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A Stochastic Formulation For Asset Stock Delineation& Interpretation Viz Euler–Maruyama Method

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***Abstract*—Fluctuations or randomness, are driven by various processes such as the Wiener diffusion path, Poisson or Compound process, etc.**

It would become imperative to apply stochastic methods in practical delineation of finance models, thus SDE expressions are potentially useful for delineation of asset stock price and volatility, the Euler-Maruyama method has been considered, and proposed here.

Keywords—Asset Stock Price, Stochastic process, Noise, Ito lemma, Euler-Maruyama.

1. Introduction

Stochastic variations are frequently encountered in diverse number of processes. Fluctuations or randomness are encountered in a number of practical applications in biology, physical sciences, atmospheric science and oceanography, finance, etc, which are driven by various processes such as the Wiener diffusion path, Poisson or Compound process, etc.

The Black-Scholes model has been previously applied in explaining, &analyzing the European option pricing, several other models are available, each with its own distinguishing, &characterizing feature. The Black-Scholes model is a static model, & a call for dynamicity, which might require some modalities, and consideration to incorporate dynamics as typical of fast fluctuation, and rapidly volatile models, thus stochastic model, is tremendously usefull, and should be explored, with suitable modifications, and theoretical approach in addition to the pre-existing empirical models.

It would become imperative to apply stochastic method in practical delineation of finance models, thus SDE expressions are potentially useful for delineation of asset stock price and volatility.

2. Euler–Maruyama method

In mathematics, more precisely in Ito calculus, the **Euler–Maruyama method**, also called simply the **Euler method**, is a method for the approximate numerical solution of a stochastic differential equation (SDE). It is a simple generalization of the Euler method for ordinary differential equations to stochastic differential equations. It is named after Leonhard Euler and Gisiro Maruyama. Unfortunately the same generalization cannot be done for the other methods from deterministic theory [1],e.g. Runge–Kutta schemes.

Consider the stochastic differential equation (see Itō calculus)

$dX_t = a(X_t)dt + b(X_t)dW_t$,.(i)

with initial condition $X_0 = x_0$, where W_t stands for the Wiener process, and suppose that we wish to solve this SDE on some interval of time [0, T]. Then the **Euler–Maruyama approximation** to the true solution X is the Markov chain Y defined as follows:

- partition the interval [0, T] into N equal subintervals of width$\Delta t > 0$:

$0 = \tau_1 < \tau_2 < .\tau_N = T$,and $\Delta t = {}^{T}/_{N}$.

- set $Y_0 = x_0$;
- recursively define, Y_n for $1 \le n \le N$ by

$Y_{n+1} = Y_n + a(Y_n)\Delta t + b(Y_n)\Delta W_n$,.(ii)

,where

$\Delta W_n = W_{\tau_{n+1}} - W_{\tau_n}$,.(iii)

The random variablesΔW_n are independent and identically distributed normal random variables with expected value zero and variance, Δt.

3. Discussion

Two naïve approaches to stochastic process comprises, the standard Weiner process and Poisson process. Where S(t) should be defined as a diffusion process with constant drift, suppose;

$dS(t) = \lambda dt + \sigma dW(t)$. (iv)

W(t) is a standard Weiner process [2-4].

Another approach; also suppose, $dS(t) = dN_\lambda(t)$.(v)

Uses a Poisson noise process [5]. As in equation(vi), S(t) also has expectationλt in equation(v). These two stochastic processes could be applied in principle to delineate the situation.

3.1 Stochastic Differential Equation

A typical stochastic differential equation is of the form

$dX_t = \mu(X_t, t)\, dt + \sigma(X_t,t)dB_t$.(vi)

where B denotes a Weiner process (standard Brownian motion).

For functions fεN, the Ito integral is

$F[f](\omega)= \int_S^T f(t,\omega)dB_t(\omega)$. (vii)

In integral form, the equation is;

$X_{t+s} - X_t = \int_t^{t+s} \mu(X_\mu, u)du + \int_t^{t+s} \sigma(X_u, u)dBu$. (viii)

For functions fεN, the Ito integral is

$F[f](\omega)= \int_S^T f(t,\omega)dB_t(\omega)$. (ix)

where B_t is 1-dimensional Brownian motion.

The Stratonovich integral, is an alternative to the Ito integral. Unlike the Ito calculus, the chain rule of ordinary calculus applies to Stratonovich stochastic integrals [6]. and the two can be converted viz the other for convenience as demanded.

A comparison of Ito and Stratonovich integral could be done.

The white noise equation;

$$\frac{dx}{dt} = b(t, X_t) + \sigma(t, X_t)W_t \text{ . (x)}$$

has the solution, X_t given as:

$$X_t = X_o + \int_o^t b(s,x)ds + \int_o^t \sigma(s, X_s)dB_s \text{ . (xi)}$$

Conversion between the Ito and Stratonovich integrals may be performed using the formula;

$$\int_o^T \sigma(X_t) o dW_t = \frac{1}{2}\int_o^T \sigma^1(X_t)dt + \int_o^T \sigma(X_t)dW_t$$

. (xi)(b)

Where X is some process, σ is a continuously differentiable function with derivative, σ^1 and the last is an Ito integral.

W(t) is used in deriving a stochastic differential equation.

4. Further Discussion

4.1 Stochastic Differential Equation for Asset Stock Price.

Write up:

$\Delta S = rS\Delta t + \sigma\sqrt{\Delta t}Z$. (xii)

; is an SDE i.e stochastic differential equation. Stochastic differential equations are no doubt encountered in diverse applications in biology and physical sciences, climate and oceanography, etc. Finance is an inevitable area of application, for instance asset or stock price can be delineated extensively by applying an SDE, stochastic differential equations follow a random pattern, obviously the SDE includes a noise or random variable or term. Noise term takes different definitions among; White noise, Poisson and Compound noise, etc, which has been previously discussed in our discussion.

The asset or stock price follows a random trajectory obvious from the stochastic differential equation(xii). It is proposed and therefore inevitable that the asset or stock price can be extensively delineated following a stochastic formulation. In the real practical scenario or market as frequently experienced in finance, there abound various forms of fluctuations from different variables or market factors, the asset or stock price does not practically follow a definite pattern or fixed predetermined law. Thus, these variations would best be explained by a stochastic differential equation.

S equivalently S(t) denotes the asset or stock price, r is the interest rate, □is the volatility, Δt is the time step or partition, Z is a stochastic random variable or noise.

Analogous to the method adopted in derivation of the expression or formula for the mass of a fish larva in an uncapped rate stochastic process, the SDE (xii) can be solved explicitly [7], and thus obtain the following expression;

$S = S_0 \exp(-\left(r - \frac{1}{2}\sigma^2\right)t + \sigma W_t)$.(xiii)

, where W_t defines a White noise or equivalently some noise or random variable [8-9].

With equation (4.0.xii), the asset or stock price can be delineated and thus sketch its path or trajectory.

Following the Euler-Maruyama method, alternative to equation (xii), it is proposed that,

$S_{n+1} = S_n + a(S_n)\Delta t + b(S_n)\Delta W_n$.(xviv)

This expression (xviv) would be quite suitable to delineate the asset stock defining a & b as appropriate parameters or constants of choice, apart from the one in (xiii).

Various pricing options comprises the European option and American option also the Asian , these can be explained a stochastic formula given the variables among the asset price(S), interest rate(r) or risk-free interest rate, expiration date(T), the exercise price (K), etc.

4.2 Pricing Options: Concise discussion:

European Option Pricing.

For example, a European call option has a payoff max(S(T)-X,0) at expiry. Assuming a log-normal process, S has the form;

$S = S_0 e^{\left(r - \frac{1}{2}\sigma^2\right)T} + \sigma\sqrt{T}Z)$.(xv)

,where Z is a standard normal N(0,1) random variable. Thus the call option can be valued by sampling S(T) if Z is generated.

4.3 American Option Pricing.

Formulation:

A general class of American pricing options can be formulated by specifying a Markov process$\{X(t), 0 \leq t \leq T\}$ representing relevant financial variables such as an underlying asset price, an option payoff h(X(t)) at time t, an instantaneous short rate process$\{r(t), 0 \leq t \leq T\}$, and a class of admissible stopping times □ with values in $[0, T]$.

The American option pricing formulation is to find the optimal expected discounted payoff

$$supE^{\sup E}_{\tau \epsilon \mathcal{T}}[e^{-\int_0^{\mathcal{T}} r(u)du} h(X(\mathcal{T})].$$

It is implicit that the expectation is taken with respect to the risk -neutral measure. In this course we assume that the short rate constant, $r(t) = r$, a non-negative constant for, $0 \leq t \leq T$.

For example, if the option can be only be exercised at times; $0 < t_1 < t_2. < t_m = T$ (this type of option is often called (Bermudan option), then the value of an American put can be written as;

$$supE_{i=1,,m}[e^{-rt_i}(K - S_i)^+].$$

,where K is the exercise price ,S_i is the underlying asset price S(t_i), r is risk-free interest rate.

More details on option pricing can be found elsewhere. However, we have given a detailed insight into the stochastic formulation of asset or stock price with requisite factors or variables mentioned.

5. CONCLUSION

A rhobust stochastic formulation would be of tremendous boost with stochastic differential equations for interpretation of asset stock price and volatility in finance, precisely asset stock issues, the Euler-Maruyama method promises to be exciting.

A stochastic formulation following Euler-Maruyama method has been proposed for asset stock interpretation.

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