

Sequencing N – Jobs On A Single Machine For A Flow Production Process(A Case Study Application)

Tobinson A. Briggs

Dept. of Mechatronic Engineering
University of Port Harcourt
Port Harcourt, Nigeria

E-mail: tobinson.briggs@uniport.edu.ng

Avery Tekena

Dept. of Mechanical engineering
University of Port Harcourt
Port Harcourt, Nigeria

E-mail: tekena.avery@uniport.edu.ng

Abstract—This work used production sequencing theory to access industrial performance in an existing system (a paint manufacturing industry); the objective is to measure the performance by adopting a comparative approach to identify the best performing sequencing rule among the following - FCFS, SPT, EDD, and FPFs. Computer codes were written in MATLAB 7.0 which is used to sequence the given jobs in accordance with the demands of these rules. Results indicate that SPT followed by EDD performed better than the other rules adopted in the comparative analysis.

Keywords—sequencing-rule, comparative analysis, Makespan, processing time, production planning and control

I. INTRODUCTION

As an all important part of manufacturing and service industries, production planning is aimed at finding an optimal usage of scarce resources. To manage productivity, modern industries adopt effective sequencing and scheduling ensure on-time delivery, reduce inventory, cut lead times and improve the utilization of bottleneck resources.

Several industrial practitioners over the years have largely contributed and adopted various techniques to control production. Beginning with the likes of Fredrick Taylor (1911), who defined the key planning functions and created a planning office to Henry Gantt (1916) who proposed the Gantt chart a tool for improving production scheduling and S.M. Johnson (1954), who modelled several mathematical equations to analyze the scheduling process of production operations by analyzing the properties of an optimal solution for a two-stage, three-stage or more flow shop developing an intelligent algorithm that generates optimal solutions for the different types of flow shop mentioned above. His since then has stimulated several other researchers who had taken a look at different aspects of the production scheduling problem to further understand the nature of this problem. For instance, Jackson and Smith (1956), referenced Johnson's work in their independent research on the single and two-machine scheduling problems with due dates.

Following the work of Johnson, a great deal of research effort have been geared towards the development of best possible job schedules to optimize production and countless scholarly publications published to this effect in several scientific and engineering journals. Needless to say, there still exists a gap between scheduling theory and practice; even though in most cases researchers have used better problem solving to improve real-world production scheduling. Although results so far obtained and published in such scholarly journals may help give a better understanding on how to go about handling real and practical systems, there still exists a large disparity between the theory of scheduling and its practice. This work therefore, attempts to use production scheduling theory to access industrial performance in a real world production environment.

II. OBJECTIVE OF THE STUDY

This paper is a study conducted on a paint manufacturing industry in Nigeria, majored in the production of different colour of paints (water based or oil based – emulsion or gloss). The production line is divided into primary and secondary lines. The primary line (where this research is carried out) concentrates on the production of water based paints consisting of the following operations/processes:

- Material Weighing,
- Materials Mixing,
- Materials Dissolution,
- Materials filtering,
- Product inspection, and quality control and
- Product packaging.

III. ASSUMPTIONS

The following assumptions generally apply to building single machine sequencing models.

- a) The machine is always available throughout the production process without breakdowns and stops for maintenance.
- b) Jobs are processed on the machine, one after the other.
- c) Job processing time on the machine is known and does not depend on previous jobs.
- d) The time taken to setup a particular job is included in its processing time.

- e) Pre-emption is not allowed while performing any of the operations.
- f) Job related information including due date (D_j) and release time (R_j) is known in advance.
- g) All jobs arrive the production line at time zero ($R_j = 0$, for all j).

IV. TERMINOLOGIES USED IN THIS PAPER

The basic information required to describe the job orders (customers' request) are:

1. Processing time (P_j): The time required to complete processing job, j .
2. Ready time (R_j): The time at which job, j is available.
3. Due date (D_j): The time when job, j is expected by the customer.
4. Completion time (C_j): The time at which processing of job, j is completed.
5. Lateness (L_j): The time by which completion time of job, j exceeds its due date, given by ($C_j - D_j$).
6. Makespan (C_{max}): The time taken to finish processing of the last job in the system.

V. GENERAL FORMULATION

We intend to determine the effectiveness of the company's adopted sequencing rule, First-Pay-First-Served (FPFS) compared to First-Come-First-Served (FCFS) rule, Shortest Processing Time (SPT) rule, and Earliest-Due-Date (EDD) rule evaluating these rules for efficiency in the utilization of company's resources (materials, labour, equipment and storage space) and response to customers demand. For a production floor faced with the challenge of scheduling a set of jobs, $J = \{1, 2, \dots, n\}$ on a single machine to minimize total completion time. Taking into consideration our assumptions, we can formulate guiding equations for our objective functions.

Let the j job in the schedule to be sequenced be represented by j . Let the time it takes to arrive at the machine be R_j . The time required to process j on the machine is P_j units of time and that starts processing at S_j . From the assumption in (Section 3.2g), since release time for all jobs is assumed as zero, then,

$$S_j \geq R_j \quad (1)$$

Therefore, completion time of j becomes

$$C_j = S_j + P_j \quad (2)$$

Since $R_j = 0$, for all jobs, total completion time of

$$C_{tot} = \sum (C_1 + C_2 + \dots + C_n) = \quad (3)$$

The problem in scheduling language or Graham's notation is written as

$$1|r_j|\sum C_j \quad (4)$$

Here, we present tools and methods used to generating job processing time (P_j), due dates (D_j), in our MATLAB 7.0 code for the computation of job start time (S_j), completion time (C_j), make-span (C_{max}) and job lateness (L_j).

- Generating Processing Times (P_j) From Minimum and Maximum Limits

Our industrial partners gave us range of production limits regarding job processing times on their primary production line obtained in the factory record or production time book. We are told the time taken to process customers order based on quantity (small or large orders) in hours runs between 1.5 to 9.5 hours considering batching for large orders. These range of hourly values (maximum and minimum) of job processing time (P_j) were used to generate a set of 50 jobs, representing 50 customers order. These values are used to derive and evaluate the needed performance measures under consideration with respect to the proposed sequencing rules.

The generated processing time (P_j) falls within the range of 1hour 30mins to 9hour 30mins (i.e., 1.5 - 9.5 hours) and was generated using MATLAB 7.0, random number function:

$$P = (P_{max} - P_{min}) * rand(50,1) + P_{min} \quad (5)$$

P_j = Processing time of job j ;

P_{max} = Maximum limit of processing time used for the iteration;

P_{min} = Minimum limit of processing time used for the iteration;

rand = command for random number generation;

50 = Number of desired iterations.

- Generating Due Dates, (D_j) from Minimum and Maximum Limits

Since the information regarding due date (D_j) and release time (R_j) should be known in advance of production sequencing. We generated values for due dates in MATLAB 7.0 within the range of processing times (P_j) given by the equation of Oyetunji and Masahudu (2009), in their study Single Machine with Release Time. The equation adopted is given below

$$[(R_j + P_j) - (R_j + 2 * P_j)] \quad (6)$$

R_j = Release date, and

P_j = Processing time among the set of jobs for which due date is found.

Coding in MATLAB 7.0 the range of due dates is generated using equation (6), replacing processing times P with due date D .

$$D = (P_{jmax} - P_{jmin}) * rand(N, 1) + P_{jmin} \quad (7)$$

D_j = Due date of job, j, the time taken for j to complete processing

P_{jmax} = Maximum range of processing time for which due date is determined.

P_{jmin} = Minimum range of processing time for which due date is determined.

rand = command for random number generation;
 N = Number of iterations.

- Computing Start Times (S_j) of Jobs

To compute the start time we develop our model based on the fact that production does not start immediately an order is placed due to time required for the flow of information from one department to another which is involved in the production process – customer service desk, accounting, bank confirmation, production, etc., hence, job start time precedes its release date expressed as:

$$S_j \geq R_j \quad (8)$$

$$S_j = R_j + W_j \quad (9)$$

$$S_j = W_j \quad (10)$$

R_j = Release time of job j which equal zero for all jobs.

W_j = Job waiting time (amount of time job j, waits before machining).

For a non-preemptive schedule, job 1, sequenced in position J_1 , will finish processing on the machine before job 2, in position J_2 , in the order (J_1, J_2, \dots, J_n) meaning start time for job J_2 , becomes

$$S_2 = (R_1 + W_1) + P_1 \quad (11)$$

Substituting 1, for j, in equation (10) gives $S_1 = (R_1 + W_1)$ and transforms equation (11) to

$$S_2 = S_1 + P_1 \quad (12)$$

and

$$S_3 = S_2 + P_2 \quad (13)$$

Similarly, we compute the series from S_4, S_3, \dots, S_{50} .

- Computing Completion (C_j) and Total Completion Times of Jobs ($\sum_{j=1}^n C_j$)

Completion time of a job, j- the time at which its processing is completed. For a non-preemptive schedule, the completion time for job, j is,

$$C_j = R_j + P_j \quad (14)$$

From assumption (Section 3.3e), once a job begins processing, it is not interrupted until its processing is complete, then,

For, $j = 1$, equation (14) becomes

$$C_1 = R_1 + P_1 \quad (15)$$

For, $j = 2$,

$$C_2 = (R_1 + P_1) + P_2 \quad (16)$$

Substituting $C_1 = (R_1 + P_1)$, transforms equation (17) to equation (18) below

$$C_2 = C_1 + P_2 \quad (17)$$

and, for $j = 3$,

$$C_3 = C_2 + P_3 \quad (18)$$

Completion time for other jobs in the sequence is derived similarly in the order of C_4, C_5, \dots, C_{50} .

Total completion time for the processing of a complete job sequence becomes

$$C_{Total} = C_1 + C_2 + C_3 + \dots + C_n = \sum_{j=1}^n C_j \quad (19)$$

- Computing Total and Mean Flow Time (F_j) of Jobs

Also known as the turnaround time, it is the time a job spends in a given system (the difference between completion time and release time). For a zero single machine problem with zero release time, it is equivalent to the job completion time C_j , of individual jobs. The flow time in the given system is computed as

$$F_j = C_j - R_j \quad (20)$$

Substituting 1 for j in equation (20) gives

$$F_1 = C_1 - R_1 \quad (21)$$

Also, putting $j = 2$,

$$F_2 = C_2 - R_2 \quad (22)$$

For, $j = 3$,

$$F_3 = C_3 - R_3 \quad (23)$$

Values for F_4, F_5, \dots, F_{50} are computed in the same order.

For zero release time ($r = 0$), equations (21), (22), and (23) reduces to

For, $j = 1$,

$$F_1 = C_1 \quad (24)$$

For, $j = 2$,
 $F_2 = C_2$ (25)

and $j = 3$,
 $F_3 = C_3$ (26)

The total and mean flow time is calculated as

$$F_{Total} = (F_1 + F_2 + \dots + F_n) \equiv$$

$$(C_1 + C_2 + \dots + C_n) = \sum_{j=1}^n C_j \quad (27)$$

$$F_{Mean} = \frac{1}{n} \sum_{j=1}^{50} F_j \quad (28)$$

- Computing Lateness (L_j) of Jobs

The time by which completion of job, j , exceeds its due date or time is referred to as the job lateness. Mathematically, it is represented,

$$L_j = C_j - D_j \quad (29)$$

For, $j = 1$,
 $L_1 = C_1 - D_1$ (30)

For, $j = 2$,
 $L_2 = C_2 - D_2$ (31)

For, $j = 3$,
 $L_3 = C_3 - D_3$ (32)

Values of $j_4, j_5, \dots, j_{50}, L_4, L_5, \dots, L_{50}$ are calculated as above, and total lateness is expressed as

$$L_{Total} = L_1 + L_2 + \dots + L_n = \sum_{j=1}^{50} L_j \quad (33)$$

VI. ASSIGNING PRIORITY WEIGHTS (W_j) TO JOBS.

Assigning weights to production orders is not an easy task, because weights have to be near reality as possible. For weights that are too big increases unnecessarily the value and effect of the criteria under assessment, which will have an adverse effect on the coding under consideration; causing its values to vary widely from those of other sequencing rules. On the other hand, weights that are too small would also affect the result of the coding. For this, the value of the weights used in this work has to be near reality as possible.

For this, we refer to the study of Reeves (1995) who proposed a simple and straight forward method for attaching weights to jobs. He assumed that at stage, k for a sequence of processing times (P_j), the effective release times, r_j^{eff} for jobs are calculated as

$$r_j^{eff} = \max(r_j, r_{[k]}^{ff} + t_{[k]}) \quad (34)$$

where $[k]$ is the k^{th} job. The weight-factor of job, j is

$$W_j = 2R_j^{eff} + (1 - \alpha)P_j \quad (35)$$

For α between 0 and 1 ($0 \leq \alpha \leq 1$), in which case α was proved to be equivalently equal to $2/3$ ($\alpha = 2/3$). For more optimal solution, α was modified to fall between 0.80 and 0.90 or 0.85 and 0.95 for better result. Thus, in this study we choose to adopt α as $2/3$ (0.67) to bring the value of the FPFS codes as close as possible to the others. Recalling that release date r_j for all jobs is zero ($r_j = 0$), the weight-factor equation becomes reduced to

$$W_j = (1 - \alpha)P_j \quad (36)$$

VII. SOLUTION METHODOLOGY

To obtain values for the formulations above, we wrote simple codes in MATLAB 7.0 taking advantage of its analytical capabilities to analyze our chosen performance measures of the proposed sequencing rules (FCFS, SPT, EDD, FPFS) with the aid of graphs to access the performance of the FPFS sequencing rule adopted by the company under study.

A. First Pay First Serve (FPFS) Algorithm

The algorithm for the FPFS model is given below

Step 1: Initialization

- Enter number of machines m and jobs n to be processed.
- Enter job processing times P_j , due dates D_j , and release dates R_j .

Step 2: Using Reeves equation, compute job weights $W_j = 0.34P_j$ processing times P_j in Step 1.

Step3: Rearrange job weights W_j in non-decreasing order $W_1 \leq W_2 \leq \dots, W_n$

Step 4: Rearrange processing times P_j , in non-decreasing order based on their given weights.

Step 5: Evaluate the desired objective functions and terminate.

B. First Pay First Served (FPFS) data

Data for our model analysis were obtained from the company under study. Processing times were given within the range of average minimum/maximum values within the range of $1\frac{1}{2} - 9\frac{1}{2}$ hours. Using MATLAB 7.0 random number generator, we generated a set 50 job processing times for a production line. With this job due dates, weights, start times, completion times and lateness are generated using appropriate

equations. Table 1A – C below shows the data used for the FPFs analysis.

TABLE 1(A): DATA FOR FIRST PAY FIRST SERVE (FPFS) RULE

S/N o.	Processing Times P_p	Start Times S_p	Job Weights	Job Due Dates	Job Completion Times	Latenes s
1	7.80	0.00	3.06	14.0	7.80	-6.20
2	6.70	7.80	2.94	12.0	14.50	2.50
3	4.60	14.5	2.88	5.00	19.10	14.10
4	7.90	19.1	2.87	9.00	27.00	18.00
5	8.30	27.00	2.86	10.0	35.30	25.30

TABLE 1(B): DATA FOR FIRST PAY FIRST SERVE (FPFS) RULE

S/No	Processing Times P_p	Start Times S_p	Weights	Due Dates	Completion Times	Latenes s
6	5.90	35.3	2.86	8.00	41.20	33.20
7	4.90	41.2	2.80	9.00	46.10	37.10
8	7.90	46.1	2.76	12.0	54.00	42.00
9	1.70	54.0	2.73	2.00	55.70	53.70
10	8.10	55.7	2.71	13.0	63.80	50.80
11	7.10	63.80	2.70	14.00	70.90	56.90
12	2.80	70.9	2.69	9.0	73.70	64.70
13	8.00	73.7	2.65	10.0	81.70	71.70
14	6.70	81.70	2.61	11.0	88.40	77.40
15	3.00	88.40	2.61	3.00	91.40	88.40
16	9.20	91.40	2.49	15.0	100.60	85.60
17	7.60	100.6	2.47	12.0	108.20	96.20
18	5.40	108.2	2.47	7.00	113.60	106.60
19	6.90	113.6	2.42	11.0	120.50	109.50
20	2.90	120.5	2.41	5.00	123.40	118.40
21	2.50	123.4	2.41	4.00	125.90	121.90
22	9.10	125.9	2.19	14.0	135.00	121.00
23	9.00	135.0	2.07	11.0	144.00	133.00
24	5.00	144.0	2.06	10.0	149.00	139.00
25	5.10	149.0	2.02	6.00	154.10	148.10
26	7.40	154.1	1.99	10.0	161.50	151.50
27	2.60	161.5	1.96	3.00	161.10	161.10
28	3.70	161.1	1.94	7.00	167.80	160.80
29	1.80	167.8	1.78	3.00	169.60	166.60
30	9.20	169.6	1.72	14.0	178.80	164.80
31	2.30	178.8	1.69	3.00	181.10	178.10
32	1.90	181.1	1.62	3.00	183.00	180.00
33	8.70	183.0	1.56	8.00	191.70	183.70

TABLE 1(C): DATA FOR FIRST PAY FIRST SERVE (FPFS) RULE

S/N o.	Processing Times P_p	Start Times S_p	Weights	Due Dates	Completion Times	Latenes s
34	4.00	191.7	1.51	4.00	195.70	191.70
35	8.80	195.7	1.43	10.00	204.50	194.50
36	6.50	204.5	1.34	7.00	211.00	204.00
37	2.30	211.0	1.19	3.00	213.30	210.30
38	9.20	213.3	1.13	11.00	222.50	211.50
39	7.60	222.5	1.06	11.00	230.10	219.10
40	6.70	230.1	1.02	13.00	236.80	223.80
41	9.30	236.8	1.00	15.00	246.10	231.10
42	7.10	246.1	0.93	8.00	253.20	245.20
43	3.70	253.2	0.89	7.00	256.90	249.90
44	1.80	256.9	0.89	2.00	258.70	256.70
45	7.20	258.7	0.80	9.00	265.90	256.90
46	4.60	265.9	0.79	8.00	270.50	262.50
47	9.20	270.5	0.78	11.00	279.70	268.70
48	7.50	279.7	0.70	12.00	287.20	275.20
49	8.80	287.2	0.66	11.00	296.00	285.00
50	5.40	296.00	0.57	6.00	301.40	295.40

C. FCFS ALGORITHM

For the FCFS model, the following algorithm is given below:

Step 1: Initialization

- Enter number of machines M and jobs N to be processed.
- Enter job processing times P_j , due dates D_j , weights W_j , and release dates R_j for all jobs.

Step 2: Arrange jobs for processing based on the order of processing times P_j , which they arrived the system as in Step 1.

Step 3: Evaluate the desired objective functions and terminate.

The table below consists of the processing times, due dates, weights, completion times and lateness used in the FCFS analysis. They are a re-arrangement of figurative values tabulated in Table 1A – C above.

Table 2 contains information regarding job processing times, due dates and weights all arranged in FCFS order. From this data set jobs start times, completion times and lateness are obtained and entered in the table as shown.

TABLE 2: DATA FOR FIRST COME, FIRST SERVE (PFPS) RULE

S/N o.	Processing Times P_F	Start Times P_F	Weights s ,	Due Dates,	Completion Times	Lateness s, L_F
1	8.00	0.00	2.65	10.00	8.00	-2.00
2	3.70	8.00	1.56	8.00	11.70	3.70
3	2.50	11.70	2.41	4.00	14.20	10.20
4	8.80	14.20	1.43	10.00	23.00	13.00
5	6.50	23.00	1.34	7.00	29.50	22.50
6	2.30	29.50	1.69	3.00	31.80	28.80
7	3.70	31.80	0.89	7.00	35.50	28.50
8	5.90	35.50	2.86	8.00	41.40	33.40
9	9.20	41.40	2.49	15.00	50.60	35.60
10	9.20	50.60	1.13	11.00	59.80	48.80
11	2.80	59.80	2.69	9.00	62.60	53.60
12	9.30	62.60	1.00	15.00	71.90	56.90
13	9.20	71.90	1.72	14.00	81.10	67.10
14	5.40	81.10	0.57	6.00	86.50	80.50
15	7.90	86.50	2.87	9.00	94.40	85.40
16	2.60	94.40	1.96	3.00	97.00	94.00
17	4.90	97.00	2.80	9.00	101.90	92.90
18	8.80	101.90	0.66	11.00	110.70	99.70
19	7.80	110.70	3.06	14.00	118.50	104.50
20	9.20	118.50	0.78	11.00	127.70	116.70
21	6.70	127.70	1.02	13.00	134.40	121.70
22	1.80	134.40	0.89	2.00	136.20	134.20
23	8.30	136.20	2.86	10.00	144.50	134.50
24	9.00	144.50	2.07	11.00	153.50	142.50
25	6.90	153.50	2.42	11.00	160.40	149.40
26	7.60	160.40	1.06	11.00	168.00	157.00
27	7.40	168.00	1.99	10.00	175.40	165.40
28	4.60	175.40	0.79	8.00	180.00	172.00
29	6.70	180.00	2.61	11.00	186.70	175.70
30	2.90	186.70	2.41	5.00	189.60	184.60
31	7.10	189.60	2.70	14.00	196.70	182.70
32	1.70	196.70	2.73	2.00	198.40	196.40
33	3.70	198.40	1.94	7.00	202.10	195.10
34	1.90	202.10	1.62	3.00	204.00	201.00
35	2.30	204.00	2.41	3.00	206.30	203.30
36	8.10	206.30	2.71	13.00	214.40	201.40
37	7.10	214.40	0.93	8.00	221.50	213.50
38	4.00	221.50	1.51	4.00	225.50	221.50
39	9.10	225.50	2.19	14.00	234.60	220.60
40	1.80	234.60	1.78	3.00	236.40	233.40
41	5.00	236.40	2.06	10.00	241.40	231.40
42	4.60	241.40	2.88	5.00	246.00	241.00
43	7.60	246.00	2.47	12.00	253.60	241.60
44	7.90	253.60	2.76	12.00	261.60	249.50
45	3.00	261.60	2.61	3.00	264.50	261.50
46	5.40	264.50	2.47	7.00	269.90	262.90
47	5.10	269.90	2.02	6.00	275.00	269.00
48	6.70	275.00	2.94	12.00	281.70	269.70
49	7.20	281.70	0.80	9.00	288.90	279.90
50	7.50	288.90	0.70	12.00	296.40	284.40

D. SHORTEST PROCESSING TIME (SPT) ALGORITHM

For the SPT model, the following algorithm is given below:

Step 1: Initialization

- Enter number of machines M and jobs N to be processed.
- Enter job processing times P_j , job due dates D_j , job weights W_j , and job release dates R_j for all jobs.

Step 2: Using Reeves equation, compute the individual job weights $W_j = 0.34P_j$ for each processing time P_j in Step 1.

Step 3: Rearrange processing times P_j , in non-decreasing order based on their given weights.

Step 4: Evaluate the desired objective functions and terminate.

Table 3, contains data used for SPT analysis. Here, processing times are arranged in order of least value to higher values.

TABLE 3: DATA FOR SHORTEST PROCESSING TIME (SPT) RULE

S/NO	Processing Times P_S	Start Times S_S	Weights	Due Dates	Completion Times	Lateness
1	1.70	0.00	2.73	2.00	1.70	-0.30
2	1.80	1.70	0.89	2.00	3.50	1.50
3	1.80	3.50	1.78	3.00	5.30	2.30
4	1.90	5.30	1.62	3.00	7.20	4.20
5	2.30	7.20	1.69	3.00	9.50	6.50
6	2.30	9.50	2.41	3.00	11.80	8.80
7	2.50	11.80	2.41	4.00	14.30	10.30
8	2.60	14.30	1.96	3.00	16.90	13.90
9	2.80	16.90	2.69	9.00	19.70	10.70
10	2.90	19.70	2.41	5.00	22.60	17.60
11	3.00	22.60	2.61	3.00	25.60	22.60
12	3.70	25.60	0.89	7.00	29.30	22.30
13	3.70	29.30	1.94	7.00	33.00	26.00
14	3.70	33.00	1.56	8.00	36.70	28.70
15	4.00	36.70	1.51	4.00	40.70	36.70
16	4.60	40.70	2.88	5.00	45.30	40.30
17	4.60	45.30	0.79	8.00	49.90	41.90
18	4.90	49.90	2.80	9.00	54.80	45.80
19	5.00	54.80	2.06	10.00	59.80	49.80
20	5.10	59.80	2.02	6.00	64.90	58.90
21	5.40	64.90	0.57	6.00	70.30	64.30
22	5.40	70.30	2.47	7.00	75.70	68.70
23	5.90	75.70	2.86	8.00	81.60	73.60
24	6.50	81.60	1.34	7.00	88.10	81.10
25	6.70	88.10	2.61	11.00	94.80	83.80
26	6.70	94.80	2.94	12.00	101.50	89.50
27	6.70	101.50	1.02	13.00	108.20	95.20
28	6.90	108.20	2.43	11.00	115.10	104.10

29	7.10	115.10	0.93	8.00	122.20	114.20
30	7.10	122.20	2.70	14.00	129.30	115.30
31	7.20	129.30	0.80	9.00	113.50	127.50
32	7.40	136.50	1.99	10.00	143.90	133.90
33	7.50	143.90	0.70	12.00	151.40	139.40
34	7.60	151.40	1.06	11.00	159.00	148.00
35	7.60	159.00	2.47	12.00	166.60	154.60
36	7.80	166.60	3.06	14.00	174.40	160.40
37	7.90	174.40	2.87	9.00	182.30	173.30
38	7.90	182.30	2.76	12.00	190.20	178.20
39	8.00	190.20	2.65	10.00	198.20	188.20
40	8.10	198.20	2.71	13.00	206.30	193.30
41	8.30	206.30	2.86	10.00	214.60	204.60
42	8.80	214.60	1.43	10.00	223.40	213.40
43	8.80	223.40	0.66	11.00	232.20	221.20
44	9.00	232.20	2.07	11.00	241.20	230.20
45	9.10	241.20	2.19	14.00	250.30	236.30
46	9.20	250.30	0.78	11.00	259.50	248.50
47	9.20	259.50	1.13	11.00	268.70	257.70
48	9.20	268.70	1.72	14.00	277.90	263.90
49	9.20	277.90	2.49	15.00	287.10	272.10
50	9.30	287.10	1.00	15.00	296.40	281.40

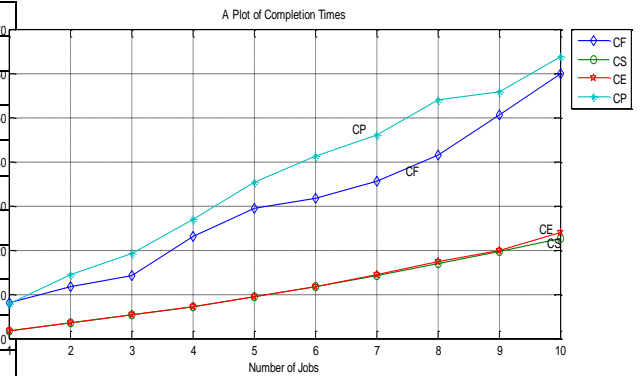


Figure 2: Graph of Completion Times against Number of jobs for n = 1 – 10

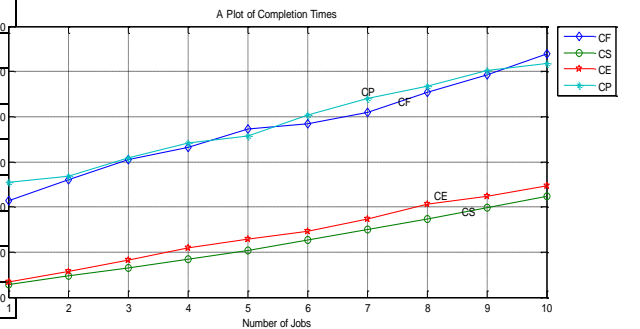


Figure 3: Graph of Completion Times against Number of jobs for n = 10 - 20

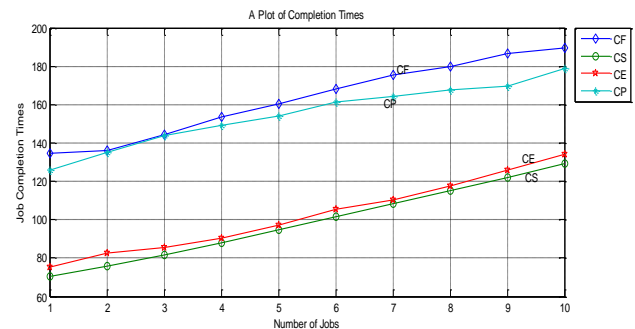


Figure 4: Graph of Completion Times against Number of jobs for n = 20

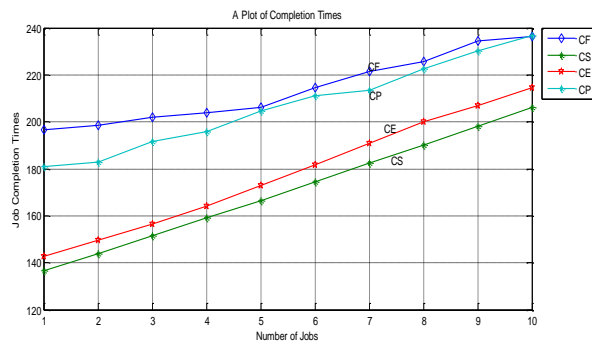


Figure 5: Graph of Completion Times against Number of jobs for n = 30 – 40

VIII. ANALYSIS AND DISCUSSION OF COMPUTATIONAL RESULTS

Below are graphs and computational results for performance measures - job completion time, makespan and lateness - obtained from our coding. Figure 1 – 6 are graphs of job completion times for the four (4) sequencing rules under investigation against number of jobs as number of jobs increases in the system from n = 10, 20, 30, 40 and 50.

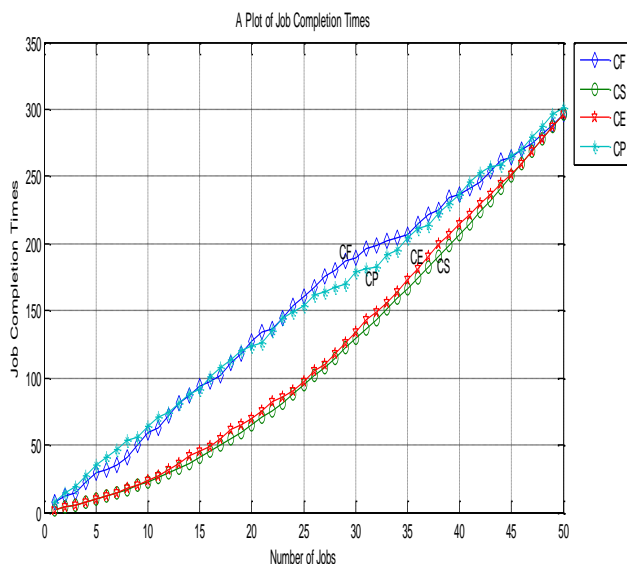


Figure 1: Graph of Completion Times against Number of jobs for n = 1 – 50

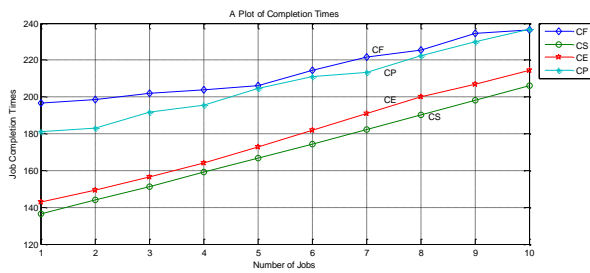


Figure 6: Graph of Completion Times against Number of jobs for $n = 40 - 50$

IX. TOTAL COMPLETION TIMES

Total completion time analysis for four (4) production stages on the service facility is shown in the graph of the effect of total completion time against number of jobs below (Fig. 7).

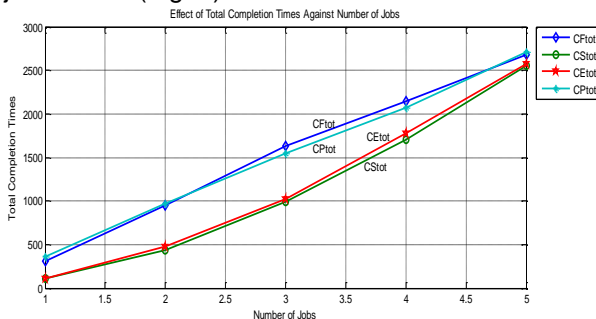


Figure 7: Graph of Total Completion Times against Number of jobs

X. ANALYSIS OF MAKESPAN

Makespan is the length of time it takes the first job to enter the machine for processing and the time required for finish processing the last job on the machine. The table below displays makespan value for the various sequencing rules under study at different stages of in the production line followed by graphical plots to study the characteristic effects of the behaviour of makespan for the various sequencing rules as increases from $n = 10$ to 50

TABLE 5: Makespan values for the various number of jobs (n)

S/ No	C_{max}	$n = 10$	$n = 20$	$n = 30$	$n = 40$	$n = 50$
1	CF_{max}	59.80	67.90	61.90	46.80	60.00
2	CS_{max}	22.60	42.30	64.40	77.00	90.10
3	CE_{max}	23.90	45.60	64.50	80.40	82.00
4	CP_{max}	63.80	59.60	55.40	58.00	64.60

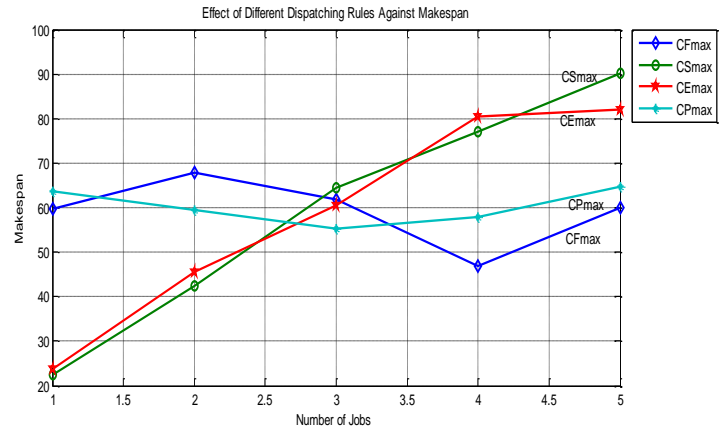


Figure 8: Graph of Makespan against Number of jobs

A. Analysis of Job Lateness

The job lateness represents the measure of an organization effectiveness resulting not only in its benefit but a factor that also ensures customers goodwill and a standard that determines its market share. The table below contains lateness for the various stages of production; as the number of jobs begins to increase along the production line.

TABLE 6: Makespan values for the various number of jobs (n)

S/N	L_{tot}	$n = 10$	$n = 20$	$n = 30$	$n = 40$	$n = 50$
1	LF_{tot}	222.50	851.30	1536.70	2068.90	2590.90
2	LS_{tot}	75.50	373.00	889.80	1596.80	2429.30
3	LE_{tot}	84.60	415.80	931.80	1668.80	2438.70
4	LP_{tot}	270.50	875.40	1467.80	1996.70	2626.60

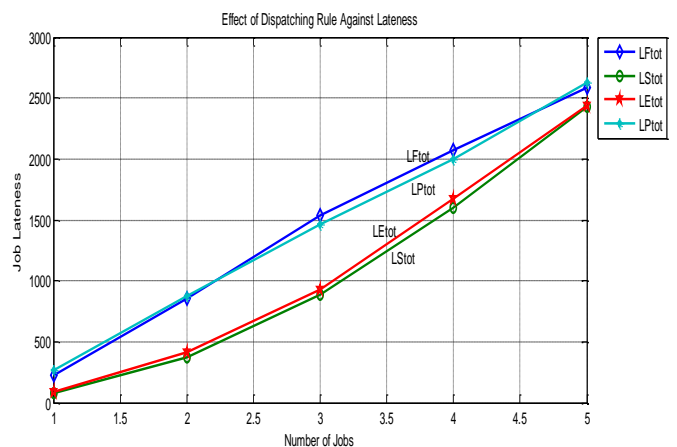


Figure 9: Graph of Lateness against Number of jobs

B. Analytical Result

Figure 1, is the graph for completion times for a set of 50 jobs under consideration. The characteristic performance of the sequencing rules at different stages of the production process considering job completion times against number of jobs processed in the system as shown in Fig. 2 – 6, is tabulated below:

TABLE 4: GRAPHICAL ANALYSIS FOR VARIOUS JOB COMPLETION TIMES

S/N	n = 10	n = 20	n = 30	n = 40	n = 50
1	CP	CP	CF	CF	CF
2	CF	CF	CP	CP	CP
3	CE	CE	CE	CE	CE
4	CS	CS	CE	CS	CS

When the number of jobs, to be processed is increases from 10 to 20, the following inequality describes the performance of the job completion times $CS \leq CE \leq CF \leq CP$. When increases from 20 to 50 jobs, the performance of the job completion times changes in the order $CS \leq CE \leq CP \leq CF$. An indication that for job completion times, SPT rule performs better than the others trailed by EDD, the company’s adopted PFPS with FCFS rule having a worst performance.

In Figure 7, the makespan for SPT and EDD rule increases from the origin of the graph along the x-axis when n is increased from 10 to 50 jobs respectively. However, for FCFS and FPFS rule the value are larger at origin and takes a dive as the graph progresses along the x-axis with the FCFS rule performing better than the FPFS rule at several points along the x-axis of the graph as the number of jobs increases from 10 to 50. As the operations tends toward the 50th job mark the makespan of FCFS and FPFS show better performance than those of the SPT and EDD rule.

The graph (Fig. 8) above is the lateness profile for the production process with SPT rule displaying a better performance than the others; followed by EDD rule. However, both the FCFS and FPFS rule results in worse performs; they produce a higher percentage of late jobs displayed in the graphs above with FCFS rule turning out more late jobs than FPFS rule.

XI CONCLUSION

The study show that, for effective production planning and control; adding starting and finishing time (sequencing) for processing particular jobs becomes necessary to ensure that production deadlines and

customer’s delivery due dates are met. However, because of the difference between sequencing theory and practice, several researches have been carried out on theoretical systems to come up with better solution methodologies to understand and improve the sequencing of real-world production systems. The results obtained may be mainly theoretical, nevertheless, they enable us understand better the behaviour of real-world systems and how to handle them in our decision making process.

REFERENCES

- [1] Fredrick Taylor, *The Principles of Scientific Management*, Harper & Brothers, 1911
- [2] Henry L. Gantt, *Industrial leadership*, New Haven: Yale University Press, 1916
- [3] S. M. Johnson, “Optimal Two and Three Stage Production Schedules with Setup Times Included” *Naval Research Logistics Quarterly*, Vol. 1, Issue 1, 1954. Pp.61-68
- [4] Jackson, J. R., “Scheduling a production line to minimize maximum tardiness”, *Research Report 43, Management Science Research Project*, University of California, Los Angeles, 1955.
- [5] Jackson, J. R., “An extension of Johnson’s results on job lot scheduling,” *Naval Research Logistics Quarterly*, 3, pp. 201-203, 1956.
- [6] SMITH, W. E., “VARIOUS OPTIMIZERS FOR SINGLE-STAGE PRODUCTION,” *NAVAL RESEARCH LOGISTICS QUARTERLY*, 3, PP. 56-66, 1956
- [7] Chinyao Low, Rong-Kwei Li, and Guan-He Wu.,”Minimizing Total Earliness and Tardiness for Common Due Date Single-Machine Scheduling with an Unavailability Interval” *Mathematical Problems in Engineering*, 12 pages, Volume 2016 (2016).
- [8] E.O. Oyetunji and M. Masahudu, "An Improved Heuristic for Minimizing Total Completion Time. *Research Journal of Mathematics and Statistics*, 1(2): Page No: 65-68., 2009
- [9] Neelam Tyagi, R. P. Tripathi and A. B. Chandramouli., “Single Machine Scheduling Model for Total Weighted Tardiness” *Indian Journal of Science and Technology*, Vol 9(31), August,2016
- [10] Du-Juan Wang, Yunqiang Yin, Shuenn-Ren Cheng, T.C.E. Cheng & Chin-Chia Wu “Due date assignment and scheduling on a single machine with two competing agents” *Journal International Journal of Production Research* Volume 54, - Issue 4, 2016