  received observation vectors for  $k = 1, 2, \dots, K$  which defined as secondary vectors and included no targets. The discrete received interference process  $\underline{n}(n)$  is considered to be a zero mean, wide-sense jointly stationary  $J \times 1$  complex base band vector consisting of  $J$  components with positive-definite covariance  $K_{uu}$ :

$$K_{uu} = \sigma_u^2 I_{J \times J} \quad (2)$$

$\underline{n}(n)$  and  $\underline{n}_k(n)$  are modeled as:

$$\underline{n}(n) = -\sum_{p=1}^M A_M^H(p) \underline{n}(n-p) + \underline{u}_n(n) \quad (3)$$

That are multichannel forward AR processes of order  $M$  with parameters  $\underline{A}$  and  $\sigma_u^2$ ,  $AR(M, \underline{A}, \sigma_u^2)$ .  $\underline{A} = [A_1 A_2 \dots A_M]$  is the AR parameter vector,  $A_M^H(p)$  is the  $p^{\text{th}}$ ,  $J \times J$  matrix coefficient for AR vector process that is dependent on unknown parameters  $\sigma^2$ ,  $\rho_{n_{i,j}}$ ,  $\lambda_{n_{i,j}}$  and  $\ell_{n_{i,j}}$  that are variance of interference Gaussian spectrum, temporal correlation between the samples received from  $i^{\text{th}}$  and  $j^{\text{th}}$  channels, spatial correlation between the samples of  $i^{\text{th}}$  and  $j^{\text{th}}$  channels and reception delay from  $i^{\text{th}}$  to  $j^{\text{th}}$ .

$\sigma_u^2$  is variance of zero-mean discrete complex white Gaussian noise driving vector  $J \times 1$   $\underline{u}_n(n)$  with covariance matrix  $K_{uu}$ . We assume that  $\underline{A}$  and  $\sigma_u^2$  are also unknown but  $\underline{A}$  can be expressed in terms of other unknown parameters which are derived from correlation function shape. In this work, at first we fix the form of the correlation function and then apply the GLR theory on unknown

parameters. We consider the correlation function  $R(l)$  which is defined as:

$$R(l) = E[n_{k,n} n_{k,n-l}^*] \begin{cases} k=0,1,\dots,K \\ n=1,2,\dots,N \end{cases} \quad (4)$$

This means a similar correlation function for interference in both primary and secondary data samples. We now consider functional forms for modeling  $R(l)$  that will capable us to obtain a variety of spectral distributions.  $R(l)$  is a matrix  $J \times J$ . We express this function for  $i^{\text{th}}$  and  $j^{\text{th}}$  channel,  $R_{n_{ij}}(\ell)$ :

$$R_{n_{ij}}(\ell) = E[n_{ij}(n) n_{ij}^*(n-\ell)] = K_{n_{ij}} f_n(\lambda_{n_{ij}}, \ell, \ell_{n_{ij}}) \exp(j\theta_{n_{ij}}(\ell)) \quad (5)$$

Here we use Gaussian form that is suitable model for our measurements. So:

$$R_{n_{ij}}(\ell) = |\rho_{n_{ij}}| \sigma^2 \left( \lambda_{n_{ij}} \right)^{\ell/\ell_{n_{ij}} - 1} \quad (6)$$

Now, the Yule-Walker equation can be used to determine the AR coefficients of the process, as could be derived:

$$\begin{bmatrix} R(0) & R(1) & \dots & R(M) \\ R(-1) & R(0) & \dots & R(M-1) \\ \vdots & \vdots & \ddots & \vdots \\ R(-M) & R(-M+1) & \dots & R(0) \end{bmatrix} \begin{bmatrix} A_1^H \\ A_2^H \\ \vdots \\ A_M^H \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \\ \vdots \\ R(M) \end{bmatrix} \quad (7)$$

The known received signal  $\underline{s}(n)$  is expressed as:

$$\underline{s}(n) = \left[ \exp(i 2\pi(n-1) \frac{2v}{\lambda \text{ PRF}}) \dots \exp(i 2\pi(n-1) \frac{2v}{\lambda \text{ PRF}} + i 2\pi(J-1) \frac{d \sin(\theta)}{\lambda}) \right]^T \quad (8)$$

where T stands for transpose. This vector corresponds to a target whose Doppler is  $\Omega$ , which is assumed to be known. In this paper,  $\alpha$  is an known complex amplitude of reflected signal from the target, where  $\lambda = c/f$  is the wavelength and  $c$  is the light velocity,  $v$  and  $\theta$  are the radial velocity and direction of target, respectively and  $d$  is the inter-element spacing.

$H_1$  and  $H_0$  denotes the hypotheses under which the signal is present or absent, respectively. Under hypothesis  $H_0$ , the multivariate joint Gaussian density can be written as the product of product of conditional densities, so that:

$$p_0 = p_x(\underline{x}_0, \dots, \underline{x}_K | H_0, \lambda_{n_{i,j}}, \rho_{n_{i,j}}, \ell_{n_{i,j}}, \sigma^2, K_{uu}, \alpha = 0) \quad (9)$$

$$= p_x(\underline{x}_0 | H_0, \lambda_{n_{i,j}}, \rho_{n_{i,j}}, \ell_{n_{i,j}}, \sigma^2, K_{uu}, \alpha = 0) \times \left\{ \prod_{k=1}^K p_x(\underline{x}_k | \lambda_{n_{i,j}}, \rho_{n_{i,j}}, \ell_{n_{i,j}}, \sigma^2, K_{uu}) \right\}$$

where:

$$\underline{x}_0(n) = \underline{y}(n) \quad (10)$$

$$\underline{x}_k(n) = \underline{y}_k(n) \quad \text{for } k=1, \dots, K$$

Under hypothesis  $H_1$  the multivariate joint Gaussian density can be written by the same routine, but:  $\underline{x}_0(n) = \underline{y}(n) - \alpha \underline{s}(n)$  and  $\underline{x}_k(n)$  has the same definition as (10). Now, the GLRT will be formulated for multiple observation:

$$L_{GLR} = \frac{\max_{\sigma_u^2, \lambda_{n_i, j}, \rho_{n_i, j}, \ell_{n_i, j}, \sigma^2, \alpha} p_1}{\max_{\sigma_u^2, \lambda_{n_i, j}, \rho_{n_i, j}, \ell_{n_i, j}, \sigma^2} p_0} \langle \eta \rangle \quad (11)$$

So, the likelihood ratio, described by:

$$\Lambda = \ln(L_{GLR}) = \max_{\sigma_u^2, \lambda_{n_i, j}, \rho_{n_i, j}, \ell_{n_i, j}, \sigma^2, \alpha} \ln(p_1) - \max_{\sigma_u^2, \lambda_{n_i, j}, \rho_{n_i, j}, \ell_{n_i, j}, \sigma^2} \ln(p_0) \quad (12)$$

$$= -JN(K+1) \left( \ln(\pi \hat{\sigma}_{u_1}^2) - \ln(\pi \hat{\sigma}_{u_0}^2) \right)$$

We name (10) as multichannel autoregressive Gaussian GLR (M-AR-GC-GLR). In (10):

$$\hat{\sigma}_{u_0}^2 = \frac{1}{JN(K+1)} \left( \underline{u} - \underline{Y} \hat{A}_0 \right)^H \left( \underline{u} - \underline{Y} \hat{A}_0 \right) \quad (13)$$

$$\underline{Y} = \left[ Y_0^T \dots Y_K^T \right]^T ; Y_k = \left[ y_{k, M+1}^T \dots y_{k, N}^T \right]^T \quad (14)$$

$$Y_{k, n} = \begin{bmatrix} y_k^T(n-1) & 0_{1 \times J(J-1)} & y_k^T(n-2) & \dots & y_k^T(n-M) & 0_{1 \times J(J-1)} \\ 0_{1 \times J} & y_k^T(n-1) & 0_{1 \times J(J-1)} & \dots & y_k^T(n-M) & 0_{1 \times J(J-2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0_{1 \times J(J-1)} & y_k^T(n-1) & 0_{1 \times J(J-1)} & y_k^T(n-2) & \dots & y_k^T(n-M) \end{bmatrix}$$

$$\underline{u} = \left[ \underline{u}_0^T \dots \underline{u}_K^T \right]^T ; \underline{u}_k = \left[ y_k(M+1)^T \dots y_k(N)^T \right]^T \quad (15)$$

Also:

$$\hat{\sigma}_{u_1}^2 = \frac{1}{JN(K+1)} \left( \underline{u}' - \underline{Y}' \hat{A}_1 \right)^H \left( \underline{u}' - \underline{Y}' \hat{A}_1 \right) \quad (16)$$

$$\underline{Y}' = \left[ Y_0'^T \dots Y_K'^T \right]^T ; Y_0' = H Y_0 \quad (17)$$

$$\underline{u}' = \left[ \underline{u}'_0^T \dots \underline{u}'_K^T \right]^T ; \underline{u}'_0 = H \underline{u}_0$$

$\underline{Y}_0$  and  $\underline{u}_0$  are defined in (14) and (15). H is defined as

$$H = I - \frac{\underline{\psi} \underline{\psi}^H}{\underline{\psi}^H \underline{\psi}} \quad (18)$$

which is the projection matrix of null space of  $\underline{\psi}$  where:

$$\underline{\psi}_n = \underline{s}(n) - \sum_{p=1}^M A_M^H(p) \underline{s}(n-p) \quad (19)$$

The structure of detector is shown in Fig. 1.

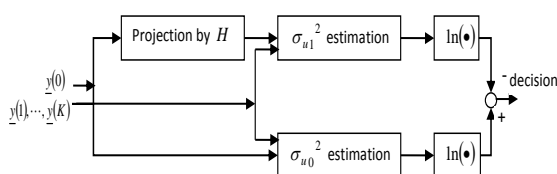


Fig. 1: The M-AR-GC-GLR block diagram [22]

### III. EXPERIMENTAL RESULTS

The sea clutter can be modeled with AR process in order of 2 [10]. We apply the M-AR-GC-GLR detector for actual sea clutter that is gathered by an HFSWR. This is a phased array radar and operates at lower half of the HF band. The sampling frequency is 5Hz.

Fig. 2 demonstrates probability of detection ( $P_d$ ) versus probability of false alarm ( $P_{fa}$ ) for several number of pulses. In HFSWR, we must receive number of pulses such as, 300 to 1000.

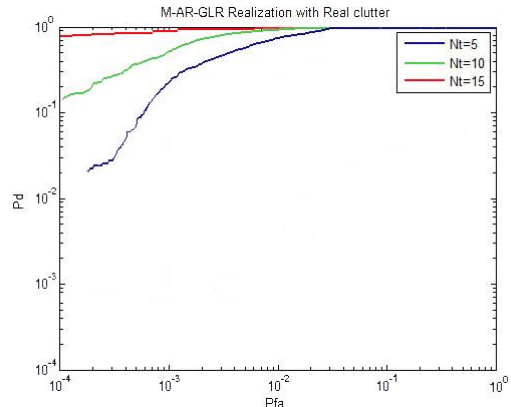


Fig 2:  $P_d$  versus  $P_{fa}$  for various numbers of received pulses

As we see, this detector needs the fewer number of pulse.

The dependence of detection performance to secondary vectors is verified in fig.3. It was observed that the performance of the M-AR-GC-GLR detector reduced with increasing numbers of secondary data that may be because of large range cell in HFSWR, with increasing numbers of secondary data, it faced by the edge of clutter and structure of clutter comes out of the uniformity. So in situations that we don't have access to secondary data, this algorithm can be effective. Therefore, the application of this

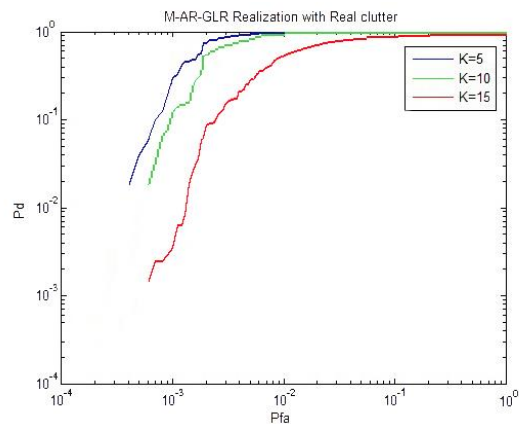
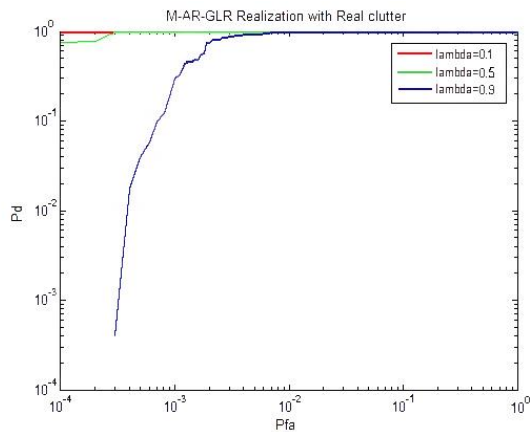


Fig 3:  $P_d$  versus  $P_{fa}$  for various numbers of secondary data

algorithm in HFSWR with the fewer number of secondary data may be useful.



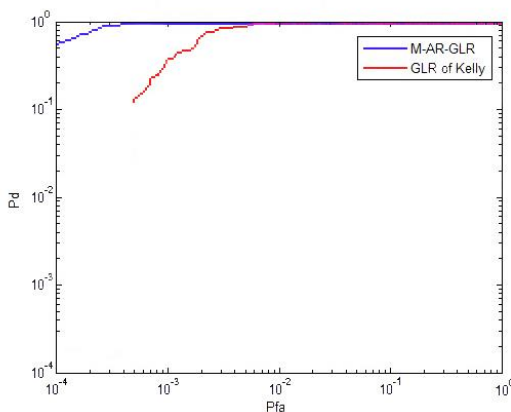
**Fig 4:  $P_d$  versus  $P_{fa}$  with versus spatial correlation**

Fig. 4 illustrates the improvement of the detection performance by decreasing the spatial correlation parameter ( $\lambda$ ).

This may be that in in single channel mode, since range cells is large (about a few kilometers), thus, a complete ring of sea clutter is received by the radar receiver. Sea waves in a part of this ring are progressive and have positive Doppler, on the other part, waves are divergent and have negative Doppler and on the other part, waves are stationary and have no Doppler. Therefore uniform structure of clutter disappears, and we have to consider larger amounts for  $\lambda$ , amounts close to one (for example 0.98). However, in multi-channel mode, range cell decreases and each cell finds its properties, hence,  $\lambda$  finds the smaller values close to zero (0.1).

We observed that the correlation function of non-adjacent elements can be ignored and the effect of two adjacent elements is just studied. So covariance matrix is converted to a  $4 \times 4$  matrix which reduces computational burden, especially on the Yule–Walker equations and AR process coefficients are obtained simply.

Fig. 5 demonstrates a comparison with Kelly's GLR and AR-GLRs detectors as  $P_d$  versus  $P_{fa}$ . We can see superiority of M-AR-GC-GLR. This superiority is the result of using a prior knowledge of being AR with a Gaussian correlation function in clutter modeling. But Kelly's GLR doesn't use this information, so it has a poor performance as compared with M-AR-GC-GLR.



**Fig 5: Comparison with Kelly's GLR and M-AR-GC-GLR**

#### IV. CONCLUSION

Targets detection in sea clutter by surface wave radar is an acute problem. Moniri et al. [22] presented M-AR-GLR detector for multi-channel airborne radar that has not been evaluated by the real data.

Studies have shown that sea clutter can be modeled by using Auto-Regressive (AR) process. In order to improve the signal to clutter ratio and to simplify of detection operation, we decided to use M-AR-GLR detector for real samples of sea clutter that is collected by surface wave radar, and with regard to the Gaussian model for correlation function of interference samples, we called that M-AR-GC-GLR, and here, we considered 2 elements ( $j = 2$ ) for interference samples of second-order AR process. We observed that in the correlation function matrix of received interference samples, the correlation function of non-adjacent elements can be ignored and the effect of two adjacent elements is just studied. So covariance matrix is converted to a  $4 \times 4$  matrix which reduces computational burden, especially on the Yule – Walker equations and AR process coefficients are obtained simply.

We showed that as much as amount of space dependence of received data is less, the detector performance will be better and the reason for this may be the fact that in single channel mode, since range cells is large (about a few kilometers), thus, a complete ring of sea clutter is received by the radar receiver. Sea waves in a part of this ring are progressive and have positive Doppler, on the other part, waves are divergent and have negative Doppler and on the other part, waves are stationary and have no Doppler. Therefore uniform structure of clutter disappears, and we have to consider larger amounts for  $\lambda$ , amounts close to one (for example 0.98). However, in multi-channel mode, range cell decreases and each cell finds its properties, hence,  $\lambda$  finds the smaller values close to zero (0.1).

It was observed that the performance of the M-AR-GC-GLR detector reduced with increasing numbers of secondary data. Probably since range cell in surface wave radar is large, with increasing numbers of secondary data, it faced by the edge of clutter and space clutter comes out of the uniformity. So in places that we don't have access to secondary data, this algorithm can be effective. Therefore, this algorithm in surface wave radar with the low number of secondary data can be useful.

This detector has a good performance in the low number of pulses and it works better than GLR detector of Kelly, and as we have seen, with increasing numbers of pulses, the detector performance is also better.

Finally, M-AR-GC-GLR detector can be appropriate for HFSWR.

#### ACKNOWLEDGMENT )

The authors are thankful to JMEST Journal for the support to develop this document.

#### REFERENCES

- [1] E. J., Kelly, "An Adaptive Detection Algorithm", IEEE Trans. AES-22, pp. 115, 1986
- [2] S. Kim, J., Lee, "Small Infrared Target Detection by Region-Adaptive Clutter Rejection for Sea-Based Infrared Search and Track", sensors, Vol. 14, No. 7, pp. 13210-13242, 2014
- [3] Y. Li, Y. Wei, R. Xu, T. Chu, Z. Wang, "Space-Time Characteristics and Experimental Analysis of Broadbanding First-order Sea Clutter in HF Hybrid Sky-surface Wave Radar", RADIOENGINEERING, Vol. 23, No. 3, pp. 831-841, 2014



- [4] A. G. Fabrizio, A. B. Gresham, M. D. Turley, "Non-Stationary Interference Cancellation in HF Surface Wave Radar", IEEE Radar Conference, pp. 672-677, 2003
- [5] L. Sevgi, A. Ponsford, H. C. Chan, "An integrated maritime surveillance system based on high-frequency surface-wave radars, part 1: theoretical background and numerical simulations", IEEE Antennas and propagation Magazine, Vol. 43, No. 5, pp. 28-43., October 2001
- [6] A. M. Ponsford, J. Wang, "A review of high frequency surface wave radar for detection and tracking of ships", Turkish Journal of Electrical Engineering & Computer Sciences, Vol.18, No.3,pp. 409-428, 2010
- [7] Z. Geng, H. Deng, B. Himed, "Adaptive Radar Beamforming for Interference Mitigation in Radar-Wireless Spectrum Sharing", IEEE Signal Processing Letters, Vol. 22, No. 4, pp. 372-382, 2015
- [8] W. Dan, H. Hang, Q. Xin, "Space-Time Adaptive Processing Method at Subarray Level for Broadband Jammer Suppression", Microwave Technology & Computational Electromagnetics (ICMTC), International Conference, May 2011
- [9] X. Zhang, Q. Yang, W. Deng, "Weak Target Detection within the Nonhomogeneous Ionospheric Clutter Background of HFSWR Based on STAP", International Journal of Antennas and Propagation, Article ID:382516,11 pages, 2013
- [10] A. G. Fabrizio, A. Farina, "Time-Varying STAP for Nonstationary Hot Clutter Cancellation", IEEE International Conference on Acoustic, Speech and Signal Processing(ICASSP), pp. 6062-6066, 2014
- [11] Y. I. Abramovich, N. K. Spencer, S. J. Anderson, A. Y. Gorokhov, "Stochastic-constraints method in Nonstationary Hot-clutter cancellation – Part I : Fundamentals and supervised Training Applications", IEEE Transactions on Aerospace and Electronic Systems, Vol. 34, No .4, pp. 1271-1292, 1998
- [12] Y. I. Abramovich, N. K. Spencer, S. J. Anderson, A. Y. Gorokhov, "Stochastic-constraints method in Nonstationary Hot-clutter cancellation – Part II: Fundamentals and supervised Training Applications", IEEE Transactions on Aerospace and Electronic Systems, Vol. 34, No. 4, pp. 1271-1292, 2000
- [13] R. Adve, T. Hale, M. Wicks, "Joint domain localized adaptive processing in homogeneous and non-homogeneous environments Part I: Homogeneous environments," IEE Proc. Radar Sonar Navig. , Vol. 147, pp. 57–65, April 2000
- [14] J. E. Yang, J. Chun, R. Adve, "A hybrid D3-Sigma Delta STAP algorithm in non-homogeneous clutter," , IEEE International Radar Conf., October 2007
- [15] D. Cristallini, W. Bürger, "A Robust Direct Data Domain Approach for STAP", IEEE International on Signal Processing, Vol. 60, No. 3, March 2012
- [16] J. Roman, M. Rangaswamy, D. Davis, Q. Zhang, B. Himed, J. Michels, "Parametric adaptive matched filter for airborne radar applications," IEEE Transactions on Aerospace and Electronic Systems, Vol. 36, no. 2, pp. 677–692 , 2000
- [17] J. Goldstein, I. Reed, L. Scharf, "A Multistage Representation of the Wiener Filter Based on Orthogonal Projections", IEEE Transactions on Information Theory, Vol. 44, No. 7, pp. 2943-2959, 1998
- [18] O. Saleh, "Adaptive Processing in High-Frequency Surface WAVE RADAR", Thesis of Master of Applied Science, University of Toronto, 2008
- [19] J. Liu, H. Li, B. Himed, "Persymmetric adaptive target detection with distributed MIMO radar", IEEE Transaction on Aerospace and Electronic Systems ,Vol. 51, No. 1, pp. 484-488, 2015
- [20] A. G. Fabrizio, A. Farina, "GLRT-Based Adaptive Doppler Processing for HF Radars Systems", IEEE International Conference on Acoustic, Speech and Signal Processing (ICASSP), pp. II949 – II952, 2007
- [21] A. Sheikhi, M. M. Nayebi, M. R. Aref, "Adaptive Detection Algorithm for Radar Signals in Autoregressive interference", IEE Proc. on Radar, Sonar Navigation, Vol. 145, No. 5, pp. 309-314. 1998
- [22] M. R. Moniri, M. M. Nayebi, A. Sheikhi, "A Multichannel Auto-Regressive GLR Detector for Airborne Phased Array Radar application", IEE Radar Conference., pp. 206-211, 2003
- [23] M. R. Moniri, M. M. Nayebi, A. Sheikhi, "Adaptive Signal Detection in Auto-Regressive Interference with Gaussian Spectrum", Iranian Journal of Electrical & Electronic Engineering, Vol.4, No. 4, pp.140-149, 2008
- [24] J. H. Michels, "Multichannel Detection Using the Discrete-Time Model-Based Innovations Approach", Ph.D. Thesis, Syracuse University, May, 1991