Extension Problem: A Note On Proper Modification Of Extended Poisson Theory

Kaza Vijayakumar
Department of Aerospace Engineering, Indian Institute of Science Bangalore-560012, India
e-mail:kazavijayakumar@gmail.com

***Abstract*—Present note is with reference to proper modification of extended Poisson theory in the exact analysis of primary extension problems within the classical small deformation theory of elasticity without the use of concept of virtual transverse displacement.**

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INITIAL SOLUTIONS OF PRIMARY EXTENSION PROBLEM

In a primary extension problem, the plate is subjected to symmetric normal stress $\sigma_{z0} = q_0(x, y)/2$, asymmetric shear stresses $[\tau_{xz1}, \tau_{yz1}] = \pm [T_{xz1}(x, y), T_{yz1}(x, y)]$ along top and bottom faces of the plate. Here, $\sigma_{z0} = q_0/2$ satisfying face condition does not participate in equilibrium equation of transverse stresses and the corresponding applied face shears $[T_{xz1}, T_{yz1}]$ are gradients of a given harmonic function $\tilde{\psi}_1$ so that $[T_{xz}, T_{yz}]= \alpha\ [\tilde{\psi}_{1,x}, \tilde{\psi}_{1,y}]$. Transverse shear stresses and normal stress satisfying face conditions are $[\tau_{xz}, \tau_{yz}] = \alpha\ z\ [\tilde{\psi}_{1,x}, \tilde{\psi}_{1,y}]$ and $\sigma_{z0} = q_0(x, y)/2$. With the inclusion of the above gradients of the known $\tilde{\boldsymbol{\psi}}_1$ in the normal stresses, in-plane equilibrium equations are, with $(v_{0,x} - u_{0,y}) = 0$

$$(E'/3)\ \alpha^2\Delta u + \mu\ \alpha\ \sigma_{z0,x} = 0 \qquad (1a)$$
$$(E'/3)\ \alpha^2\Delta v + \mu\ \alpha\ \sigma_{z0,y} = 0 \qquad (1b)$$

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in which $u = u_0 + \tilde{\psi}_1/G$ and $v = v_0 + \tilde{\psi}_1/G$. Above static equilibrium equations (1) along with two conditions

$$u = \tilde{u}_0(y) \text{ or } \sigma_{x0}(y) = T_{x0}(y) \qquad (2a)$$
$$v = \tilde{v}_0(y) \text{ or } \tau_{xy0}(y) = T_{xy0}(y) \qquad (2b)$$

along x = constant edges (with analogue conditions along y = constant edges) have to be solved for u_0 and v_0. They remain same in the z-integrated equations.

We have from constitutive relation $\varepsilon_z = -\mu\ e_0 + (1-2\ v\ \mu)\ q_0/2E$ so that $w = z\ \varepsilon_{z0}$ is known. Due to this known vertical displacement '*w*' linear in z, the above solutions with reference 3-D equations are in error due to transverse shear strain-displacement relations. To rectify this error, one considers higher order in-plane displacement terms $f_2(z)\ [u_2, v_2]$ which, in turn, induce additional $[\tau_{xz1}, \tau_{yz1}]$ other than the gradients of harmonic function $\tilde{\psi}_1$. Assumed displacements are in the form

$$w = z\ \varepsilon_{z0},$$
$$[u, v] = [u_0, v_0] + f_2\ [u_2, v_2] \qquad (3)$$

With a priory known $[u_0, v_0]$ and ε_{z0}, the corrective displacements $[u_2, v_2]$ have to be determined from satisfying both static and z-integrated equilibrium equations. Static equations are, with $(v_{2,x} - u_{2,y}) = 0$,

$$(E'/3)\ \alpha^2\Delta u_2 + \mu\ \alpha\ \sigma_{z0,x} = \tau_{xz3} \qquad (4a)$$
$$(E'/3)\ \alpha^2\Delta v_2 + \mu\ \alpha\ \sigma_{z0,y} = \tau_{yz3} \qquad (4b)$$

In order to keep $[\tau_{xz3}, \tau_{yz3}]$ as free variables in the integrated equilibrium equations, $f_3(z)$ is modified with $\beta_1 = 1/3$ as $f^*_3(z) = f_3(z) - \beta_1 z$ so that

$$\tau_{xz} = z\ (\tau_{xz1} - \beta_1\ \tau_{xz3}) + f_3\tau_{xz3} \qquad (5a)$$
$$\tau_{yz} = z\ (\tau_{yz1} - \beta_1\ \tau_{yz3}) + f_3\tau_{yz3} \qquad (5b)$$

in which $[\tau_{xz1}, \tau_{yz1}] = G\ [(\alpha\varepsilon_{z0,x} - u_2), (\alpha\varepsilon_{z0,y} - v_2]$. We have from equilibrium equations of transverse stresses

$$\alpha\ [\tau_{xz1,x} + \tau_{yz1,y}] = \sigma_{z2}, \qquad \alpha\ [\tau_{xz3,x} + \tau_{yz3,y}] = \sigma_{z4}$$

so that $\sigma_{z2} = \beta_1\ \sigma_{z4}$.

Noting that $\sigma_{z2} = G\ [(\alpha^2\ \Delta\ \varepsilon_{z0} - e_2)]$, and $\sigma_{z4} = \alpha^2\Delta\ [(E'/3)\ e_2 + \mu\ \sigma_{z0}]$ from equation (4), we get one equation governing $[u_2, v_2]$ in the form

$$\beta_1\ \alpha^2\Delta\ [(E'/3)\ e_2 + \mu\ \sigma_{z0}] = G\ [(\alpha^2\ \Delta\ \varepsilon_{z0} - e_2)] \qquad (6)$$

and the second equation is $(v_{2,x} - u_{2,y}) = 0$.

Here, it is convenient to express $[u_2, v_2]$ in the form

$$[u_2, v_2] = -\alpha\ [[\psi_{2,x} + \varphi_{2,y}, \psi_{2,y} - \varphi_{2,x}] \qquad (7)$$

so that equation (6) becomes a fourth order equation in ψ_2. This equation in ψ_2 along with $\Delta\varphi_2 = 0$ constitute a sixth order system to be solved with the following three conditions along x = constant edges (with analogue conditions along y = constant edges)

$$\psi_2 = 0 \text{ or } \tau_{xz3} = 0 \qquad (8a)$$
$$u_2 = 0 \text{ or } \sigma_{x2} = 0 \qquad (8b)$$
$$v_2 = 0 \text{ or } \tau_{xy2} = 0 \qquad (8c)$$

Note that 2-D variables [ψ_2, φ_2], thereby, [u_2, v_2] are determined from satisfying both static and integrated equilibrium equations of 3-D infinitesimal element.

With reference to solution of 3-D problem, above analysis in the determination of [w_3, u_2, v_2] is in error in the transverse shear strain-displacement relations due to [τ_{xz}, τ_{yz}] = $f_3(z)$[τ_{xz3}, τ_{yz3})], and in the constitutive relations due to $f_4(z)\sigma_{z4}$. The procedure for rectification of these errors is described in [1].

CONCLUDING REMARKS

Two term representation of displacements is mandatory for initial analysis of primary extension problems. The need for the use of higher order $f_k(z)$ polynomials for obtaining exact solutions of 3-D primary extension problems is eliminated by the use of proper Fourier series expansion of transverse stresses and displacements other than the basic variables in EPT of plates.

REFERENCE

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