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Fish Harvesting Models And Their Applications in a reservoir in Saranda, Albania

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***Abstract*—This paper studies the harvesting strategies for wrasse fish farming. There are used three logistic growth models, namely constant, proportional and periodical harvesting. For each strategy it is estimated the optimal amount of fish harvested to protect the population from extinction. The data for this paper are provided by a fish reservoir in Saranda, Albania. The use of mathematical models in the defining of the harvesting strategies is not widely applied in Albania. The objective of this study is to define the most optimal growth and reproduction of the wrasse fish. The findings can assist fish farmers to increase the supply to meet the demand for wrasse fish.**

Keywords—logistic growth model; stability; constant, proportional, periodical harvesting

I. INTRODUCTION

Nowadays Albanian people have become aware of the fish as one of the major sources of human diet-main source of protein and fat. Awareness of fish as nutritious diet implies the growth in demand for it. The greater part of fish consumed by human comes from fishing in ocean/sea, but the natural supply is not yet sufficient to satisfy the increasing consumption, so aquaculture will have great potential all over the world, in the near future. Mathematical models have been used widely to estimate the population dynamics of animals for so many years as well as the human population dynamics. Agriculture sector, especially in cattle farming, is the field in which the use of mathematical models has been extended in recent years. This has been to ensure continuous and optimum supply. The logistic growth model in term of harvesting has been used to study the fishery farming (Laham et al., 2012). Harvesting has been an area under discussion in population as well as in community dynamics (Murray 1993). The most important for successful management of harvested populations is that harvesting strategies are sustainable, not leading to instabilities or extinctions and produces great results for the year with little variation between the years [3]. Therefore, it can fulfill the market demand throughout the year. Malthus was the first to formulate theoretical treatment of population dynamics in 1798 and Verhulst formed the Malthus theory into a mathematical model called the logistic equation that led to nonlinear differential equation (Alan 1992). According to Idels and Wang (2008), constant harvesting is where a fixed numbers of fish were removed each year, while periodic harvesting is usually thought of a sequence of periodic closure and openings of different fishing grounds. Harvesting has been considered a factor of stabilization, destabilization, improvement of mean population levels, induced fluctuations, and control of non-native predators (Michel 2007). In recent years, fish farming has been developed in Albania and aquaculture production constituted 17.6% of total fish caught in 2011, around 14% in 2010 from around 1% in 2001 (INSTAT database). Among Albanian products, processed fish products are successfully emerging in regional and European markets. Canned fish was the most exported product and constituted 25% of total agriculture products exported in 2010 [6], [7]

II. METHODS

We are going to analyze some simple fishery management models in a real world situation. We consider the logistic growth equation to model a fish population in the absence of fishing [2], [3], [4]:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{M}\right), \qquad P(0) = P_0 \qquad (1)$$

where P is the size of the population, r is the growth rate due to reproduction and M is the carrying capacity of the environment.

Now consider modeling the population and harvesting some of the population using some common harvesting strategies: constant harvesting, proportional harvesting and periodic (seasonal) harvesting.

Constant harvesting is where a fixed numbers of fish were removed each year, while periodic harvesting is usually thought of a sequence of periodic closure and openings of different fishing grounds (Idels and Wang, 2008; Laham et al., 2012). In proportional harvesting, the quantity harvested is proportional to the population. Harvesting has been considered a factor of stabilization, destabilization, improvement of mean population levels, induced fluctuations, and control of non-native predators (Michel 2007). We can use qualitative analysis to estimate how many fish can be harvested and still allow the fish population to survive.

A. Constant Harvesting

One of the simplest methods is the idea of harvesting where a set limit is established for harvesting. We assume that the dynamics of the population satisfies the logistic growth model (1) and that a constant harvesting, h , is added for removing a constant number of the fish over a given time interval. The mathematical model becomes:

$$\frac{dP}{dt}=rP\left(1-\frac{P}{M}\right)-h, \qquad P(0)=p_0 \qquad (2)$$

The fixed points, P^*, are simply the solution of the equation $rP^*\left(1-P^*/M\right)=h$. The model (2) has two fixed points $P^*_{1,2}=\frac{1}{2}\left(M\pm\sqrt{M^2-\frac{4hM}{r}}\right)$ if $0<h<rM/4$; one fixed point $P^*=M/2$ when $h=rM/4$; no fixed point when $h>rM/4$.

B. Proportional Harvesting

Another common form of harvesting is when one puts in a constant effort to harvest. In this case, the quantity harvested is proportional to the population. Thus, the mathematical model can be written:

$$\frac{dP}{dt}=rP\left(1-\frac{P}{M}\right)-hP, \quad P(0)=P_0 \qquad (3)$$

where again r is the growth rate, M is the carrying capacity with no harvesting and now h is the proportional rate of harvesting.
Algebraic solution is complex and harder to interpret, thus we again turn to the geometric analysis of the model. The fixed points [1] of (3) are the solution of the equation:
$rP^*(1-P^*/M)=hP^*$, that is, $P^*=0$ and $P^*=\frac{(r-h)M}{r}$. The extinction fixed point, $P^*=0$, is unstable for values of $h<r$. As h increases, the larger equilibrium (carrying capacity) shrink, but it remains stable for $h<r$.

C. Periodic Harvesting

Another very used form of harvesting is when harvesting is done during periods of time within a year, so the fish won't become extinct during fishing time and in some periods fishing is stopped, the population of fish might be able to increase again. The mathematical model can be written:

$$\frac{dP}{dt}=rP\left(1-\frac{P}{M}\right)-h\left(1+\sin 2\pi t\right) \qquad (4)$$

The model (4) is a non-autonomous differential equation, so the solutions are periodic and have the same general trend with the previous models.

III. Results And Discussion

A. Logistic Growth Model With Constant Harvesting

The data, taken from the reservoir "Qendra sh.p.k", are considered to illustrate the above results. It cultivates wrasse fish. The reservoir's surface is 100 m^2. 1m^2 sustains 80 wrasse fish. The carrying capacity of the reservoir is 8000 wrasse fish. The period of maturity for the wrasse fish is 15 months and 80 % of which is estimated to survive to maturity [5].

Fig. 1

The fixed point for model (2), where $h=0$, are $P^*=0$ and $P^*=M=8000$.
As the harvesting increases, the two fixed point move closer to each other with the lower fixed point remaining unstable and the upper fixed point remaining stable [1] (Fig. 2).

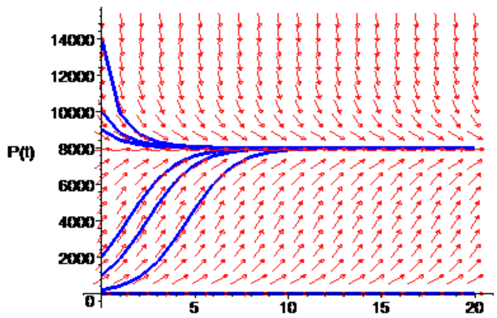

Fig. 2

For $h=1600$ (the maximum growth rate of the logistic growth equation), there is one fixed point $P^*=4000$ (Fig. 3).

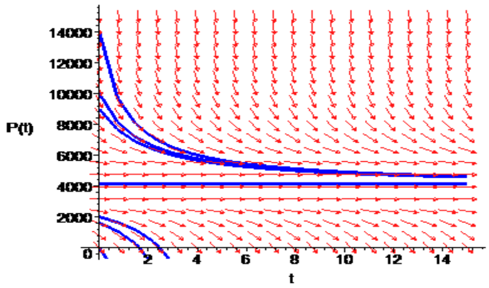

Fig. 3. h=1600

For $h > 1600$, there is no fixed point, and the model shows that the population always goes extinct. This model shows a classic example of a *saddle node bifurcation* [1]. The bifurcation values is $h = rM / 4 = 1600$ (Fig. 4).

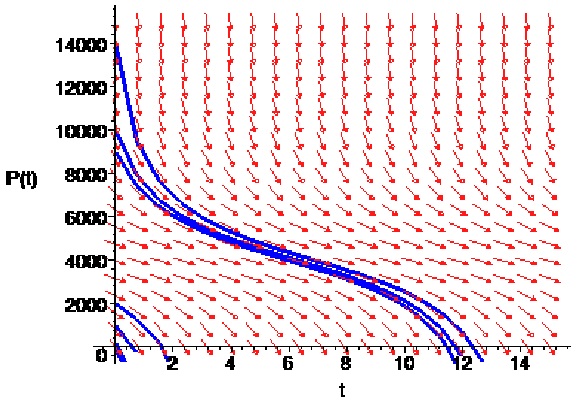

Fig. 4. h=2000

The direction field of the differential equation for some values of h indicate first the existence of two fixed point (one stable and one unstable), then no fixed point. (Fig. 4)

The purpose of simple mathematical models applied to complex problems is to offer some insight. Here, the results indicate that overfishing (in the model $h > rM / 4$) during one year can potentially result in a sudden collapse of the fish catch in subsequent years, so that fish farmers need to be particularly cautious and not overcome 1600 in fishing quotas.

With uncontrolled fish harvesting, a population could easily become extinct..

B. Logistic Growth Model With Proportional Harvesting

As h move toward 0.8 (the growth rate), the nonzero fixed point fades to zero, which implies there is extinction because the harvesting rate approaches the growth rate. When $h > 0.8$, the rate of harvesting exceeds the reproduction rate and extinction necessarily follows. This model shows a classic example of a *transcritical bifurcation*. The bifurcation point is $h = 0.8$ (Fig. 5, 6, 7)

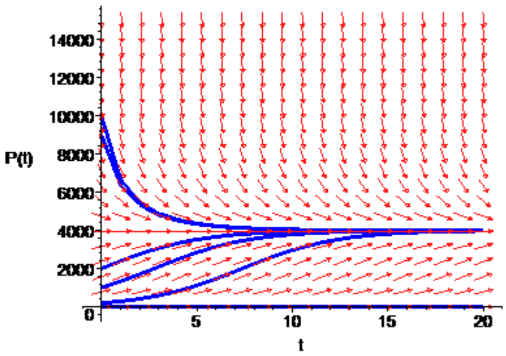

Fig. 5. h=0.4

The direction field of the differential equation for some values of h indicates the existence of two fixed point (one stable and one zero unstable), then one zero fixed point [1].

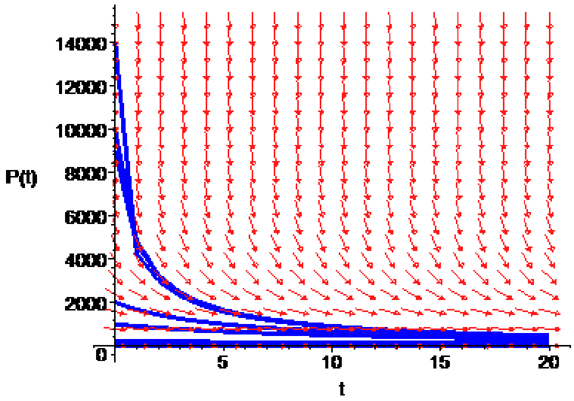

Fig. 6. h=0.8

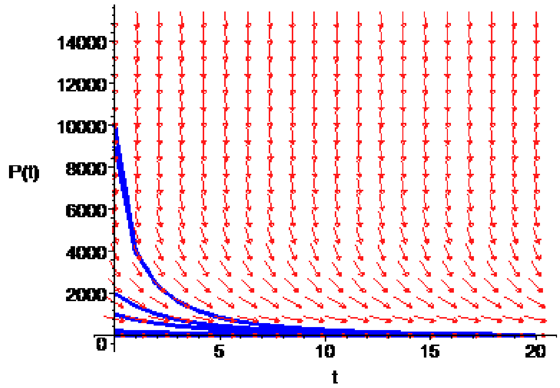

Fig. 7. h=1

The results indicate that overfishing (in the model $h > r$) during one year can potentially extinct the fish in the reservoir. That is why it is crucial not to exceed the fishing quotas.

I. Logistic Growth Model With Periodic Harvesting

The reservoir has full carrying capacity of 8000 wrasse fish as initial population. Its growth rate is $r = 0.8$. Let us assume that during the first six months we have harvested 1600 wrasse fish, and the next six months there is no harvesting. In order to ensure the population of fish is increasing, there is no harvesting in the next six months and the population of fish will increase until it approaches the carrying capacity 8000 of the wrasse fish [5].

There are two solutions that oscillate about the fixed points (Fig. 8). The solutions converge to one periodic solution that oscillates around the stable fixed point. For $h =$ 1600 , there is only one fixed point (Fig. 9). The population reaches the fixed point and stays there. As h increases more the population will extinct (Fig. 10). Therefore, this periodic equation has the same bifurcation point as model (2).

The periodic seasonal harvesting strategy optimizes the harvest while maintaining stable the population of fish. A harvesting strategy using logistic periodic seasonal harvesting strategy can be used to improve productivity, shorten investment return time and reduce risk from changes in sale price and costs of productions, particularly when comparatively short return periods are used (Laham et al., 2012).

It is common to have some months, for example 3 months, where heavy fishing is allowed and other months where only light fishing is allowed. The population still recovers to fixed point, but it takes longer for it to reach the stable fixed point because there are still a few fish that are being harvested during the rest of the year.

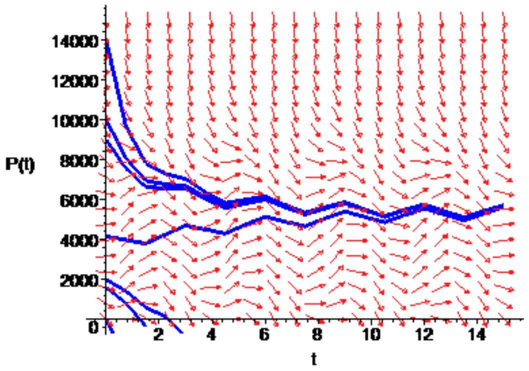

Fig. 8. h=1400

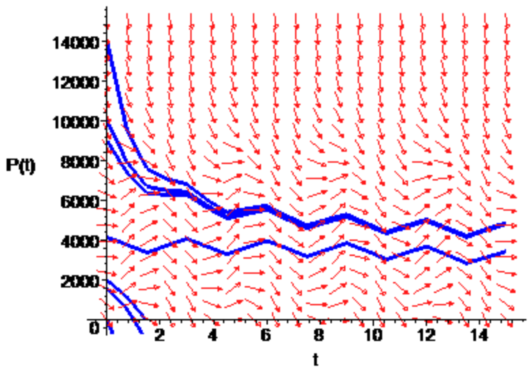

Fig. 9. h=1600

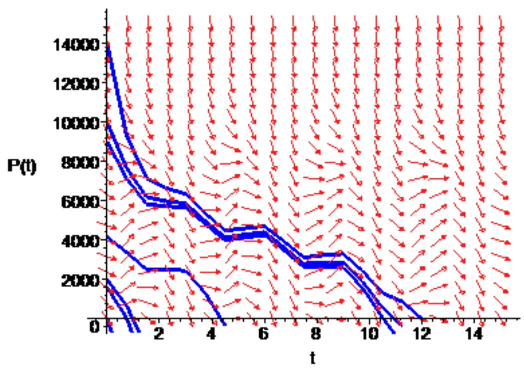

Fig.10. h=2000

IV. Conclusions

From the discussion of three harvesting strategies, results that using:

The constant harvesting strategy, the fish population does not have enough time to recover if the constant harvesting is greater than the bifurcation point.

The proportional harvesting strategy, the fish population will extinct if the proportional rate of harvesting is greater than the growth rate of the population or the bifurcation point.

The periodic seasonal harvesting strategy optimizes the harvest while maintaining stable the population of fish, if the harvesting is lower or equal with the bifurcation point.

The development of appropriate fishery harvesting strategy can help the fulfillment of market demand. Supply of fish cannot rely only on the ocean/seas fishing activities, alternatives can be found by commercializing the aquaculture.

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