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Numerical methods of control the hidden oscillations of fractional-order chaotic systems

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***Abstract*—Complex processes in nonlinear dynamic systems are accompanied by transient processes in which, along with the classical attractors occur hidden attractors. In the present article questions the attention of uncovering hidden oscillations in multidimensional fractional chaotic systems. To solve this problem using numerical methods of control and reconstruction of a dynamic systems. As an example, given the reconstruction algorithm for one-dimensional time series of fractional-order "Chen" and "Rabinovich-Fabrikant" systems. The results are shown in time series and the recurrence diagrams.**

Keywords—fractional-order chaotic system, SSA – algorithm, Poincare diagram

I. Introduction

During the control of the stability of multi-dimensional fractional order chaotic systems can appear the hidden oscillations. Such fluctuations in the form of hidden fluctuations occur during transients. For control systems (eg aircraft), such phenomena can have undesirable consequences caused by false control parameters [1]. To investigate the existence of hidden oscillations in the application of numerical control methods used Singular Spectrum Analysis (SSA), and also built Poincare recurrence plots allow to visually assess occurring processes [2,3].

II. General Provisionse

A. *Singular-spectral* analyses to identify hidden oscillations in chaotic *time series*

Let $\{x_i\}$ be a time series of values of function $f(t): x_i = f[i] = f(i\Delta t)$, $i = 1,\ldots,N$. Let number of $M < N$ - length of a window. To introduce k, $k = N - M + 1$ and construct k M - dimensional vectors $X_i = (x_i,\ldots,x_{i+M-1})^T$, $1 \le i \le k,\ X_i \in R^M$. Let's make a matrix [4]:

$$X = \begin{pmatrix} x_1 & x_2 & x_3 \ldots & x_k \\ x_2 & x_3 & x_4 \ldots & x_{k+1} \\ \ldots & \ldots & \ldots & \ldots \\ x_M & x_{M+1} & x_{M+2} \ldots & x_k \end{pmatrix} \quad (1)$$

Remark. The condition of $M < N/2$ - limit on the integer parameter M.

Let $S = XX^T \in R^{M \times M}$ - non-negative and symmetric matrix. Eigen values S are non-negative: $\lambda_1,\ldots,\lambda_M \ge 0$. Arrange the Eigen values in not increase order:

$$\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_M \ge 0.$$

$u_1,\ldots,u_M$ - ortonormality system eigenvectors of a matrix S.

$d = \max\{i : \lambda_i > 0\}$ - order singular value decomposition.

We introduce the matrix [4]:

$$V_i = \frac{1}{\sqrt{\lambda_i}} x^T u_i,\ i = 1,\ldots,d. \quad (2)$$

Than the singular value decomposition of the matrix has the form:

$$x = x_1 + x_2 + \ldots + x_d,\quad x_i = \sqrt{\lambda_i} u_i V_i^T. \quad (3)$$

It is known, that the singularly-spectral analyses is effective in a combination with wavelet-transformation [4]. It is connected by that the signal can have a changing frequency.

Further cleaned components related to trend and noise. After the restoration of a number of used wavelet-transform [5].

In this paper, for the purpose of localization and reconstruction of abnormal components of fractional dynamic chaotic multidimensional maps, it proposed the use singularly-spectral analyses in combination with lifting method [2,6].

Lifting methods of processing of the information make possible wavelet the stretching's and shifts of one function.

Advantage of the lifting scheme is:

1. the conversion process occurs quickly;
2. the set of wavelet-coefficients occupies a volume that matches the original data;

3. return transformation restores a signal very precisely.

B. Lifting scheme

Briefly, the mechanism is a follows [7]. Let the original signal s_j contains 2^j points. Transformation involves three steps (split-predict-update), which will yield two sets of points s_{j-1} and d_{j-1}.

- *Split*

From in s_j shape two new not crossed sets. We note that the division of the set into two depends on the type of wavelet. For example, Lazy wavelet distinguishes $even_{j-1}$ and odd_{j-1} samples.

Formally it looks as [7]:

$$\left(even_{j-1}, odd_{j-1}\right) = S\left(s_j\right). \tag{4}$$

- *Predict*

Here is calculated the difference between true and predicted values and defines coefficients d_{j-1} [7]:

$$d_{j-1} = odd_{j-1} - P\left(even_{j-1}\right), \tag{5}$$

where P - the predicting operator.

- *Update*

On this step, the help of the operator U, calculate coefficients s_{j-1} [7]:

$$s_{j-1} = even_{j-1} + U\left(d_{j-1}\right). \tag{6}$$

The described algorithm of transformation of data lifting-scheme is presented in figure [7].

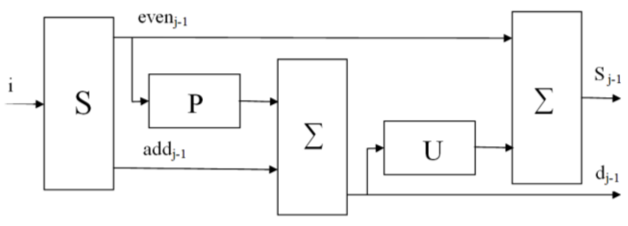


Fig. 1 Constructing the wavelet-coefficients in the lifting-scheme.

Thus, lifting schema generates two sets of coefficients s_{j-1} and d_{j-1}, each of which is less than half the length of the initial signal.

From here s_{j-1} reflects behavior of a signal in the big scale, and a coefficients d_{j-1} shows difference an initial signal from s_{j-1}.

In this paper, the realization of lifting scheme is based on the use of Haar wavelets and Doubechies [8].

C. The goal of the problems:

i - determine the influence on the function of fractional sawtooth at hyperchaotic systems.

ii - determine the stability systems.

iii - approximate result with subsequent reconstruction mapping.

iiii - construction Poincare recurrence diagram.

III. Algorithm

Step 1. Let given the fractional-order Rabinovich-Fabrikant system following [9]:

$$\left.\begin{aligned} \dot{x}_1 &= x_2\left(x_3 - 1 + x_1^2\right) + \gamma x_1, \\ \dot{x}_2 &= x_1\left(3x_3 + 1 - x_1^2\right) + \gamma x_2, \\ \dot{x}_3 &= -2x_3\left(x_1 x_2 + \alpha\right), \\ \dot{x}_4 &= -3x_3\left(x_2 x_4 + \delta\right) + x_4^2, \end{aligned}\right\} \tag{7}$$

where $\alpha = 0.14,\ \gamma = 1.1,\ -0.01 \le \delta \le 7650$.

The fractional-order Chen system as follows [10-12]:

$$\begin{aligned} \frac{d^\alpha x}{dt^\alpha} &= a\left(y - x\right) + w, \\ \frac{d^\alpha y}{dt^\alpha} &= bx - xz + cy, \\ \frac{d^\alpha z}{dt^\alpha} &= xy - dz, \\ \frac{d^\alpha w}{dt^\alpha} &= yz + rw, \end{aligned} \tag{8}$$

where $a = 35,\ b = 7,\ c = 12,\ d = 3,\ r = 0.5$ and $\alpha = 0.9$.

Step 2. Simulation of the system (1 – 2) according to the algorithm [9].

Step 3. Let $\hat{x} =_\alpha \left\{x_n\right\}_{n=0}^{N}$ is related observable two fractional-order hyperchaotic (1 – 2) systems.

Step 4. The related observable $\hat{x}$ perturb of sawtooth wave $\eta : D^q x_i \vee A\, frac\left(\frac{x}{T} + \varphi\right)$, where

$frac(x)$ is the fractional part. $frac(x) = x - [x]$, A is amplitude, T is the period of the wave, and φ is its phase.

Step 5. We determine the stability of noisy systems (1 – 2).

Step 6. Lifting step 5 produce the singular-spectrum analysis for systems (1 – 2).

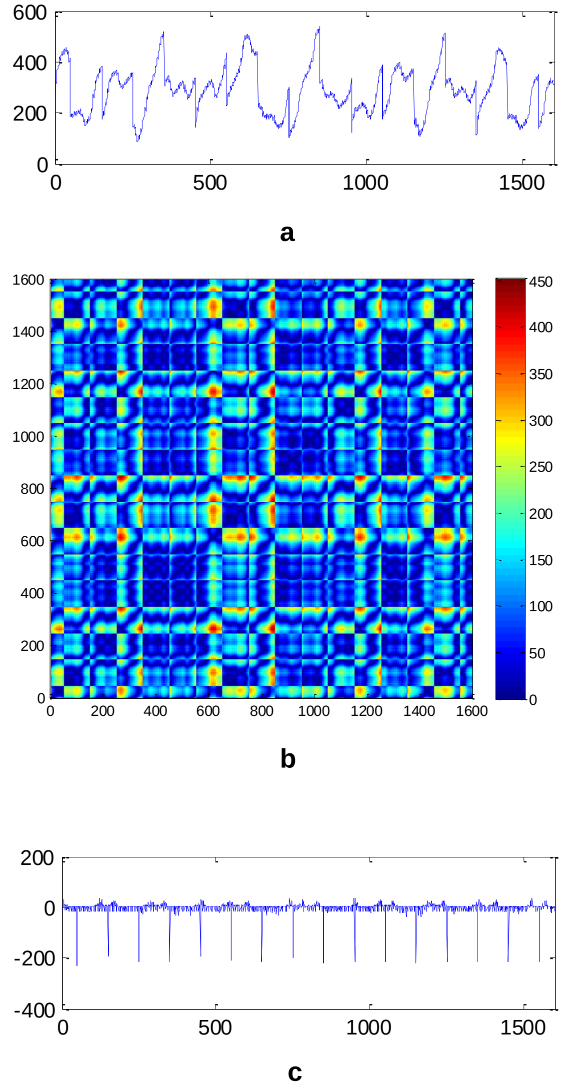
Step 7. Produce signals (*Step 6*) conversion on lifting scheme.

Step 8. Determine the overage return time of Poincare.

Step 9. We make visualization of the results.

IV. VISUALIZATION OF SIMULATION

Below is a visualization of the main stages of the implementation of the algorithm of time series of fractional-order chaotic systems Chen and Rabinovich-Fabricant. Figures 2 and 3 show the marked time series chart relevant systems and their Poincare recurrence diagrams



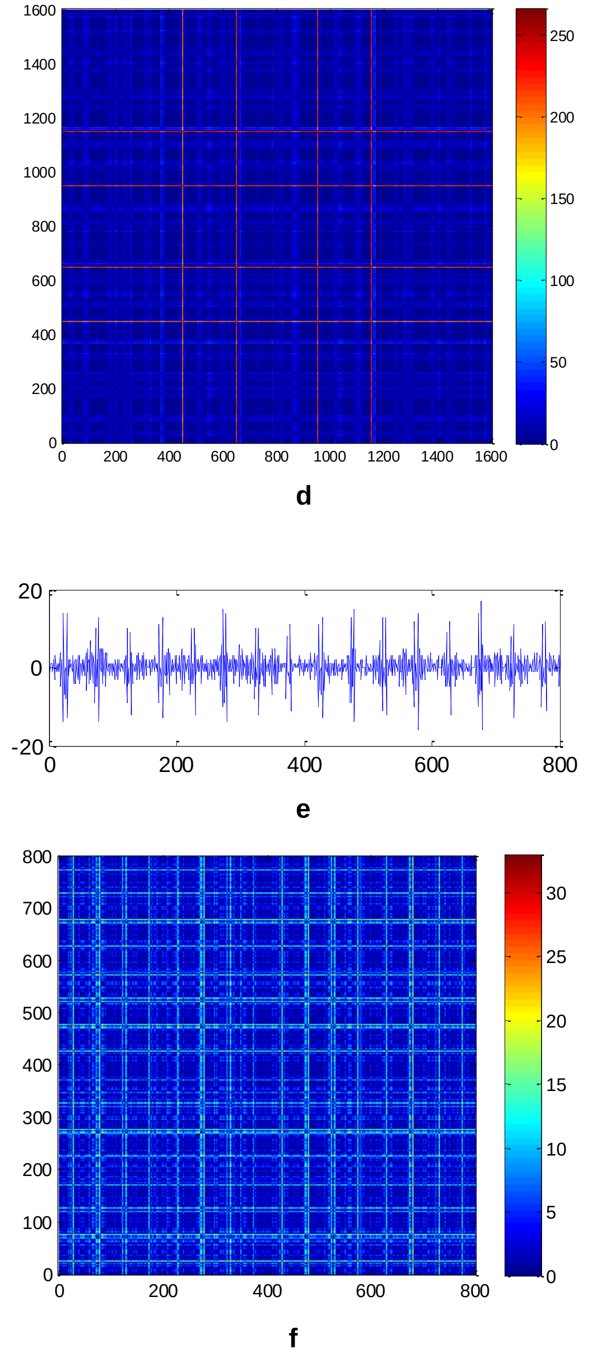
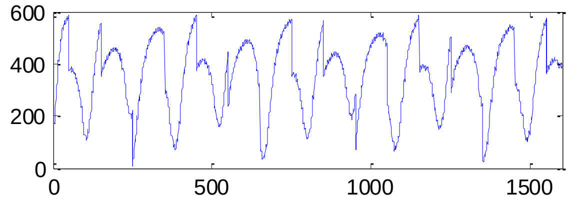
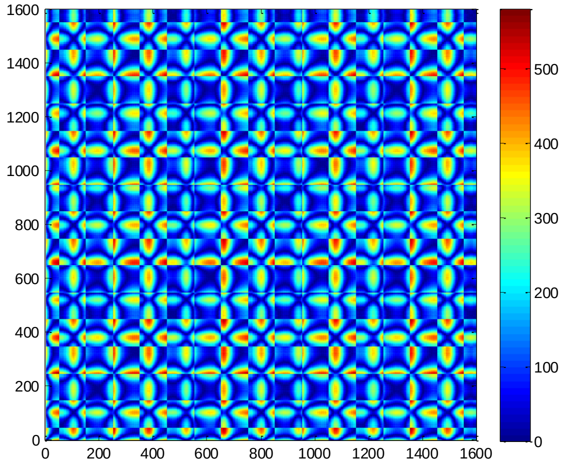


Fig. 2 Fractional-order chaotic Chen system;
a - signal with noise, c - stability, e – reconstruction signal, b, d, f - corresponding Poincare recurrence diagrams.

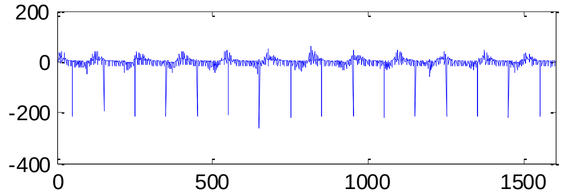
The following figure shows the main pieces of proposed algorithm performance in relation to the time series of fractional-order chaotic system Rabinovich-Fabricant.



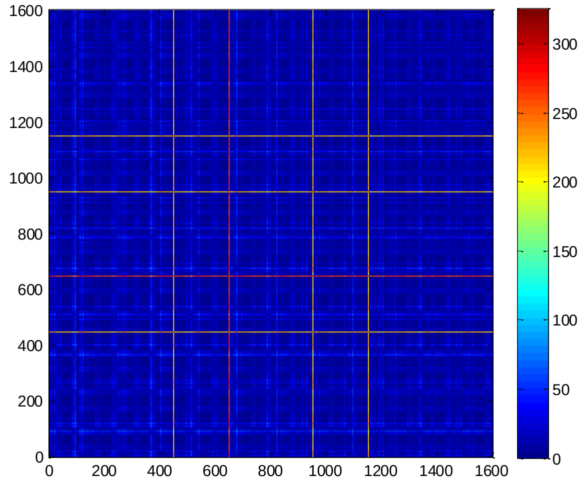
a



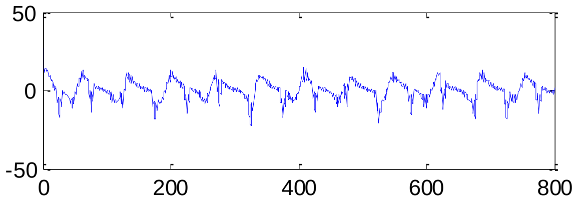
b



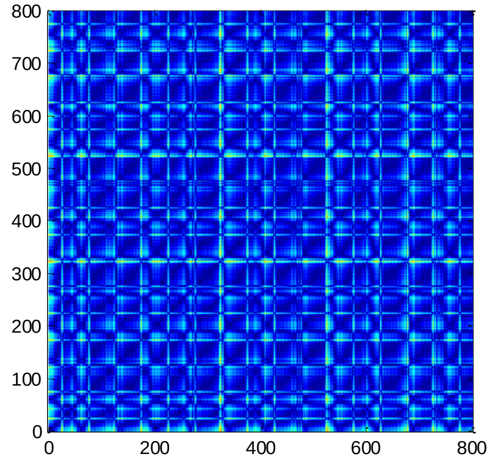
c



d



e



f

Fig. 3 Fractional-order chaotic Rabinovich-Fabricant system; a - signal with noise, c - stability, e – reconstruction signal, b, d, f - corresponding Poincare recurrence diagrams.

V. Conclusion

The results showed that when using the Haar wavelet hold the transient type “hyperchaos – chaos” and wavelet Doubechies “hyperchaos – quasiperiodic” transient. The proposed algorithm can be used in micro – control systems. Visualization of transients implemented software MATLAB. It shows the numerical values of the overage Poincare return time.

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