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Markov Chain Analysis Of Electromagnetic Scattering By A Random One-Dimensional Rough Surface

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***Abstract*—Several methods have been developed for electromagnetic scattering analysis by random rough surfaces. Such methods include the classic asymptotic model, numerical computation using monte –Carlo method. But to our knowledge only one paper exist in the literature that used Markov chain method to study electromagnetic scattering by rough surface. On that paper one dimensional rough surface is generated by calculating the probabilities of tossing a number of coins. But for our study one-dimensional rough surface is generated using MATLAB and it is partitioned in seven different levels. The surface levels correspond to the states in the Markov transition matrix. Ray tracing is used to investigate the electromagnetic scattering by this surface and the scattering field is plotted**

Keywords—Markov chain, Electromagnetic scattering, Rough surface, Transition matrix, Scattering coefficient, Ray tracing.

I. INTRODUCTION

The studies of wave scattering from random rough surface have attracted researchers and the problem of calculating its statistical properties has received consideration in recent years. Several methods has been used for rough surface scattering. The classical analytic method of Kirchhoff approximation, small perturbation, and Rayleigh-Rice perturbation theory [1],[2], [20] have been utilized to solve problems. Some applications of this topic are surface physics, satellite remote sensing, and radar data interpretation. But Kirchhoff approximation and small perturbation methods are restricted in domain of validity. For an important incident angle the shadowing effects will happen and then the classical Kirchhoff method will not be useful.

Monte Carlo simulations of wave scattering have become a popular approach due to the advent of modern computers and the development of fast numerical methods [3], [4]. When a wave travels from one medium to another, it is scattered in different direction. The scattered wave depends not only on the incident angle but also on the wavelength. At the end point, the field can be composed of direct rays and wave reflected from the terrain. Electromagnetic wave scattering by smooth surface is computed efficiently, but if the surface is rough which is usually the case then an efficient method is needed to investigate [5].

In this paper, we study one-dimensional rough surface which is generated using MATLAB and the surface is partitioned in different levels. The surface levels correspond to the states in the Markov transition matrix. Ray tracing is used to investigate the electromagnetic scattering by this surface and the scattering coefficient is plotted against the scattered angle. From that plot, the scattering coefficient distribution increases with scattered angle up to a stationary point where it starts decreasing.

II. THEORY AND NUMERICAL ANALYSIS

A. Random rough surface

From electromagnetic point of view there are two main criteria which determine the surface roughness - the Rayleigh and the Fraunhofer, respectively [5], [6], [7].Let consider electromagnetic wave transmitted onto a rough surface with local incident angle θ (see Figure 1). Then the phase difference between two rays scattered is calculated according to the formula:

$$\Delta\emptyset = \frac{4\pi H}{\lambda} cos\theta \qquad (1)$$

where $\lambda\ and\ H$ are the wavelength and the standard deviation respectively of the roughness height. According to Rayleigh definition a surface is smooth when $\Delta\emptyset < \frac{\pi}{2}$ and for Frauenhofer the surface is smooth when $\Delta\emptyset < \frac{\pi}{8}$

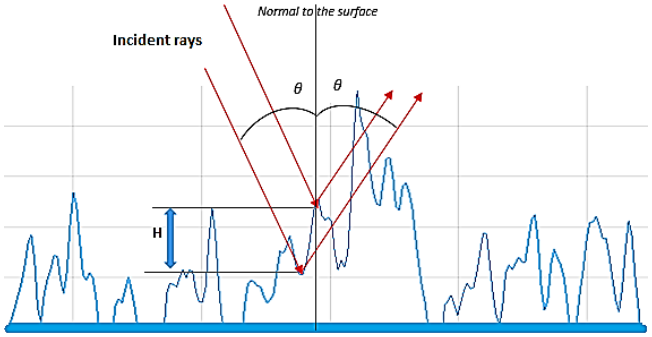

Figure 1. Field Scattering on Rough surface

B. *Parameters for one-dimensional(1D) rough surface:*

Several methods exit for generating random rough surfaces. These include Convolution method and Discrete Fourier Transform (DFT). In general a surface is characterized by its spectrum density, correlation length, autocorrelation function and its autocovariance. Those parameters that characterize the surface are computed analytically [8], [9].

The Gaussian height probability is given by:

$$P\big(f(x)\big) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(f(x)-\mu)^2}{2\sigma^2}} \quad (2)$$

where $f(x)$ represent the height function of random rough surface, μ its mean, and σ is the standard deviation of the height.

The spectrum density for Gaussian spectrum is expressed as follow:

$$W(K) = \frac{cl\sigma^2}{2\sqrt{\pi}} e^{-\left(\frac{Kcl}{2}\right)^2} \quad (3)$$

where cl represents the correlation length.

From this equation we derive the autocorrelation function (ACF) which is given by

$$ACF = \int_{-\infty}^{+\infty} W(K) e^{iKx} dK = h^2 e^{\frac{-x^2}{cl^2}} \quad (4)$$

The autocovariance function is described by a Gaussian function:

$$ACV = \langle f(x_1) | f(x_2) \rangle = \sigma^2 e^{-\frac{|x_1 - x_2|^2}{cl^2}} \quad (5)$$

where x_1 and x_2 are two points on the surface.

C. *One-dimensional rough surface algorithm*

As we mentioned before, we used the Discrete Fourier Transform (DFT) method to generate the surface [9]. This method requires the complex type of DFT and random number generator. For the complex type of 1D array the Discrete Fourier Transform is defined by

$$F = DFT(f) \text{ with } f = f_p + if_q = (f_0, f_1 \dots\dots f_{N-2}, f_{N-1})$$

Then $F = F_p + iF_q = (F_0, F_1 \dots\dots F_{N-2}, F_{N-1})$

$$F\big(K_{x_n}\big) = \sum_{n=0}^{N-1} f_n(x) e^{-i K_{x_n} x} \quad (6)$$

where $K_{x_n} = \frac{2\pi n}{N}$ and N represent the length of surface.

We get the height function $z = f(x)$ by taking the discrete inverse Fourier transform of $\big(K_{x_n}\big)$.

$$f_n(x) = \frac{1}{N} \sum_{x=0}^{N-1} F\big(K_{x_n}\big) e^{i K_{x_n} x}, \; n = 0,1,2 \dots . N-1 \quad (7)$$

D. *Simulation of 1D rough surface :*

One dimensional random rough surface f(x) has been generated with N surface points.
The surface has a Gaussian height distribution and an exponential autocovariance function, where N-number of surface points, rL-length of surface, h-rms height, cl-correlation length. The MATLAB program is used to generate the simulation results. Figure 2 shows the surface generated and table1 presents the values of the parameters used to generate this rough surface.

Table 1. Rough surface parameters values.

Number of surface points (N)	Length of surface (rL)	Rms height(h)	Correlation Length(cl)
500	1λ	0.01λ	0.001λ

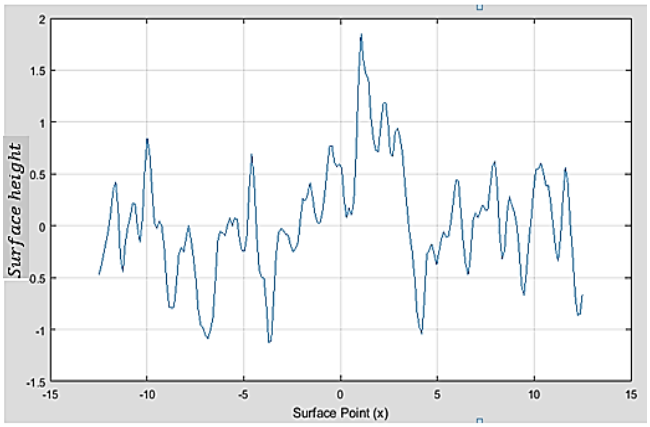

Figure 2. Random Rough Surface

III. MARKOV CHAIN

Markov chain is used in our study in order to determine the probability of a random element to transit from any level of the surface to another level and the probability to remain at the same level of the surface. Here Markov chain is defined first and then we describe the model of Markov that will be used in our study [10], [11], [12], [13].

1. Definition

Markov Chains are named after the Russian mathematician Andrey Markov who invented them.
A stochastic process (x_t) has Markovian property if $P(x_{t+1} = q | x_0 = s_0, x_1 = s_1, \ldots\ldots, x_t = p) =$

$$P(x_{t+1} = q | x_t = p), p, q \in X, t = 0,1,2, \ldots. \quad (8)$$

This is a stochastic process for which future states are conditionally independent of their past states but dependent only upon the present states.
The conditional probabilities $P(x_{t+1} = q | x_t = p)$ are called one step transition probabilities.
If for each $p \text{ and } q$, $P(x_{t+1} = q | x_t = p) = P(x_1 = q | x_0 = p)$ for all $t = 0,1,2, \ldots.$, then the one step transition probabilities are said to be stationary and are denoted by P_{pq}.
The stationary transition probability simple means that the transition probabilities do not change over time.
And it implies that

$$P(x_{t+n} = q | x_t = p) = P(x_n = q | x_0 = p) \text{ for each } p, q, and\ n = 0,1,2 \ldots \quad (9)$$

This is generally denoted as n-step transition probabilities ${P_{pq}}^n$

- If $n = 1$, ${P_{pq}}^1 = P_{pq}$
- ${P_{pq}}^n \geq 0$
- $\sum_q^M {P_{pq}}^n = 1$ for all $p, q, and\ n = 0,1,2..$ (10)

A notation to represent the n- step transition probabilities is given by

$$\boldsymbol{P^n} = \begin{bmatrix} P_{00} & P_{01} & P_{02} & \cdots \\ P_{10} & P_{11} & P_{12} & \cdots \\ P_{20} & P_{21} & P_{22} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \quad (11)$$

2. Calculation of the transition probabilities

In order to determine the transition probabilities, the joint probabilities and conditional properties have been utilized and some of those properties are [11], [12]: for continuous random variables $X \text{ and } Y$ the function $f(x, y)$ is joint density function if

i. $f(x, y) \geq 0$ for all (x, y)

ii. $P[(X, Y) \in A] = \iint_A f(x, y)dxdy$ for any region A (12)

The conditional distribution of the random variable, discrete or continuous $Y given\ that\ X = x$ is

$$f(y|x) = \frac{f(x,y)}{g(x)}, \text{ where } g(x) > 0 \quad (13)$$

Similarly, the conditional distribution of the random variable, discrete or continuous $X\ given\ that\ Y = y$ is

$$f(x|y) = \frac{f(x,y)}{h(y)}, \text{ where } h(y) > 0 \quad (14)$$

Also we used the following property which is the area under the curve [a, b] (see figure 3).

$$P(a < X < b) = \int_a^b f(x)dx \quad (15)$$

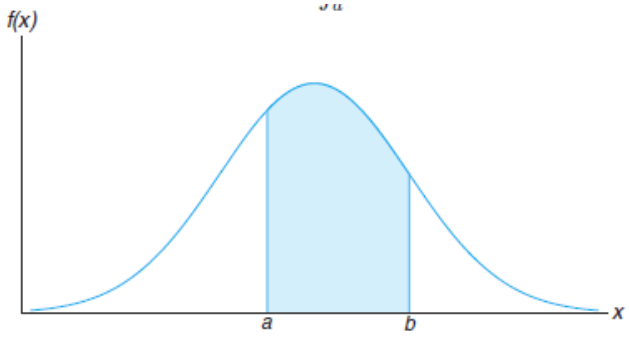

Figure 3. Probability distribution function with area under the curve

3. Method of analysis: Finite-state Markov (FSMC)

Several model of Markov analysis exist such as Gibbs sampler, Metropolis Hastings, Hidden Markov model. Here the finite-state-Markov Chain (FSMC) is used [16]. Usually this method of Markov Chain is used to model radio Communication Channel. In this method the states correspond to partitioning the random rough surface heights (amplitude) probability distribution into k nonoverlapping regions such as

$[h_0, h_1), [h_1, h_2), \ldots [h_{k-1}, \infty)$ where the h_k represent surface high. Figure 4 shows the surface height partitioned into seven regions and figure 5 presents the transition probabilities between different states. For our study this method is used:

- First a random rough surface is generated
- Second the surface height (amplitude) probability distribution is plotted.
- Third the random rough surface height probability distribution is portioned into k nonoverlapping regions where each region corresponds to state space $S = \{1, \ldots, N\}$. And we assigned the midpoints of the intervals as the representative value of each state, r_k,

$k = 1,2, \ldots \ldots N$ .And from that the Markov transition matrix is obtained [14], [15].

$$\tilde{P}_{p,q} = P(R_t = q | R_{t-1} = p)$$

where

$$P(R_t = q | R_{t-1} = p) = \int_{\mathbf{h_{q-1}}}^{\mathbf{h_q}} \int_{\mathbf{h_{p-1}}}^{\mathbf{h_p}} \mathbf{f(r_2 | r_1) dr_1 dr_2}$$

then $$\widetilde{\mathrm{P}}_{\mathrm{p,q}} = \frac{\int_{\mathbf{h_{q-1}}}^{\mathbf{h_q}} \int_{\mathbf{h_{p-1}}}^{\mathbf{h_p}} \mathbf{f(r_2, r_1) dr_1 dr_2}}{\int_{\mathbf{h_{p-1}}}^{\mathbf{h_p}} \mathbf{f(r) dr}} \quad (16)$$

For $1 \leq p, q \leq N$, where $f(r_1, r_2)$ is the two-dimensional unit-variance Gaussian probability distribution function (pdf).The transition probability matrix must be adjusted such that the rows sum to exactly one [15]. This is because of the finite high representation of the model. Each row is uniformly scaled by its sum.
Therefore

$$P_{pq} = \frac{\tilde{P}_{p,q}}{\sum_{q=1}^{N} \tilde{P}_{p,q}} \quad (17)$$

For $n - step$ transition probabilities, if n is large enough all the rows or column of the matrix have identical entries. The steady state equation is given by

$\hat{\pi}_q = \sum_{q=0}^{N} \hat{\pi}_p P_{pq}$ $for\ q = 0,1,.,N$ and $\sum_{q=0}^{N} \hat{\pi}_q = 1$ (18)

$$\mathrm{Lim}_{n \to \infty} {P_{pq}}^n = \hat{\pi}_q > 0 \quad (19)$$

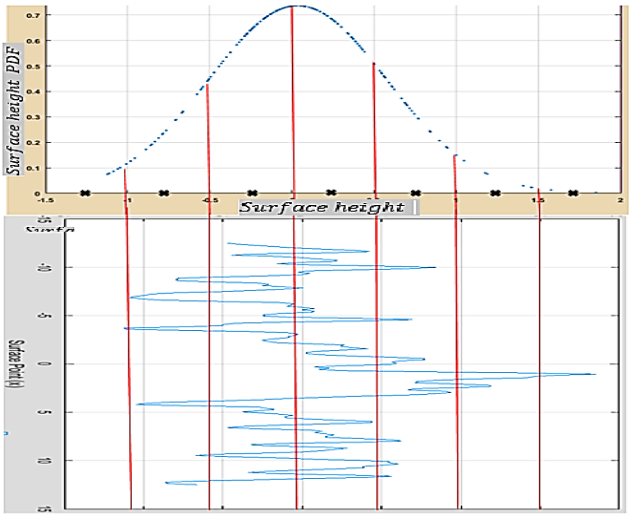

Figure 4. Relationship between rough surface amplitude partition and FSMC

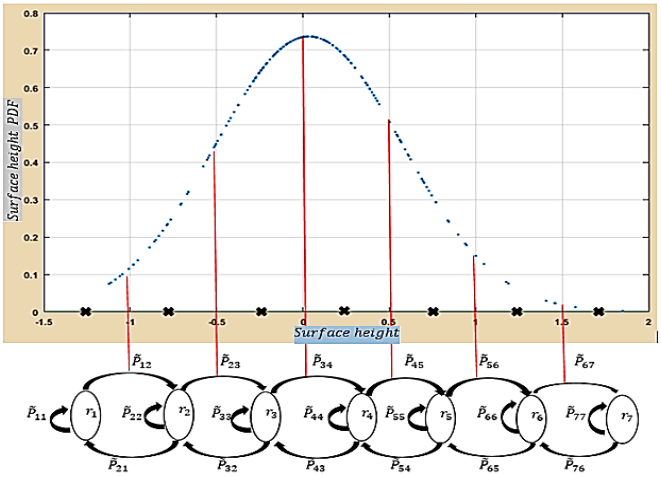

Figure 5. Rough surface amplitude probability distribution with different transition probabilities

Table 2. Rough surface height values

h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7
-1.5	-1	-0,5	0	0.5	1	1.5	2

The random rough surface is divided in seven level (or height).
$Level\ r_1\ represents\ the\ midpoint\ of the\ interval\ betwen\ h_0\ and\ h_1.$
$Level\ r_2\ represents\ the\ midpoint\ of\ the\ interval\ betwen\ h_1\ and\ h_2$
$Level\ r_3\ represents\ the\ midpoint\ of\ \ the\ interval\ betwen\ h_2\ and\ h_3$
$Level\ r_4\ represents\ the\ midpoint\ of\ \ the\ interval\ betwen\ h_3\ and\ h_4$
$Level\ r_5\ represents\ the\ midpoint\ of\ \ the\ interval\ betwen\ h_4\ and\ h_5$
$Level\ r_6\ represents\ the\ midpoint\ of\ the\ interval\ betwen\ h_5\ and\ h_6$
$Level\ r_7\ represents\ the\ midpoint\ of\ \ the\ interval\ betwen\ h_6\ and\ h_7$

IV. SIMULATION AND NUMERICAL CALCULATION

A. *Transition Matrix numerical calculation*

We have determined each value of $\tilde{P}_{pq}$ where $p, q = 1,2,,N$ and MATLAB is used to do the computation. If $p = q$ then $\tilde{P}_{11}$ represent the area under the curve of the surface height Pdf from $h_0 = -1.5\ to\ h_1 = -1$ and this area $\tilde{P}_{11} = 0.026$. Similarly $\tilde{P}_{22}$, $\tilde{P}_{33}$, …. , $\tilde{P}_{77}$ are computed. But $p \neq q$ then the computation is based on equation (16). For instance based on equation (16), the probability of transiting from state 1 to state 2 is given by

$$\tilde{P}_{1,2} = \frac{\int_{h_1}^{h_2}\int_{h_0}^{h_1} f(r_2,r_1)dr_1dr_2}{\int_{h_0}^{h_1} f(r)dr} \quad \text{therefore} \quad \tilde{P}_{1,2} = 0.618$$

.

After all the probability calculation we got the following transition matrix:

Table 3. Transition Probability

	0.026	0.618	0	0	0	0	0
	0.119	0.134	0.773	0	0	0	0
	0	0.33	0.313	0.652	0	0	0
$\widetilde{P_{pq}} =$	0	0	0.617	0.331	0.369	0	0
	0	0	0	0.773	0.158	0.14	0
	0	0	0	0	0.653	0.034	0.036
	0	0	0	0	0	0.372	0.003

From this transition matrix (see table 3) we have for instance $\tilde{P}_{11} = 0.026\ \ and\ \tilde{P}_{12} = 0.618$.
As mentioned before this transition probability matrix must be adjusted such that the rows sum exactly to one. To do so, the equation (17) is used. Therefore we get the following matrix, which represents our state transition matrix (table 4). As it can be notified each row sum to one.

Table 4. Transition probability matrix: where each row is uniforly scaled by its sums

	0.04	0.96	0	0	0	0	0
	0.116	0.13	0.754	0	0	0	0
	0	0.255	0.241	0.504	0	0	0
$P_{pq}{}^{1} =$	0	0	0.468	0.251	0.281	0	0
	0	0	0	0.722	0.147	0.131	0
	0	0	0	0	0.903	0.047	0.0495319
	0	0	0	0	0	0.991	0.0087039

In addition, we observe from this matrix that some of probabilities are zero. It simply means that there is no probability to connect some levels from another level. For example $P_{13} = 0$, then the probability that a random element will connect level 1 (stater_1) to level 3 (state r_3) is zero.

In order to avoid the zero probability, the process is left running for a long period of time (up to 6-step), so that the process reached at the point where any level can be reached from all other level (see figure 5). It means there is probability that a random element can link all the levels each other.

Table 5. 6-step Transition probability matrix (where $n = 6$)

	Output (6-step) Transition Matrix						
	0.021	0.125	0.39	0.322	0.13	0.011	0.0006634
	0.015	0.136	0.337	0.373	0.12	0.018	0.0005706
	0.016	0.114	0.358	0.35	0.144	0.018	0.0010195
$P_{pq}{}^{6} =$	0.012	0.117	0.325	0.382	0.139	0.023	0.0009815
	0.013	0.097	0.344	0.358	0.165	0.022	0.0015404
	0.008	0.103	0.292	0.411	0.152	0.032	0.0013161
	0.009	0.064	0.337	0.349	0.213	0.026	0.0028469

Table 6. Steady State transition matrix (where $n = 22$)

	Output (22-step) transition matrix						
	0.014	0.115	0.341	0.366	0.142	0.021	0.001
	0.014	0.115	0.341	0.366	0.142	0.021	0.001
	0.014	0.115	0.341	0.366	0.142	0.021	0.001
$\hat{\Pi p}$	0.014	0.115	0.341	0.366	0.142	0.021	0.001
	0.014	0.115	0.341	0.366	0.142	0.021	0.001
	0.014	0.115	0.341	0.366	0.142	0.021	0.001
	0.014	0.115	0.341	0.366	0.142	0.021	0.001

After running the process for certain period of the time, we reach the steady state matrix, where each element of each column has the same value (see table 6). For example the column two has probability of 0.115.For our study, after 22-step of process the steady state is reached.

The probability that electromagnetic field will transit from one level to another random level is given in matrix L_{pq} [6] (see table 9).

$$L_{pq} = \pi_p P_{pq}{}^{n} \qquad where\ \ n = 1,2,..$$

$$\pi_p = P_p\delta_{pq} \quad \text{with} \quad \delta_{pq} = \begin{cases} 0\ \ for\ p \neq q \\ 1\ \ for\ p = q \end{cases} \qquad (20)$$

where π_p is a diagonal Matrix (see table 8)
This is called Kronecker delta function.

Also we have $\sum_{q=1}^{k}\sum_{p=1}^{k} L_{pq} = 1$, where k is the maximum number of levels and here $k = 7$

Table 8. Diagonal Matrix

	0.014	0	0	0	0	0	0
	0	0.115	0	0	0	0	0
$\prod_p = P_p\delta_{pq}$	0	0	0.341	0	0	0	0
	0	0	0	0.366	0	0	0
	0	0	0	0	0.142	0	0
	0	0	0	0	0	0.021	0
	0	0	0	0	0	0	0.001

Table 9. Transition probability where the scattered field will transit from p level to level q

	0.0003	0.00175	0.00545	0.00451	0.00182	0.00016	9.3E-06
	0.00174	0.01571	0.03886	0.04296	0.01377	0.00213	6.6E-05
	0.00541	0.03886	0.12192	0.11918	0.04892	0.00603	0.00035
$L_{pq} = \prod_p {P_{pq}}^6$	0.00448	0.04295	0.11918	0.13974	0.05102	0.00848	0.00036
	0.0018	0.01377	0.04892	0.05102	0.02347	0.00314	0.00022
	0.00016	0.00213	0.00603	0.00849	0.00315	0.00066	2.7E-05
	9.2E-06	6.6E-05	0.00035	0.00036	0.00022	2.7E-05	2.9E-06

This matrix shows the probability that the scattered field will transit from p level to level q. And it presents all the different possibilities. For instance let consider the first row, if the electromagnetic wave is scattered at the first state (level 1), there is a possibility that it will stay in the same level, or it can rise to the second state and so on up seven level. Also if the wave scattered at the second level (the second row), there is possibility that it will scatter at the first level, or remain in the second level, or that the wave will rise to the third or fourth.

For example the probability that a wave scatters at level r_1 without any transition to the next level is given by $L_{11} = 0.0003$ with $p, q = 1$. And the probability that scattered field will go from r_1 to r_2 is given by $L_{12} = 0.00175$ where $p = 1\ and\ q = 2$. We have state $q\ (level\ r_2)$ is said to be reachable from state $p\ (level\ r_1)$, if there exists $n \geq 1$ so that ${P_{pq}}^n > 0$.

B. Trace of the Matrix L_{pq}

By definition trace of a matrix (L_{pq}) is the sum of the diagonal elements of that matrix and denoted by $tr(L_{pq})$ [6]. Let set an integer $I \in [-r_k, r_k]\ interval$ with $I = q - p$ and r_k the number of levels. For our study the number of level is 7. Then the probability that I is an integer between -7 and 7 is given by the sum of all matrix elements L_{pq}
And that probability is denoted $tr_{I=q-p}(L_{pq})$
For instance if $p = 1\ and\ q = 2$ then $I = 1$

Therefore $tr_{I=1}(L_{pq}) = L_{12} + L_{23} + L_{34} + L_{45} + L_{56} + L_{67} = 0.00175 + 0.03886 + 0.11918 + 0.05102 + 0.00314 + 2.7 * 10^{-5}$ (21)
(21)

then $tr_{I=1}(L_{pq}) =$ 0.2139770

V. RAY TRACING TECHNIQUE

This method is limited to certain roughness parameters but it is also the one which considers both shadowing and multiple scattering. This method is utilized here because of its easy implementation for computation purpose [17], [18]. In ray tracing method, the transmitted rays traverse the air before hit the surface boundaries and from there they get scattering in different direction.

When the beam rich the rough surface several phenomena may happen:

- The ray may just scattering once on the surface and leaves

- It may have multiple scattering before it leaves

- Also the shadowing effect is one the important phenomena that can occur.
The shadowing is when a part of the surface doesn't get electromagnetic wave from the incident direction.

1. The electromagnetic scattering by inclined surface

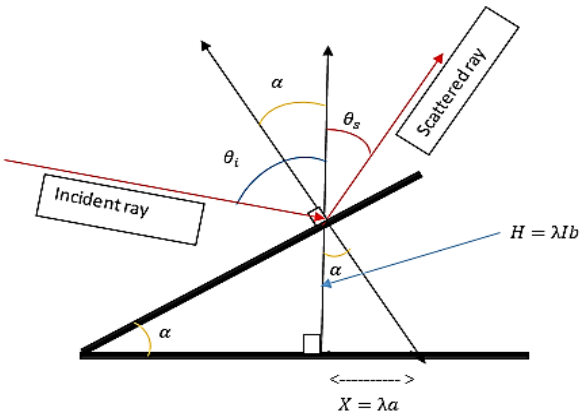

Figure 6. Field scattering by a surface with inclination α

Let consider an electromagnetic field scattered by a surface with inclination α figure 6.

Then the scattering field is given by [18]

$$E_s(\theta_s) = E_0 * F_s(\theta_i, \theta_s, n) * \exp(-j\varphi) \quad (22)$$

Where $\exp(-j\varphi)$ correspond to phase shifting due to the height H of the local facet with respect to the global mean value which is usually $at\ H = 0$

$$\varphi = \frac{4\pi}{\lambda} H * \cos(\theta_i)$$

where θ_i represent the incident angle

- Let set $\alpha = \frac{\theta_i + \theta_s}{2}$ then $\theta_i = 2\alpha - \theta_s$

From the Figure 6 we have $\tan\alpha = \frac{\lambda a}{\lambda I b} = \frac{a}{(q-p)*b}$ where $I = q - p$

Then $\alpha = \text{atan}(\frac{a}{(q-p)*b})$

Therefore we get $\theta_i = 2 * \text{atan}(\frac{a}{(q-p)*b}) - \theta_s$

Let substitute θ_i into $\varphi = \frac{4\pi}{\lambda} H * \cos(\theta_i)$ by its expression and we get

$$\varphi = \frac{4\pi}{\lambda} H * \cos(2\text{atan}(\frac{a}{(q-p)*b}) - \theta_s) \quad (23)$$

Therefore the electromagnetic scattering equation (22) becomes [7]:

$$E_s(\theta_s) = E_0 * F_s(\theta_i, \theta_s, n) \frac{1}{tr_I(L_{pq})} \sum_{I=q-p} L_{pq} \exp(-j(4\pi b(q-1) * \cos(2\text{atan}(\frac{a}{(q-1)*b}) - \theta_s))) \quad (24)$$

From equation (24) the scattering coefficient is deduced. It is the ratio between the scattered power in a solid angle $d\theta$ around θ_s and the incident power.

$$\sigma(\theta_s) = |E_s(\theta_s)| \, |E_s(\theta_s)|^* \quad (25)$$

2. *Plot of scattering coefficient σ distribution*

Case 1:

- $tr_{I=1}(L_{pq})$ where $p = 1 \; and \; q = 2$ then $I = 1$

This shows that the scattering field will go from level 1 to 2, from level 2 to 3, from level 3 to 4, from level 4 to 5, from level 5 to 6, and from level 6 to 7. Here the scattering field will not skip any level it will just scatter at the closest superior level of the surface by ascending each time to the next closest level.

Case 2:

- $tr_{I=-2}(L_{pq})$ where $p = 3 \; and \; q = 1$ then $I = -2$

Here the scattering field will go from level 3 to level 1 by skipping level 2, from level 4 to 2 by skipping level 3, from level 5 to 3 by skipping level 4, from level 6 to 4 by skipping level 5, and from level 7 to level 5 by skipping level 6. In this case the scattering field will not rise from one level to another instate it will descend by skipping each time one level.

Let plot the scattering field for $tr_{I=1}(L_{pq})$ and $tr_{I=0}(L_{pq})$

Let set $E_0 = 1 \; V/m$; n = 1 + 7j And $tr_{I=1}(L_{pq}) = 0.2139770$

$$E_s(\theta_s) =$$

$$\text{For } \begin{cases} p = 1 \\ q = 2 \end{cases} \quad F_s(\theta_i, \theta_s, n) \frac{1}{0.2139770} L_{12} * \exp(-j(4\pi b * \cos(2\text{atan}(\frac{a}{b}) - \theta_s)))$$

$$+$$

$$\text{For } \begin{cases} p = 2 \\ q = 3 \end{cases} \quad F_s(\theta_i, \theta_s, n) \frac{1}{0.2139770} L_{23} * \exp(-j(8\pi b * \cos(2\text{atan}(\frac{a}{2b}) - \theta_s)))$$

$$+$$

$$\text{For } \begin{cases} p = 3 \\ q = 4 \end{cases} \quad F_s(\theta_i, \theta_s, n) \frac{1}{0.2139770} L_{34} * \exp(-j(12\pi b * \cos(2\text{atan}(\frac{a}{3b}) - \theta_s))$$

$$+$$

$$\text{For } \begin{cases} p = 5 \\ q = 6 \end{cases} \quad F_s(\theta_i, \theta_s, n) \frac{1}{0.2139770} L_{56} * \exp(-j(20\pi b * \cos(2\text{atan}(\frac{a}{5b}) - \theta_s))$$

$$+$$

$$\text{For } \begin{cases} p = 6 \\ q = 7 \end{cases} \quad F_s(\theta_i, \theta_s, n) \frac{1}{0.2139770} L_{67} * \exp(-j(24\pi b * \cos(2\text{atan}(\frac{a}{6b}) - \theta_s)$$

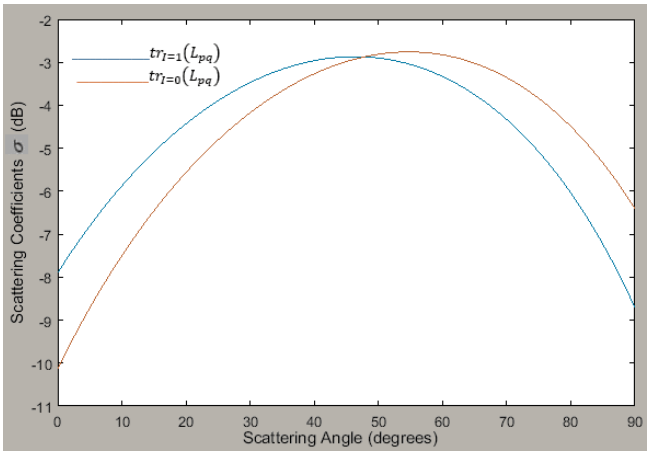

Figure 7. Scattering Distribution of σ for different trace ($tr_{I=1}(L_{pq})$ and $tr_{I=0}(L_{pq})$)

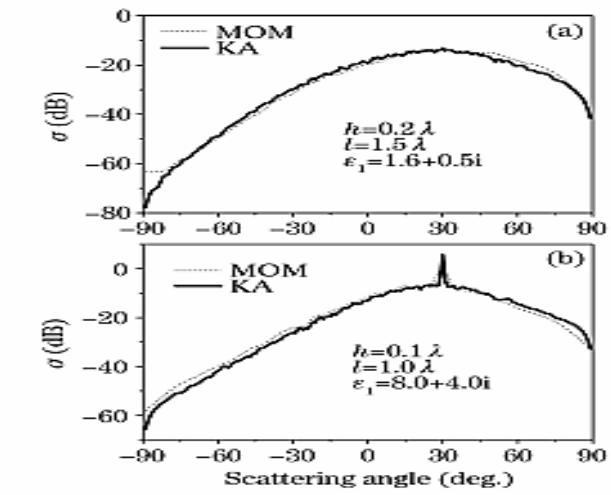

Figure 8. Comparison of scattering coefficients calculated by MOM and KA for (a) HH polarization and (b) VV polarization. Reference [19]

VI. CONCLUSION

This paper has presented Markov Chain analysis of electromagnetic wave scattering by one-dimensional rough surface. The generated one dimensional rough surface is partitioned in different levels where each level represent the state of Markov chain. The probability that a random field will scattering by one level and its probability of transiting from one level to another are calculated using equation (16). And from that Markov transition matrix is built (see Table 3).The calculations of the different probabilities are performed on MATLAB. In our simulation the distribution of scattering coefficient σ is plotted against the scattering angle (see figure 7).Our results is compared with the distribution of scattering coefficient calculated and plotted by MOM and KA (see figure 8) [19]. This comparison has shown an agreement between our work and those of others. These include the method of Moment (MOM) and Kirchhoff approximation (KA).For the future study this work will be extended to two-dimensional rough surface scattering by electromagnetic wave.

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