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Modeling The Stirling–Ringbom Engine Cycle

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***Abstract*— The main goal of this article is to present a complete “isothermal” mathematical model of the hybrid Stirling-Ringbom engine cycle and an example of a computational procedure, TUS-SRSim, developed in Matlab/Simulink environment. The TUS-SRSim computational procedure is validated against the Stirling-Ringbom Simulator of the JLB Enterprises. Simulation results about the time variation of several key parameters of a hybrid Stirling-Ringbom engine cycle under various conditions are presented and analysed.**

***Keywords*— Stirling - Ringbom engine, Stirling cycle, mathematical modeling, computational procedure**

I. Introduction

In 1816 Robert Stirling patented a machine that produces a driving force by heated air [1]. He invented a safer substitute for the steam engines. At that time they often exploded due to the improper material implementation and caused serious injuries to people working with them. For comparison, if a heated part of the Stirling engine is damaged (part of the hot cylinder), it simply will stop.

Stirling engine operates on an external heat source. It consists of two connected cylinders with the corresponding pistons (displacer and power) and a flywheel. Its cycle is composed of four processes - isothermal compression, isochoric heat addition, isothermal expansion and isochoric heat rejection.

The type of binding of the two pistons with the flywheel defines the p-V diagram of the Stirling engine, [9, 10]. For example, on Fig. 1 is presented the p-V diagram of a Stirling engine with kinematically linked pistons [8].

The Stirling-Ringbom engine, presented on Fig. 2, is a hybrid type Stirling engine. This modification is patented by Ossian Ringbom in 1905. It is termed ”hybrid’’ because the displacer is a free piston while the power piston is kinematically linked with a flywheel [4, 5, 6]. The expansion space is object of a permanent heating, while the compression space is object to a permanent cooling.

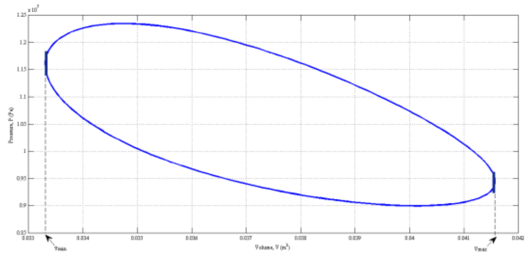

Fig. 1. *p-V diagram of the Stirling engine with kinematically binded pistons, [8]*

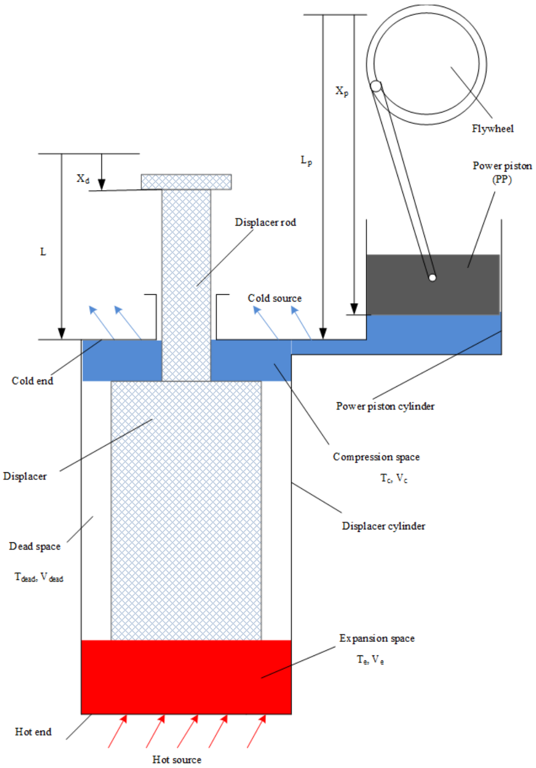

Fig. 2. *Schematic diagram of the Stirling- Ringbom engine*

In the hybrid engine designed by Ossian Ringbom the working fluid pressure pushes up the displacer from the hot end to the cold end (Fig. 2) while the power piston is moving to its own dead point. The displacer is returned back to the bottom due to its own weight. The difference between the ambient pressure and the working fluid pressure over the cross-section of the displacer rod is moving the displacer backward and forward. The working fluid pressure varies around the ambient pressure (B) and preconditions almost equal moving power over the displacer rod in both directions.

On Fig. 3 is presented the sequence of events for one cycle of the hybrid Stirling-Ringbom engine following Chen [5]. At the initial moment of time the displacer is at its bottom dead end and the power piston is at its top dead end (Fig. 3.a). At this moment the pressure of the working fluid is lower than the ambient pressure. The flywheel moves down the power piston (Fig. 3.b), which contracts the working fluid and pushes it to the displacer cylinder [5]. The working fluid is cooled at constant volume in the compression space. The power piston is moving toward its bottom dead end, the working fluid volume contracts and its pressure increases at constant temperature. The fluid pressure becomes greater than the ambient pressure near to the bottom dead end of the power piston. Than the working fluid starts to move up the displacer (Fig. 3.c). The working fluid enters the expansion space where it is heated and increases the pressure. The displacer reaches to its top dead end and holds in this position because the pressure of the working fluid has a maximal value (Fig. 3.d). The power piston starts to move to its top dead end, expands the working fluid and decreases its pressure (Fig. 3.e). The working fluid temperature decreases gradually in the compression space, the working fluid volume increases and its pressure decreases. When the gas pressure is lower than the ambient pressure the displacer starts to move down until it reaches its bottom dead end (Fig. 3.f). On Fig. 4 is presented the p-V diagram of a hybrid Stirling-Ringbom engine [8].

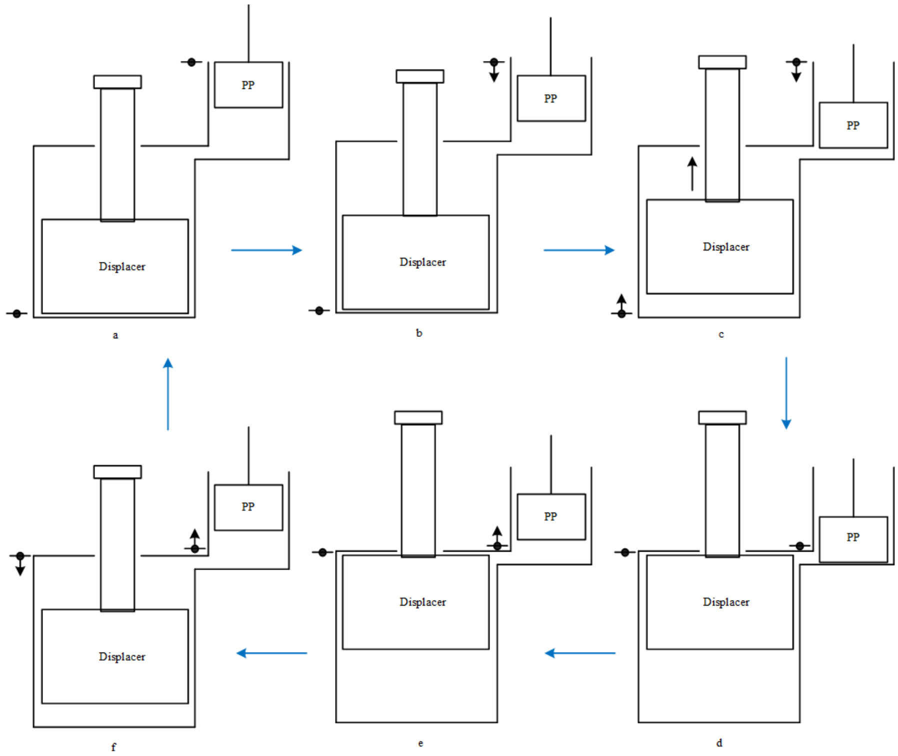

Fig. 3. *Sequence of events for one cycle of Stirling-Ringbom engine*

II. Mathematical model and computational procedure

James Senft presented some of the basic ideas of the "isothermal" hybrid model of the Stirling-Ringbom engine cycle in 1981 [5, 7]. Here it is followed the same approach. This means that the engine volume is divided into three spaces (Fig. 2): expansion space (with temperature T_e and volume V_e), compression space (T_c and V_c) and dead space (T_{dead} and V_{dead}). The dead space includes all possible interconnecting ducts and the annular gap.

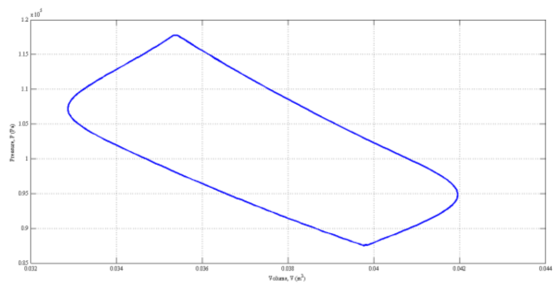

Fig. 4. *p-V diagram of a hybrid Stirling-Ringbom engine cycle, [8]*

The isothermal model is based on the following assumptions (A):

A1. The mass of the working fluid is constant;

A2. The working fluid behaves as an ideal gas;

A3. The working fluid temperature in each of the three spaces is constant;

A4. The working fluid temperature in the expansion space is equal to the heat source temperature T_{Hot};

A5. The working fluid temperature in the compression space is equal to the heat sink temperature T_{Cold};

A6. The ambient pressure is constant;

A7. The working fluid pressure in each of the three spaces is equal and varies only in time;

A8. The mean pressure of the working fluid across the cycle is equal to the ambient pressure;

A9. The dead volume is constant across the cycle;

A10. The displacer motion is physically limited by rigid stops;

A11. The power piston motion is periodical and is governed by a sinusoidal law;

A12. The gravity and viscous forces are ignored;

A13. The only force acting on the displacer when not held against one of its stops is the pressure difference between the ambient and the working fluid pressure across the displacer rod.

The mathematical model of the Stirling-Ringbom engine operation includes the relations given below.

The power piston position variation with time is described by:

$$X_p = L_p \sin(\omega t), \tag{1}$$

where:

L_p- power piston half-stroke;

ω – flywheel angular velocity, rad/s.

The variation with time of the displacer position (X_d) is calculated by (3) through the integration over time of its speed (U_d), where the U_d is obtained in terms of the following equation:

$$U_d = \frac{\mathrm{d}X_d}{dt} \tag{2}$$

$$X_d = \int_0^t U_d \,\mathrm{dt} \tag{3}$$

The time variation of the speed of the displacer piston is calculated by integration of its acceleration:

$$U_d = \int_t^{t+\Delta t} a_d \,\mathrm{dt} \tag{4}$$

The acceleration of the displacer piston is calculated by:

$$a_d = \frac{(B-P)A_r}{M_d}, \tag{5}$$

where:

B - ambient pressure;

P – working fluid pressure;

M_d – mass of the displacer assembly;

A_r – cross section of the displacer rod.

The model of the displacer motion is the following:

- if the displacer position is $X_d \geq L$ and $B \geq P$, then $X_d = L$;
- if $X_d \leq -L$ and $B \leq P$, then $X_d = -L$.
- in other cases the displacer acceleration is calculated by (5).

The expansion volume V_e and the compression volume V_c are calculated with the positions of the pistons as follows:

$$V_e = \mathrm{A}(\mathrm{L} - X_d) \tag{6}$$

$$V_c = (A - A_r)(L + X_d) + A_p(L_p - X_p) \tag{7}$$

where:

A, A_p – cross section of the displacer and the power piston, respectively;

L – displacer half-stroke;

X_d, X_p – displacer position and power piston position at time t.

The total volume occupied by the working fluid is calculated by:

$$\mathrm{V} = V_e + V_c + V_{dead} \tag{8}$$

The working fluid pressure is calculated from the ideal gas equation of state:

$$P = \frac{M.R}{\left(\frac{V_e}{T_e} + \frac{V_c}{T_c} + \frac{V_{dead}}{T_{dead}}\right)} \tag{9}$$

where:

M- mass of the working fluid;

R – ideal gas constant of the working fluid.

According to the assumptions A3, A4, and A5 the working fluid temperature in the expansion space T_e is equal to the temperature of the heat source, the working fluid temperature in the compression space T_c is equal to the temperature of the heat sink and working fluid temperature in the dead space, which is constant across the cycle, and is calculated by:

$$T_{dead} = \frac{T_e + T_c}{2} \tag{10}$$

The rate of change of the expansion volume is obtained from the displacer piston speed:

$$\frac{dV_e}{dt} = -\mathrm{A}\frac{dX_d}{dt} \tag{11}$$

The compression volume time variation along the cycle is given by:

$$\frac{dV_c}{dt} = (\mathrm{A} - A_r)\frac{dX_d}{dt} - A_p\frac{dX_p}{dt} \tag{12}$$

where the power piston speed U_p=dX_p/dt is calculated by:

$$\frac{dX_p}{dt} = L_p * \omega * cos(\omega t) \tag{13}$$

The mathematical model comprising assumptions A1-A13 and equations (1) - (13), given above, is used for the development of a computational procedure in Matlab/Simulink environment. The principle structure of this procedure named TUS-SRSim is presented on Fig. 5.

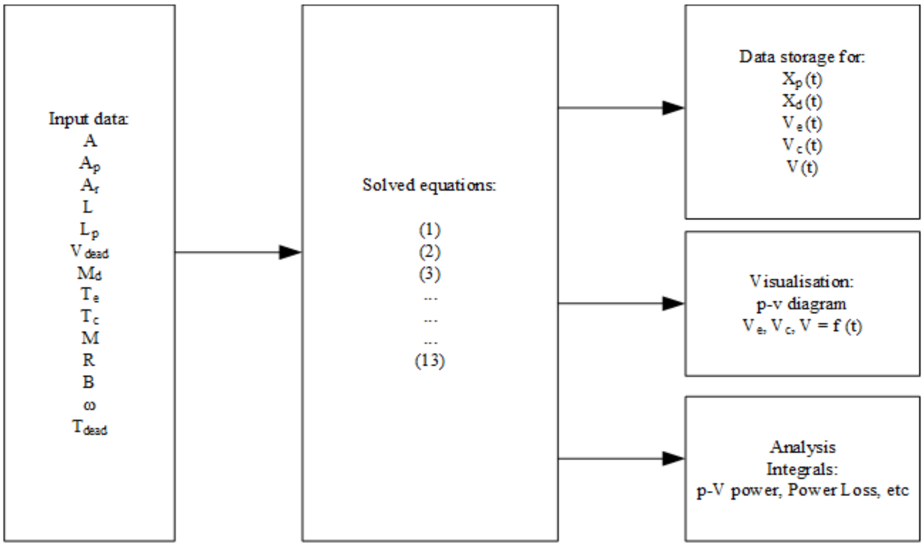

Fig. 5. *The principle structure of TUS-SRSim computational procedure*

III. RESULTS AND DISCUSSIONS

The TUS-SRSim procedure is used to investigate the hybrid Stirling-Ringbom engine presented in [8]. The required input parameters according to Fig. 5, taken from [8], are presented in Table I. Further in this text these conditions will be referred to as design conditions. Thus, two tasks are completed. First, the results obtained for the design conditions are compared with the results obtained by Stirling-Ringbom Simulator of JLB Enterprises [9]. Second, the work of the engine is investigated under various ambient pressures differing from the design one.

TABLE I. DESIGN CONDITIONS FOR THE STIRLING-RINGBOM ENGINE

Parameter	**Dimension**	**Value**
Cross section of the displacer piston, A	m^2	0.01319167
Cross section of the power piston, A_p	m^2	0.00082958
Power piston half-stroke, L_p	m	0.015
Dead volume, V_{dead}	m^3	0.0000145
Expansion space temperature, T_e	K	313.15
Dead space temperature, T_{dead}	K	302.15
Frequency, f	Hz	16.67
Mass of working fluid (air), M	kg	1.93E-04
Cross section of the displacer rod, A_r	m^2	7.13306E-05
Displacer half-stroke, L	m	0.00523
Mass of the displacer assembly, M_d	kg	0.006
Compression space temperature, T_c	K	291.15
Air gas constant, R	J/kgK	287
Air heat capacity at constant volume, c_v	J/kgK	717.5
Ambient pressure, B	Pa	101325

The p-V diagram of the Stirling-Ringbom engine cycle obtained in both models is presented on Fig. 6. Here V is the total working fluid volume, calculated by (8).

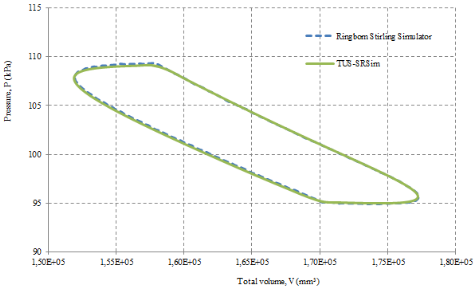

Fig. 6. *p-V diagrams of the simulated Stirling-Ringbom engine*

The time variation of the power piston position X_p obtained in both models is presented on Fig. 7.

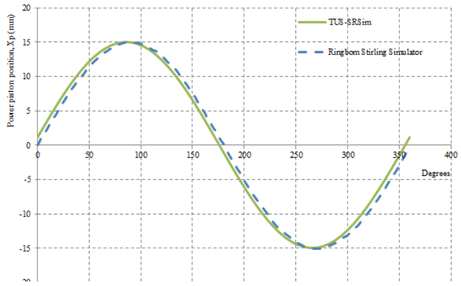

Fig. 7. *Time variation of the power piston position*

The time variation of the displacer piston position X_d obtained in the both models is presented on Fig. 8.

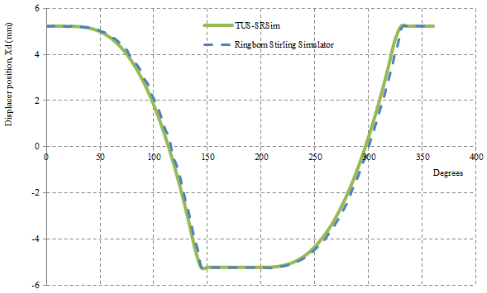

Fig. 8. *Time variation of the displacer piston position*

The results visualized on Fig. 6, 7 and 8, show that the presented here mathematical model of the hybrid Stirling-Ringbom engine corresponds well with the one described in [9].

The flywheel helps the power piston to move continuously while the displacer motion depends on the difference between the working fluid pressure and the ambient pressure and hence the displacer stops for a while at both the top dead end and the bottom dead end. Fig.9 shows the time variation of the position of the power piston and the displacer obtained by the TUS-SRSim. The character of the movement of both pistons and the phase shift between them is clearly demonstrated on this figure.

The time variation of the three volumes (total, expansion, and compression) is shown on Fig.10. The expansion volume V_e depends only on the displacer position and its variation follows the displacer's position. The compression volume V_c depends on the positions of the two pistons. According to (8) the total volume V depends on the variation of both volumes.

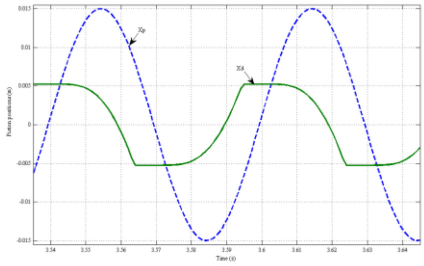

Fig. 9. *Time variation of the power piston X_p and the displacer X_d*

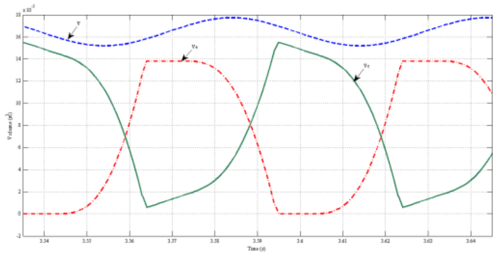

Fig. 10. *Time variation of the expansion volume V_e, compression volume V_c and the total volume V*

The efficiency of the hybrid Stirling-Ringbom engine depends strongly on the ambient pressure. When the ambient pressure differs from the design one, but the working fluid mass is the same as at the design conditions then assumption A8 is not met and the shape of the p-V diagram and its area (Fig. 11) differ substantially from those presented on the Fig. 6.

On Fig. 12 the p-V diagrams of the investigated Stirling-Ringbom engine are presented for four different values of the ambient pressure in the range from 93 kPa to 98 kPa. In this particular case the working fluid mass is corrected to meet the condition of assumption A8. Fig. 11 and Fig. 12 show that in order to ensure correct operation (maximum power) of the investigated engine under varying ambient conditions, therefore the mass of the working fluid must be changed correspondingly.

On Fig. 13 the time variation of the displacer piston motion is visualized under design conditions (ambient pressure of 101.325 kPa) and with the same mass of the working fluid at ambient pressure of 98 kPa. The dotted line shows the displacer motion at B = 101.325kPa at which the Stirling-Ringbom engine operates properly. The solid line denotes the displacer motion at B = 98kPa. Under this condition the displacer does not reach the upper dead end. It makes small movements near its bottom dead end.

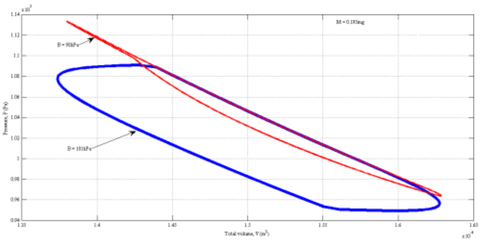

Fig. 11. *p-V diagrams of the investigated hybrid Stirling-Ringbom engine working with the design working fluid mass under different ambient pressures*

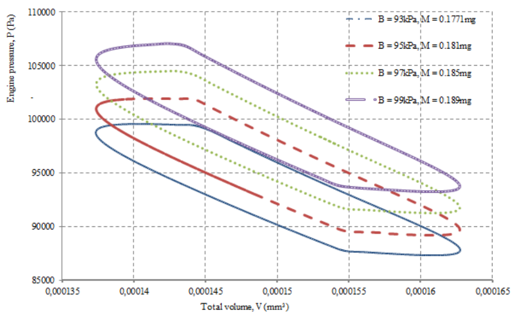

Fig. 12. *p-V diagrams of the Stirling-Ringbom engine at varying ambient pressure B and the corresponding working fluid mass M*

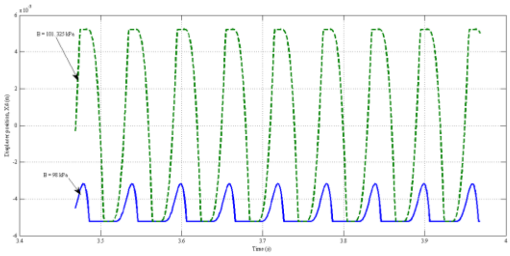

Fig. 13. *Displacer piston motion under design conditions and at ambient pressure of 98 kPa*

Fig. 14 presents the variation of the cycle net power of the investigated engine with the change in the ambient pressure. On Fig. 15 the power loss is presented according to the variations of the ambient pressure. Both quantities increase linearly with the increase of the ambient pressure. The power loss is associated with the acceleration of the displacer: during the engine operation, the displacer stays for a while at its upper and lower dead point. Therefore, part of the mechanical work is used for accelerating this piston. With the increase of the ambient pressure, the energy required for accelerating the displacer increases.

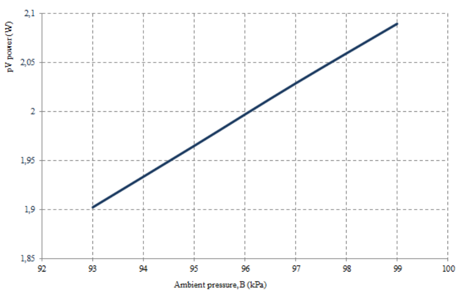

Fig. 14. *Cycle net power variation with the ambient pressure*

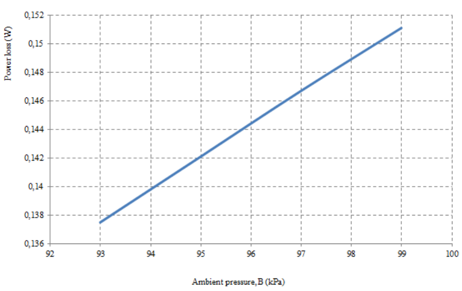

Fig. 15. *Variation of power loss with the ambient pressure*

IV. Conclusion

The main goal of the present work was to investigate the created isothermal model TUS-SRSim and to validated it against preliminary established similar model described in [9]. For that purpose the main characteristics of the Stirling-Ringbom engine were compared: the actual p-V diagram and the pistons motions. In addition, the variations in the three volumes V_e, V_c ang V during the engine operation were studied as well as the engine's behavior while operating at different ambient pressure conditions with constant fluid mass and with modified fluid mass. The main conclusions can be summarized as follows:

1. The results obtained by the TUS-SRSim are the same as the results obtained by the Stirling Ringbom Simulator of JLB Enterprises. The capacity of the mathematical model described here, composed of assumptions A1-A13 and equations (1)-(13), is validated.

2. The results obtained for the different ambient pressure conditions show that the TUS-SRSim can be used for detailed analysis of the hybrid Stirling-Ringbom engine cycle.

3. The cycle net power of the hybrid Stirling-Ringbom engine depends strongly on the mass of the working fluid and the ambient pressure.

4. In order to ensure correct operation (maximum power) of the hybrid Stirling-Ringbom engine under varying outdoor conditions, the mass of the working fluid must be changed correspondingly.

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