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A Discrete Least Squares Approximation Based Algorithm For Empirical Model Adaptation

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***Abstract*—This study proposes an algorithm for terrain-specific empirical model adaption, termed the Least Squares Approximation based Algorithm (LSAA). Using path loss data obtained from the rural area between the cities of Jos and Abuja, Nigeria, the proposed LSAA is statistically compared for adaptation accuracy with two existing techniques. The first technique has to do with the use of the Okumura G_{AREA} (Gain due to type of environment) obtained from the Okumura Curves to adapt the Okumura model to a given terrain, while the second uses a computed Root Mean Squared Error (RMSE) as correction factor. Results of the Okumura empirical propagation model adaptation to the terrain in question using the three techniques show that the proposed LSAA gives the least adaption error of 2.18dB, compared with 8.95dB for the Okumura G_{AREA} and 9.41dB for the commonly used RMSE adaptation technique. Furthermore, tests for capacity to generalize using new data sets indicate that the LSAA adapted Okumura model with a geometric mean prediction error of 4.53dB, offers greater path loss prediction accuracy than its counterparts.**

Keywords—Least Squares Approximation; Okumura Model; Path Loss Predictin; Empirical Model Adaptation

I. INTRODUCTION

Adequate knowledge of radio propagation characteristics across a specific terrain is an essential requirement in the planning of a wireless telecommunications network. Since path loss serves as the dominant factor for the characterization of a radio link, there is need for accurate path loss prediction so that the radio path can be optimally engineered. Path loss varies from one environment to the other according to the physical nature, dimensions and geometries of the various obstacles that perturb radio propagation. Hence, it is of high necessity to create prediction models that are not only very accurate, but also computationally efficient.

Although empirical models are quite simple to implement, they are not universally applicable due to terrain diversity across the globe, in spite of the availability of correction factors. As a result, empirical model adaptation or optimization is one of the numerous methods used by researchers to develop terrain-specific radio propagation models. The most popular technique for adapting empirical models has to do with the introduction of a computed error as correction factor, into empirical model expressions, as demonstrated by [1,2,3,4]. However, these correction factors in most cases only modify the constant within an expression, disregarding the slope coefficient, which is a dominant factor in determining how well an empirical model fits (or is adapted to a given terrain). By implication, if the slope of the best fit curve through measured path loss points significantly differs from that of the empirical model expression, such a technique will be highly inaccurate. As a result, these techniques are limited in terms of ability to accurately adapt empirical models to terrains due to terrain diversities. Hence, it is necessary to develop a technique that does not only provide the best possible fit for the empirical model, but also ensures that the adapted empirical model is robust when tested with new data.

In this study, a Least Squares Approximation based Algorithm (LSAA) for adapting empirical models is proposed. The LSAA provides the best possible fit for an empirical model by directly fitting the empirical model onto the best fit curve through measured path loss points, thereby ensuring greater correlation with the measured data. Using the Okumura empirical model, the new technique is statistically compared for adaptation accuracy with two existing techniques: i) the use of Okumura G_{AREA} (Gain due to type of environment) obtained from the Okumura Curves to adapt the Okumura model to a given terrain, and ii) the use of a computed Root Mean Squared Error (RMSE) as correction factor. Furthermore, the LSAA adapted Okumura model is analytically compared for path loss prediction accuracy with those adapted using the mentioned existing techniques. As case study, at an operating frequency of 900MHz, the rural area between the Nigerian cities of Jos and Abuja is considered. This terrain matches Hata's description of a suburban area.

II. THE OKUMURA MODEL

The Okumura model [5,6] was formulated based on empirical data collected in the city of Tokyo, Japan. It is one of the most widely used empirical propagation models for path loss prediction across various terrain types, classified as urban, suburban, quasi-open and open areas. This model is applicable for frequencies in the range 150 MHz to 1920 MHz (although it is typically extrapolated up to 3000 MHz) and distances ranging from 1 km to 100 km. The Okumura model path loss equation is given by (1)

$L = L_{FSL} - A_{MU} - H_{MG} - H_{BG} - G_{AREA}$ (1)

Where,

- L = Median path loss in Decibels (dB)
- L_{FSL} = Free Space Loss in Decibels (dB)
- A_{MU} = Median attenuation in Decibels (dB)
- H_{BG} = Base station antenna height gain factor given by 20log(h_b/200) for 30m<h_b<100m
- H_{MG} = Mobile station antenna height gain factor given by 10log(h_m/3) for h_m<3m
- G_{AREA}=Gain due to type of environment

The Gain due to type of environment and Median attenuation are obtained from the Okumura curves in shown in Fig. 1.

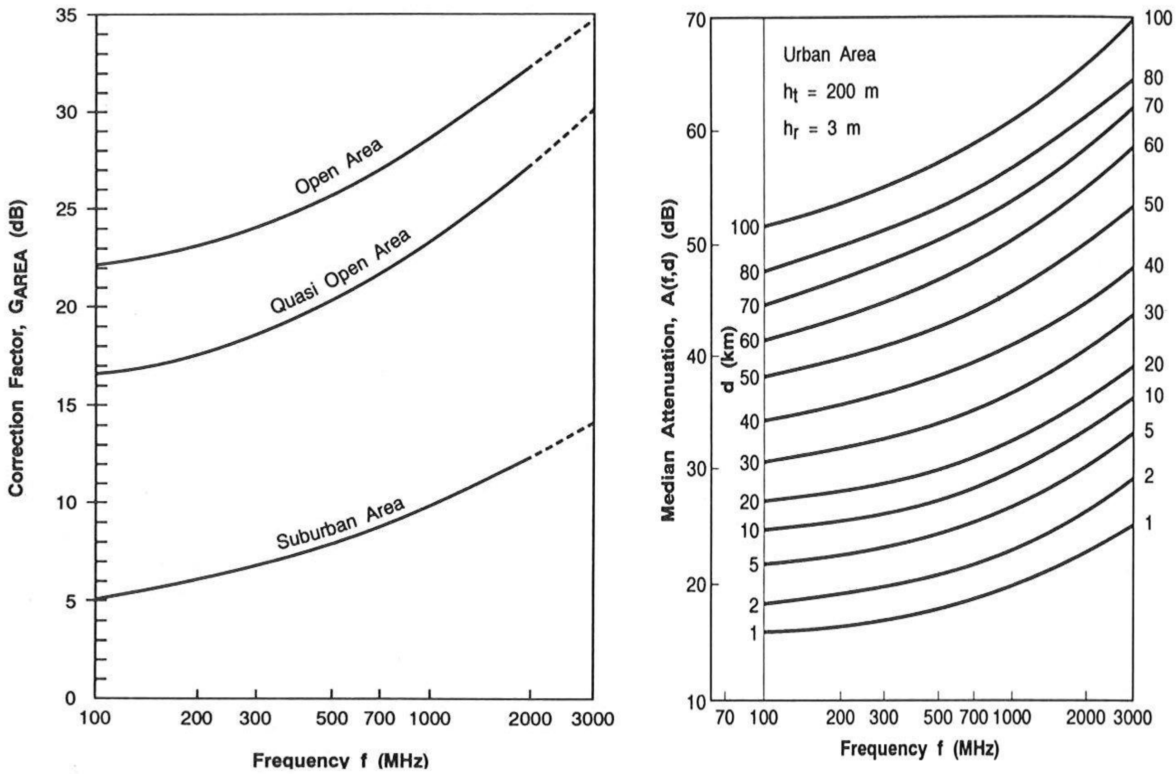

Fig. 1. *Okumura Curves for Correction Factor (G_{AREA}) and Median Attenuation*

III. DISCRETE LEAST SQUARES APPROXIMATION

As described by [7], least square approximation involves fitting a polynomial function $P(x)$ to a set of data points (x_i, y_i) having a theoretical solution depicted by (2).

$y = f(x)$ (2)

The procedure involves minimizing the squares of errors, taking into consideration a data set that satisfies the theoretical solution to equation (2) as (x_1, y_1), (x_2, y_2) . (x_n, y_n). If the polynomial to be fitted to these set of data points is denoted by $P(x)$, the curve or line represented by $P(x)$ is considered the best fit to $f(x)$, if the difference between $P(x_i)$ and $f(x_i)$, where i = 1, 2, . . . , n, is least. That is, the sum of the differences $e_i = f(x_i) - P(x_i)$, where i = 1, 2, . . . , n, should be the minimum. The sum of the differences e_i may add up to zero, thereby given the wrong error for the approximating polynomial. As such, the square of these differences are preferable. In other words, the sum of the squares of the deviations to get the best fitted curve is considered. Thus the required equation for the sum of squared errors that requires minimization is then written as (3)

$S = \sum_{i=1}^{n} [f(x_i) - P(x_i)]^2$ (3)

Where,

$P(x) = a_0 + a_1 x + a_2 x^2 +. + a_n x^n$ (4)

In order to derive the discrete function that best fits the given data points, equations (2) and (4) are substituted in (3) to give (5).

$$S = \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i + a_2 {x_i}^2 +. + a_k {x_i}^k)]^2 \quad (5)$$

To minimize S, (5) is differentiated with respect to a_i and equated to zero. Therefore, differentiating (5) partially with respect to a_0 , a_1,. a_k and equating each to zero, gives (6), which can be rewritten as (7).

Solving (7) to determine a_0 , a_1 ,. a_k , and substituting into (4) gives the best fit curve to (2). The set of equations (7) are called the Normal Equations of the Least Squares Method.

$$\left.\begin{aligned}
\frac{dS}{da_0} &= -2\sum_{i=1}^{n}[y_i - (a_0 + a_1x_i + a_2{x_i}^2 + \cdots + a_k{x_i}^k)] = 0\\
\frac{dS}{da_1} &= -2\sum_{i=1}^{n}[y_i - (a_0 + a_1x_i + a_2{x_i}^2 + \cdots + a_k{x_i}^k)]x_i = 0\\
\frac{dS}{da_2} &= -2\sum_{i=1}^{n}[y_i - (a_0 + a_1x_i + a_2{x_i}^2 + \cdots + a_k{x_i}^k)]{x_i}^2 = 0\\
&\vdots\\
\frac{dS}{da_k} &= -2\sum_{i=1}^{n}[y_i - (a_0 + a_1x_i + a_2{x_i}^2 + \cdots + a_k{x_i}^k)]{x_i}^k = 0
\end{aligned}\right\} \quad (6)$$

$$\left.\begin{aligned}
\sum_{i=1}^{n} y_i &= na_0 + a_1\sum_{i=1}^{n} x_i + a_2\sum_{i=1}^{n} x_i^2 + \cdots + a_k\sum_{i=1}^{n} x_i^k\\
\sum_{i=1}^{n} x_i y_i &= a_0\sum_{i=1}^{n} x_i + a_1\sum_{i=1}^{n} x_i^2 + a_2\sum_{i=1}^{n} x_i^3 + \cdots + a_k\sum_{i=1}^{n} x_i^{k+1}\\
\sum_{i=1}^{n} x_i^2 y_i &= a_0\sum_{i=1}^{n} x_i^2 + a_1\sum_{i=1}^{n} x_i^3 + a_2\sum_{i=1}^{n} x_i^4 + \cdots + a_k\sum_{i=1}^{n} x_i^{k+2}\\
&\vdots\\
\sum_{i=1}^{n} x_i^k y_i &= a_0\sum_{i=1}^{n} x_i^k + a_1\sum_{i=1}^{n} x_i^{k+1} + a_2\sum_{i=1}^{n} x_i^{k+2} + \cdots + a_k\sum_{i=1}^{n} x_i^{2k}
\end{aligned}\right\} \quad (7)$$

IV. REVIEW OF ADAPTATION TECHNIQUES

A. The Okumura Adaption Technique using G_{AREA}

The Okumura adaption technique using G_{AREA} (Gain due to type of area) is quite simple and straight forward. It has to do with subtracting the G_{AREA} value obtained from the Okumura Correction Factor curves in figure 1, from the Okumura Model expression (1).

B. Adaption by Computed Correction Factor

A widely used technique for adapting an empirical propagation model to a given terrain involves the use of a computed correction factor to compensate for differences between measurements and the empirical propagation model prediction. For example, as stated earlier, the use computed Root Mean Squared Error (RMSE) as correction factor was demonstrated by [1,2,3,8]. The RMSE error is given by

$$RMSE = \sqrt{\sum_{i=1}^{N}\frac{(M-P)^2}{N-1}} \quad (8)$$

Where, M is Measured Path Loss, P is Predicted Path Loss and N is Number of paired values.

Adaptation is achieved by subtracting the RMSE from the model expression if the model overestimates the path loss; else it is added to the model expression if the reverse is the case.

V. MATERIALS AND METHODS

A. Received Power Measurement and Path Loss Computation

Received power measurements were obtained at an operating frequency of 900MHz from ten MTN (Mobile Telecommunications Network) Base Stations situated within the rural area between the cities of Jos and Abuja, Nigeria. The instrument used was a Cellular Mobile Network Analyser (SAGEM OT 290) capable of measuring signal strength in decibel milliwatts (dBm), held at an average height of 1.5meters. Mobile network parameters obtained from MTN include the following: Mean Base Station height is 33meters, Mean Effective Isotropic Radiated Power (EIRP) is 46dBm.Received power (P_R) readings were recorded at intervals of 0.3km away from the Base Station, after an initial separation of 0.1kilometer. Corresponding path loss values (L_P), were computed using (3),

$$L_P = EIRP - P_R \quad (9)$$

The EIRP was determined from the expression

$$EIRP = P_T - L_F + G_T \quad (10)$$

Where, P_T is Transmitted power, L_F is Feeder Loss and G_T is Transmitter gain.

B. The Least Squares Approximation based Algorithm

The proposed Least Squares Approximation based Algorithm (LSAA) provides the best possible fit for an empirical model by fitting the model curve onto the best fit least squares curve derived from the polynomial representing measured path loss data points. The technique has to do with the use of a quotients function as a multiplication factor to adapt an empirical model to a given terrain. It involves multiplying the empirical model expression by the quotients function, which is dependent on transmitter – receiver separation in kilometres. This distance dependent function represents the best fit least squares polynomial representing the quotient points, which are obtained by dividing the best fit Least Squares polynomial representing measured path loss points, by the empirical model expression at various transmitter-receiver separations. The algorithm is as follows:

Step1: Obtain the best fit Least Squares quadratic function for representing measured path loss points by solving (11), derived from (7), while replacing y_i with measured path loss L_i, and $x_i$with transmitter-receiver separation (in kilometres) d_i, where N is the number of path loss data points.

$$\left.\begin{aligned} \sum_{i=1}^{N} L_i &= Na_0 + a_1 \sum_{i=1}^{N} d_i + a_2 \sum_{i=1}^{N} d_i^{\,2} \\ \sum_{i=1}^{N} d_i\, L_i &= a_0 \sum_{i=1}^{N} d_i + a_1 \sum_{i=1}^{N} d_i^{\,2} + a_2 \sum_{i=1}^{N} d_i^{\,3} \\ \sum_{i=1}^{N} d_i^{\,2}\, L_i &= a_0 \sum_{i=1}^{N} d_i^{\,2} + a_1 \sum_{i=1}^{N} d_i^{\,3} + a_2 \sum_{i=1}^{N} d_i^{\,4} \end{aligned}\right\} \quad (11)$$

Solving the (11), the coefficients $\boldsymbol{a_0}, \boldsymbol{a_1},., \boldsymbol{a_2}$ are obtained and used to formulate the best fit Least Squares function (12), representing the path loss data points.

$L(d) = a_0 + a_1 d + a_2 d^2$ (12)

Step2: Using (13), the quotients $Q_1, Q_2,. Q_N$ are obtained at intervals $d_1, d_2,. , d_N$ respectively, by dividing the Least Squares function value *L(d*$_i$*)*, by the empirical model predicted path loss value $L_P(d_i)$

$Q(d_i) = \frac{L(d_i)}{L_P(d_i)}$ (13)

Step3: Obtain the optimal Least Squares function $Q(d) = a_0 + a_1 d$ representing the quotient points $Q_1, Q_2,. Q_N$ using (14)

$$\left.\begin{aligned} \sum_{i=1}^{N} Q_i &= Na_0 + a_1 \sum_{i=1}^{N} d_i \\ \sum_{i=1}^{N} d_i\, Q_i &= a_0 \sum_{i=1}^{N} d_i + a_1 \sum_{i=1}^{N} d_i^{\,2} \end{aligned}\right\} \quad (14)$$

The function $Q(d)$, is the adaptation factor.

Step4: Multiply the empirical model expression by the adaptation factor $Q(d)$ to obtain the adapted model expression.

V. RESULTS AND DISCUSSION

The Okumura model is adapted to the terrain under investigation as follows: The measured path loss data obtained from the ten Base Stations is split into two sets: Base Stations 1 to 5 - adaptation set, and Base Stations 6 to 10 – generalization set. The geometric mean (GM) of the adaptation set values at each receiver-transmitter separation is obtained using (15).

GM=$\sqrt[n]{X_1 . X_2 . X_{3,} ., X_n}$ (15)

In performance evaluation, the geometric mean is preferred to the arithmetic mean because it is less sensitive to extreme values [9]. The Okumura model is then adapted to the computed GM using the earlier mentioned three techniques, i.e., the use of the Okumura G$_{AREA}$ as Correction Factor, the use of a Computed RMSE as Correction Factor, and the proposed LSAA technique. Subsequently, the adapted Okumura models are tested for path loss prediction accuracy, (i.e., capacity to generalize) through comparisons with path loss data from the generalization set. The statistical bases for comparison are the Root Mean Squared error (RMSE) and the Coefficient of Determination (R^2) given by

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y}_i)^2} \quad (16)$$

where y_i is the measured path loss, $\hat{y}_i$ is the predicted path loss and $\bar{y}_i$ is the mean of the measured path loss.

A. Comparison of Adaptation Techniques

From the Okumura Median Attenuation curves the median attenuation $A_{MU}(f,d)$ for this terrain within the range 1 to 4.3km at 900MHz is 20dB. Likewise, the G$_{AREA}$ for suburban areas at 900MHz is 9dB according to the G$_{AREA}$ curves. Hence, from (1), the G$_{AREA}$ adapted Okumura model path loss equation for this terrain is given by:

$L_{OKM} = L_{FSL} + 20 - H_{MG} - H_{BG} - 9$

$= L_{FSL} + 11 - H_{MG} - H_{BG}$ (17)

Where,

- $L_{FSL} = 32.45 + 20log900 + 20logd$
- $H_{MG} = 10\log(\frac{1.5}{3})$
- $H_{BG} = 20\log(\frac{33}{200})$

Fig. 2 shows that the G$_{AREA}$ adapted Okumura model (17) underestimates the path loss while Table 1 shows that the model does so by an RMSE of value of 8.95dB. Hence, computed correction factor is 8.95dB. Therefore, the RMSE-adapted Okumura model expression from (17) is given by

$L_{RMSE\_adapted} = L_{FSL} + 11 - H_{MG} - H_{BG} + 8.95$

$= L_{FSL} + 19.95 - H_{MG} - H_{BG}$ (18)

Fig. 2 shows that the RMSE adapted Okumura model (18) overestimates the path loss while Table 1 shows that the model does so by an RMSE value of 9.41 dB.

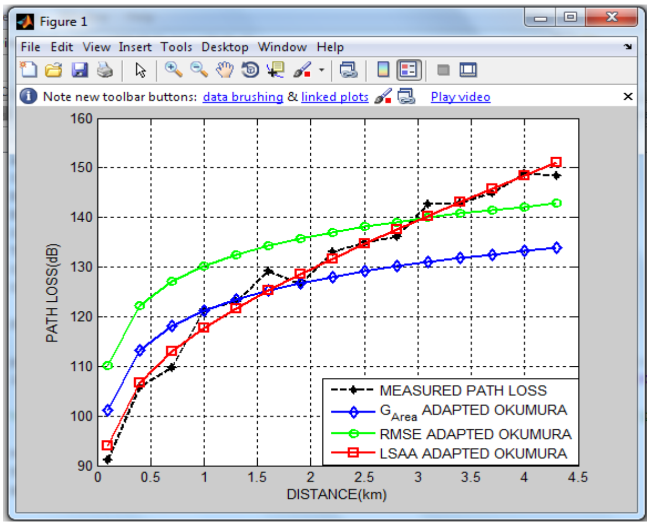

Fig. 2. *Comparison of Adaptation Techniques*

Table 1: Adaptation Accuracy Comparison

PERFORMANCE INDICES	Okumura G_{AREA} Adaptation	RMSE Correction Factor Adaptation	LSAA Adaptation
RMSE(dB)	8.95	9.41	2.18
R^2	0.69	0.66	0.97

According to the proposed LSAA, the Okumura model is adapted to the terrain as follows:

Step1: The ***coefficients*** of the best fit Least Squares function representing the geometric mean of path loss points are obtained by solving (11). The coefficients obtained are as follows: $a_0 = 94.33$, $a_1 = 23.13$, $a_2 = -2.55d$. Hence the best fit least squares equation for measured path loss is

$L_{LS} = 94.33 + 23.13d - 2.55d^2$ (19)

Step2: Quotients computed at various intervals using (20) are shown in Table 2.

$Q_i = \frac{L_{LS}(d_i)}{L_{OKM}(d_i)}$ (20)

Step3: Coefficients obtained for best fit curve through quotients are $a_0 = 0.9238$ and $a_1 = 0.0475$.

Therefore, the adaptation function is given by

$Q(d) = 0.9238 + 0.0475d$ (21)

Step 4: The LSAA-adapted Okumura Model for this terrain is obtained by multiplying the Okumura model expression (1) by the adaptation function $Q(d)$ as follows:

$L_{LSAA-OKM} = (0.9238 + 0.0475d) \,.\, L_{OKM}$ (22)

As shown in Fig. 2, the convergence of the LSAA-adapted Okumura Model (21) with measurements is an indication of the effectiveness of the technique. Results in Table 1 buttress this fact with the LSAA technique having the lowest adaptation error of 2.18dB and the best fit indicator of 0.97.

B. Generalization Test for Adapted Okumura Models

Each of the three adapted Okumura models is tested for path loss prediction accuracy using data from the generalization set (Base Stations 6 to 10). Figs. 3 to 7 graphically show the performance of each of the three adapted Okumura models on each of the Base Stations. Results in Table 3 clearly show that the LSAA-adapted Okumura model gives the most accurate predictions across all the Base Stations with the least prediction errors. On the average, the LSAA-adapted Okumura model outperforms its counterparts in prediction accuracy by a margin greater than 3.62dB in RMSE. Furthermore, the impressive fit of value 0.9 is a testimony to the high correlation between the LSAA- adapted model with the test data.

Table 2: Quotients obtained at various intervals between Base and Mobile Stations

Intervals (km)	0.1	0.4	0.7	1.0	1.3	1.6	1.9	2.2	2.5	2.8	3.1	3.4	3.7	4.0	4.3
Quotients	0.96	0.98	0.93	0.95	0.98	1	1.02	1.04	1.06	1.08	1.09	1.09	1.1	1.1	1.1

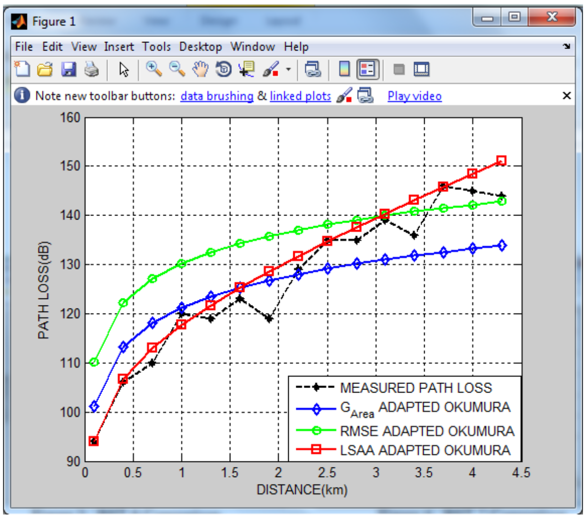

Fig.3. *Base Station 6 Comparison*

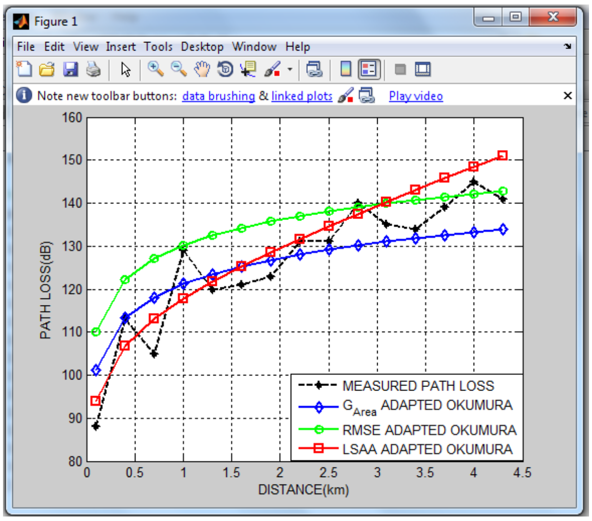

Fig. 4. *Base Station 7 Comparison*

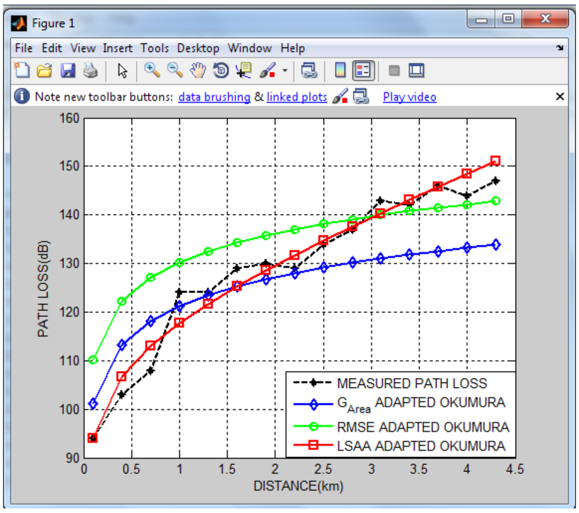

Fig.5. *Base Station 8 Comparison*

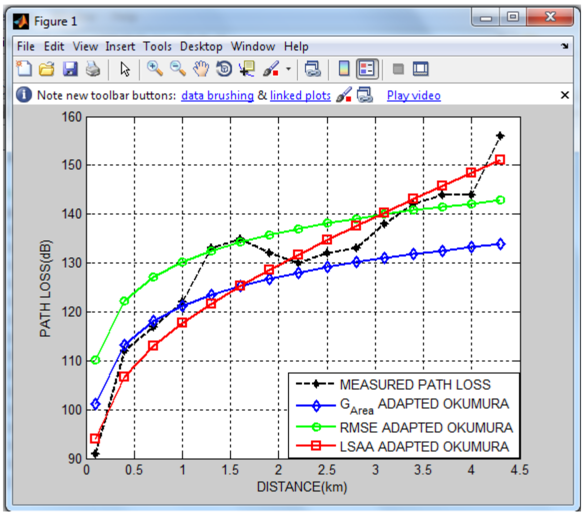

Fig.6. *Base Station 9 Comparison*

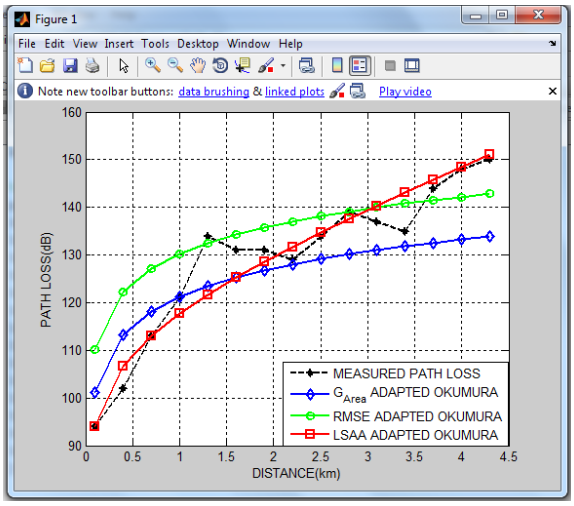

Fig.7. *Base Station 10 Comparison*

Table 3: Performance Comparison of Adapted Okumura Models

MODEL	STATS.	BST6	BST7	BST8	BST9	BST10	GEOM. MEAN
G_{AREA} Adapted Okumura	RMSE(dB)	7.39	7.36	8.48	9.05	8.65	8.16
	R^2	0.76	0.76	0.71	0.64	0.68	0.71
RMSE Adapted Okumura	RMSE(dB)	10.71	10.97	9.48	8.16	9.11	9.44
	R^2	0.49	0.45	0.63	0.71	0.65	0.59
LSAA Adapted Okumura	RMSE(dB)	4.04	6.36	3.18	5.10	4.58	4.53
	R^2	0.93	0.82	0.96	0.89	0.91	0.90

VI. CONCLUSION

A Least Squares Approximation based Algorithm for adapting an empirical model to a given terrain was formulated and compared for adaptation accuracy with two exiting techniques: i) the Okumura model adaptation using Okumura G_{AREA} (Gain due to type of environment) obtained from the Okumura Curves, and ii) the use of a computed Root Mean Squared Error (RMSE) as correction factor. It was discovered that the proposed technique gave the highest adaptation accuracy based on data obtained from the rural area between the cities of Jos and Abuja, Nigeria. The LSAA technique gave the lowest adaptation error of 2.18dB, compared with 8.95dB for the Okumura G_{AREA} and 9.41dB for the commonly used RMSE adaptation technique. Furthermore, tests for generalization of adapted Okumura models based on the three adaptation techniques indicated that the LSAA-adapted model is more robust than its counterparts with a mean prediction error of about 4.53dB, compared with 8.16dB for the G_{AREA} Adapted Okumura model, and 9.44dB for the RMSE adapted Okumura model.

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