Saturation Modelling: Using The Waxman-Smits Model/Equation In Saturation Determination In Dispersed Shaly Sands

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Abstract-The occurrence of shale in reservoir rocks can result in erroneous values of water saturation and porosity as calculated from well logs. While Archie's equation describes the electrical conductivity behaviour of shale-free rocks, Waxman-Smits (and Dual-Water) models account for the dual conductivity pathways formed by pore brine and clay mineral exchange cations in shaly sands. The determination of water saturation in shaly sand cannot be accurately achieved by use of classical water saturation models/equations. Also, the resistivity in shaly sands is lower than in clean sands having the same porosity and hydrocarbon saturation. This is as a result of additional conductivity of the clay. Hence, use of Archie's equation would result in very low hydrocarbon saturation. In a dispersed clay distribution, Waxman-Smits equation is more suitable for hydrocarbon saturation determination. This paper present a description of use of Waxman-Smits inaccurate determination of hydrocarbon saturation in dispersed shaly sands. The analysis show that the application of Waxman-Smits model/equation yield more hydrocarbon than the application of Archie's model/equation in dispersed shaly sands. The result show that the maximum absolute increase in hydrocarbon saturation is 0.25 (fraction of pore volume). This will be the case if the formation water is relatively fresh (low R_w), and/or high temperatures (high B), and/or if Q_V is high (much clay).

Keywords—Waxman-Smits, Archie, Shaly sand, Hydrocarbon saturation, Water Saturation

I. INTRODUCTION

One of the most important common constituents of rocks in log analysis in formation evaluation are shales. Their importance stems from their electrical properties, apart from their effects on porosity and permeability, which have a great influence on the determination of fluid saturations. Shale effect in reservoir rock also act as one of the most controversial problems in formation evaluation. Subject to many uncertain parameters is the accurate determination of formation porosity and fluid

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saturation in shaly sand, which are all induced by the existence of shale in pay formation. The occurrence of shale in reservoir rocks can result in erroneous values of water saturation and porosity as calculated from well logs.

The way shaliness affects log responses depends on a number of factors:

- The proportion of shale
- The physical properties of the shale, and
- The way the shale is distributed in the host layer.

Shaly materials can be distributed in the host layer in three ways:

- 1. Laminar existing in form of laminae between which are layers of sand.
- 2. Structural existing as grains or nodules in the formation matrix. This matrix shale is termed as structural shale.
- Dispersed existing as a material dispersed throughout the sand, partially filling the intergranular interstices. Dispersed shale in the pores markedly reduces the permeability of the formation.

All these shale forms may occur simultaneously in the same formation. However, shale in only one form is frequently predominant and simplified models can provide reasonable porosity and water saturation (1) (2) (3) (4). Figure one shows the various distribution of shaly materials in host layers.

Determination of water saturation in shaly sand cannot be accurately achieved by use of classical water saturation model/equation. It is necessary to either modify the saturation equation or develop a new equation. Corrections of all shaly sand tend to reduce the water saturation relative to that which can be calculated if the shale effect is ignored in the evaluation process. А large number of models/equations have been developed over the years for shaly sand relating fluid saturation to resistivity. These have been developed according to the geometry of existing shales (dispersed, laminar, structural). All the models/equations are reduced to clean sand model when the volume of shale is insignificant, as the models are composed of shale term and sand term (the shale term may be

independent or not of the sand term). Most of the shale models might yield quite similar results for relatively small volumes of shale (6) (7) (5).



Figure 1: Forms of shale classified by mode of distribution in host layers (5)

Archie's empirical law (8) are what most electrical resistivity interpretation methods are based on. This works well for the estimation of nonconducting constituent and water saturations in the pore space for clay-free sands i.e. clean sands. However, for the determination of water saturation in shaly sand (where conductive clay minerals are present), Archie's law is not accurate. (Shaly sand is the term used for a claybearing sand). Hence, the determination of water saturation in the pore space using electrical conductivity log data is not accurate if the clay material resistivity is not accounted for. To correct for clay conductivity on the resistivity of the formation, a number of clay models/equations have been proposed over the years (9). Comparison of the various clay models/equations for water saturation show that:

- Simandoux or Indonesia model/equation (10) is essentially applicable to laminated clay models, with some adaptation for non-linear behaviour of shale electrical properties.
- 2. The dual-water model/equation (11) is based on the assumption that the exchange cations contribute to the conductivity of clay bound water that is spatially separated from the bulk water.
- 3. Waxman-Smits or Dual-Water model (11) is essentially designed for the case of dispersed or structural clay models and as they account for the effects occurring in the pore space.

They also provide lower water saturation than laminated models (6) (10) (12) (13).

It is known that the resistivity in shaly sands is lower than that in clean sand having the same porosity and hydrocarbon saturation. The cause of this is known to be the additional conductivity of the clay compared to the clean sand. As a result of this the use of Archie's equation would result in a very low hydrocarbon saturation. If we have a dispersed clay distribution i.e. if the clay is coating the sand grains homogeneously, we need to use the Waxman-Smits equation instead of the Archie's equation.

In this paper, we review and describe how to accurately determine the hydrocarbon saturation in dispersed shaly sands using the Waxman-Smits model/equation. This approach is analytical and has been theoretically applied to values of borehole parameters gotten from oil wells in the Niger Delta fields in Nigeria.

2. Saturation Modelling

Saturation models are models which relate measured resistivity to water saturation from which hydrocarbon content can be determined. Saturation models like Archie, Waxman-Smits, and other shaly sand models i.e. Dual-Water model, Indonesia Model, etc., are used to calculate the hydrocarbon saturation from resistivity log.

Archie's equation, introduced in 1942 (8) is entirely based on laboratory experiments on clean sands, and therefore an empirical model. In the 1960s, it became apparent that in the case of shaly sands there is need to make corrections. As a result, many models were proposed (9) such as the Indonesia model and the Schlumberger's Dual-Water model. In 1968, M. Waxman and L. Smits published their model, the Waxman-Smits Equation. Beside the Dual-Water by Schlumberger, model the Waxman-Smits equation/model became one of the most important equations for shaly sands. The Waxman-Smits equation/model is based upon an integrated approach of theoretical considerations, new experiments and the old empirical Archie's equation. Thus, the Waxman-Smits equation/model is a semi-empirical model.

As important as the Waxman-Smits equation/model is for shaly sands, people still started looking for more complicated models in the 1980s. The reasons being that:

- 1. People wanted to derive a model from basic physics rather than relying on semi-empirical models. This search has led to the various so called Effective Medium Models (EMM) (14).
- 2. There arose some doubts about the general applicability of the Waxman-Smits equation/model vis-à-vis the assumptions upon which that model is based.

The Waxman-Smits equation/models and others like Archie's equations contain empirical constants e.g.

Archie's m and n constants, and have to be calibrated against cores in the laboratory, which yield such measurements as the so called resistivity index (I-S_W) curve. Using this laboratory derived curve to translate field resistivity into hydrocarbon saturation produce some difficulties. This is because it is difficult to measure the curve in the laboratory at simulated in situ conditions of high pressure and temperature, and using field fluid and reaching a sufficiently low water saturation. Few laboratory experiments, are however, carried out at simulated in situ condition, but they are very expensive and time consuming to achieve. Also, most of these experiments can only be carried out for water saturations down to 20% or 30% due to limited capillary pressures that can be reached in such experiments. As a result, in many cases, extrapolation of the laboratory I-S_w curves is necessary to lower S_w saturations to different temperature and pressure and fluid type conditions. This is the reason why saturation models like Waxman-Smits and Archie are used depending on the sand type.

Extrapolation is not difficult in the case of Archie's equation, as the $I\text{-}S_W$ curve is a straight line on a double-logarithmic plot. But in shaly sands, the $I-S_W$ relationship on a double-logarithmic plot is no longer linear. As a result the extrapolation is not as simple or straightforward. In the case of shaly sands, especially dispersed shaly sandstones, the extrapolation has to achieved the Waxman-Smits be usina equation/model.

3. Archie Equation/Model

In 1942, Archie, from empirical observations, suggested that the resistivity of brine-saturated rock, R_o, was related to brine resistivity, R_w. He established that the ratio of the resistivity of $R_{\rm O}$ to $R_{\rm w}$ was a constant for every given rock sample. The name, resistivity formation factor (F) was given to this proportionality constant. Hence, according to Archie:

$$= \frac{R_O}{R_{out}}$$

 $R_0 = FR_W$

or (1) Archie (1942) also showed that there was a strong linear relationship between the logarithms transforms of F and porosity (ϕ) in sand stones i.e. F depends only on porosity:

$$F = \frac{1}{\phi^m}$$
(2)

where m, the porosity exponent, takes different values for a variety of sandstones and limestones. Archie estimated m to be approximately 2. Combining equations (1) and (2) gives the well-known Archie's equation expressed as the electrical resistivity of water saturated sediments (R_o) as:

$$R_{O} = \frac{aR_{W}}{\phi^{m}}$$
(3)

where a and m are Archie constants which can be derived empirically, with m commonly called the cementation factor.

Archie (8), also showed that assuming that hydrocarbon partially saturates the pore space, he

suggested multiplying Ro by a factor called the resistivity index I, to obtain true resistivity, R_t

$$R_t = IR_0$$
 (4)
which led him to proposed

$$I = \frac{1}{S_w^n}$$
(5)

The combination of these equations led to the Archie's equation for water saturation (S_W) in a formation:

$$S_{W} = \left(\frac{aR_{W}}{\phi^{m}R_{t}}\right)^{\frac{1}{n}}$$
(6)

4. The Waxman-Smits Equation/Model

The Waxman-Smits equation/model is a semiempirical extension of the Archie's equation, taking into account the additional conductivity caused by shale. The Waxman-Smits equation/model is mostly used for dispersed shaly sandstones. In case of laminated shaly sandstones, either the Archie or the Waxman-Smits equation/model can be used in combination with specialist software. In this section, we describe the Waxman-Smits equation/model for both water-bearing shaly sandstone and hydrocarbonbearing shaly sandstone.

4.1 The Waxman-Smits equation/model for water-bearing shaly sandstone

The actual Waxman-Smits equation is the general one for educative, informative and instructive purposes. But it is easier to start with the equation for fully waterbearing sandstone.

The Archie's equation for clean water-bearing

$$R_0 = \varphi^{-11} R_W$$
 (7)
where, $R_0 =$ resistivity of the fully brine saturated

sandstone R_w = resistivity of the brine

 Φ = total porosity

m = cementation exponent

Equation (7) can be rewritten in terms of the Formation Resistivity Factor, F,

$$F = \frac{R_0}{R_{\rm ev}} = \varphi^{-m} \tag{8}$$

To arrive at the Waxman-Smits equation, it is easier to work with conductivity, preferably than resistivity. Conductivity is the reciprocal of resistivity i.e. $C = \frac{1}{p}$.

Therefore, $C_{O} = \phi^{m} C_{W}$

(9)

(Rewriting equation (7) in terms of conductivities instead of resistivities)

where C_0 = conductivity of the fully brine saturated sandstone = $\frac{1}{R_0}$

 C_W = conductivity of brine = $\frac{1}{R_W}$ Waxman-Smits began with equation (9) but replaced C_{W} by an equivalent water conductivity ($C_{w} + C_{e}$), thus taking the additional clay conductivities into account. This takes place via the Clay Bound Water Layer. (Note that C_e is the clay conductivity and C_w has its usual meaning as already defined above).

The tortuosity factor, ϕ^{-m^*} acts on the clay conductivity (C_e) in the same way as it acts on the pore i.e. brine conductivity. This is because all clay is supposed to be in the pores, lining the pore walls. Waxman-Smits use a constant m^{*} rather than m as its cementation exponent. The Waxman-Smits equation for water-bearing shaly sandstone thus becomes,

$$C_{O} = \phi^{-m^{*}} (C_{W} + C_{e})$$

= $\phi^{-m^{*}}_{*} (C_{W} + B.Q_{V})$ (10)

 $= \Phi^{-m}$ (C_W + B.Q_V) (10) where m is the cementation exponent in the Waxman-Smits equation.

In equation (10), $B.Q_V$ was substituted for C_e . An important factor in clay conductivity is the mobility of the cat-ions. This gives rise to an equivalent conductance per cat-ion, called B. B for a given type of

brine is solely dependent on the brine salinity and temperature (see figure 2). B is expressed in mho.cm²/meq (1 mho is the unit of conductance, it is 1/Ohm).

The clay conductivity is,

 $C_e = B.Q_V$ (which is what is used in equation (10) above).



Figure 2: "B" Chart – B as a function of water resistivity (coupled to salinity) and temperature.

The clay conductivity is B times Q_V, with mho/cm as unit, where Q_v is the Cation Exchange Capacity (meq/ml). The number of positive ions i.e. the cat-ion concentration, attracted to the clay surface depends on the amount of clay and the type of clay. This number is Q_v.

Therefore, writing equation (10) in terms of resistivities instead of conductivities gives,

$$R_{O} = \phi^{-m^{*}} R_{W} / (1 + R_{W}.B.Q_{V})$$
(11)

If the Q_V did not depend on porosity, the doublelogarithmic plot of F (= R_0/R_W) versus ϕ would be a straight line. However, Q_V decreases with increasing porosity (less surface to volume ratio). Thus, the double-logarithmic plot of F versus ϕ is curved, especially for low porosities (see figure 3).

Waxman-Smits, therefore, introduced a modified Formation Resistivity Factor, F

$$F^* = \frac{R_0^*}{R_w} = \Phi^{-m^*}$$
 (12)

where,
$$R_0^* = R_0.(1 + R_W.B.Q_V)$$
 (13)

The double-logarithmic plot of F^{*} versus φ is a straight line – by definition of F².



Figure 3: Determination of Q_v from log responses across the water-bearing zone. Shows the effect of shale on the Resistivity.

Waxman-Smits 42 The equation for hydrocarbon bearing shaly sandstone

Again, we start with the equation for clean hydrocarbon-bearing sandstone just as we did for a fully water-bearing sandstone above. The Archie's equation for clean hydrocarbon-bearing shalv sandstone is, R

$$R_t = \varphi^{-m} S_w^{-n} R_w$$
(14)

Where, R_t = resistivity of the partly hydrocarbonbearing sandstone

 R_w = resistivity of brine Φ = total porosity m = cementation exponent n = saturation exponent

In terms of the Resistivity Index, I, equation (8) can be rewritten as

$$I = R_t / R_0 = S_w^{-n}$$
(15)

Just as we did for the water-bearing shaly sandstone, it is easier to arrive at the Waxman-Smits equation by working with conductivities, rather than resistivities. Therefore,

$$C_{t} = \varphi^{m} S_{w}^{n} C_{W}$$
(16)

where, C_t = conductivity of the partly hydrocarbonbearing rock = $1/R_t$

C_W = conductivity of brine = $1/R_W$

Again, Waxman-Smits began with equation (16), but replace C_W by an equivalent water conductivity (C_W + C_e/S_W), thus taking the additional clay conductivity into account. Due to the fact that the surface to volume ratio for the brine has now changed with this factor, the additional term S_W arises. The tortuosity factor, ϕ^{m^*} acts on this clay conductivity in the same way as it acts on the brine conductivity (just as we saw for the water-bearing sandstone).

As a result, the Waxman-Smits equation for hydrocarbon-bearing shaly sandstone becomes,

 $C_{t} = \phi^{m^{*}} S_{w}^{n^{*}} (C_{W} + C_{e}/S_{W})$ (17)

Or $C_t = \phi^{m^*} S_w^{n^*} (C_W + B.Q_V/S_W)$ (by substituting $B.Q_V$ for C_e)

where, m = cementation exponent in the Waxman-Smits equation

 n^{*} = saturation exponent in the Waxman-Smits equation.

(18)

Equation (18) is the general form of the Waxman-Smits equation.

Writing equation (18) in terms of resistivities rather than conductivities results in,

 $R_{t} = \phi^{-m^{*}} S_{w}^{-m^{*}} R_{W} / (1 + R_{W}.B.Q_{V} / S_{W})$ (19)

Therefore,

$$I = R_t/R_0$$

= $R_t/R_t(S_W = 1)$
= $S_w^{-n^*}(1 + R_W.B.Q_V)/(1 + R_W.B.Q_V/S_W)$ (20)

Equation (20) shows that the Resistivity Index curve will be non-linear (when plotted double-logarithmically).

Waxman-Smits introduced a modified Resistivity Index, $\dot{\mathsf{I}}$

$$I^{*} = R_{t}^{*}/R_{0}^{*} = S_{w}^{-n^{*}}$$
(21)

where,

 $R_t^* = R_t (1 + R_W.B.Q_V/S_W)$ (22)

The double-logarithmic plot of I^{\bullet} versus S_W is a straight line, by definition of I^{\bullet} . The Waxman-Smits procedure is summarized in figure 4.



Figure 4: The Waxman-Smits procedure.

5. Influence of Shaliness in Determination of Hydrocarbon Saturation

The Waxman-Smits equation in the hydrocarbonbearing zone can be rewritten in terms of S_W .

$$S_{W}^{n^{*}} = \frac{R_{W}}{R_{t}\phi^{m^{*}} (1 + R_{W}.B.\frac{Q_{V}}{S_{W}})}$$
(23)

A quick look at equation (23) reveals apparently two things:

- (1) Solution for S_W has to be iterative. This is/or can be taken care of using petrophysical evaluation software.
- (2) Water saturation will decrease, resulting in the hydrocarbon saturation increasing the higher term $R_W.B.Q_V/S_W$. The effect of shaliness may be ignored if this term is equal to or less than 10% i.e. 0.1. The higher the term, the more significant the effect of the shaliness. If the formation water is relatively fresh i.e. low resistivity, leading to high B, this will be the case, and/or if Q_V is high (much clay), and/or at high temperatures (high B). In such a case S_W decreases and Waxman-Smits hence yields more hydrocarbon than the use of Archie's equation (see figure 5).

It can be shown that the maximum increase in hydrocarbon saturation when using Waxman-Smits equation instead of Archie's equation is about 25% (in absolute saturation) Therefore, if Archie's equation would give, for example, $S_W = 0.50$, then Waxman-Smits would give something like $S_W = 0.75$, in the case where $R_W.B.Q_V/S_W$ would be very high, according to figure 6.



Figure 5: Plot showing how the application of Waxman-Smits gives more hydrocarbons. Waxman-Smits gives a higher hydrocarbon saturation.



Figure 6: Effect of Q_{ν} on saturation: Example of the influence of Q_{ν} on calculated hydrocarbon saturation, $S_{0}.$

6. Determination of Q_V

It is known that the number of positive ions i.e. the cat-ion concentration, attracted to the clay surface depends on the amount of clay and the type of clay. This number, the Cation Exchange Capacity (CEC) also known as the Q_V can be measured using three methods:

(1) Measuring Q_V on core samples in laboratory, using several available laboratory methods i.e. C_O - C_W , titration, etc.

(2) By fitting the Waxman-Smits equation to the resistivity log in water-bearing zone. This is similar to the Pickett procedure in Archie but now having two unknowns, other than the one unknown R_W , are fitted, namely R_W , from the intercept, and Q_V , from the curvature, as seen in figures 3 above and 7 below. Constant m^{*} has to be known from core analysis, or from regional data. As a result, the fit will give an R_W as well as a Q_V value.



Figure 7: Waxman-Smits procedure, showing how Q_V is determined from logs (Q_V from log).

(3) Use of method (2) above to determine Q_V in a shale interval. Q_V is then calculated in shaly sand by multiplying Q_V in pure shale by shale fraction Vsh. This is called the Juhasz method or the 'normalised Q_V' method (15). From gamma-ray log or density/neutron cross-plot, the Vsh parameter can be calculated (see the Thomas-Stieber analysis in figures 8 and 9).



Figure 8: The Thomas-Stieber Crossplot for Shale Typing 1. Recognition of other shale distributions from a Density/Neutron Xplot.

The neutron/density cross-plot would indicate whether the clay distribution is dispersed, laminar or structural - with the Waxman-Smits equations only applying in the case of dispersed clay.

The three methods sometimes may give rather different relationships, as a result of several reasons ranging from inaccurate laboratory method, limited resolution of resistivity logs, inaccurate shale fraction assessment, to inapplicability of the Waxman-Smits equation/model. The obtained Q_V values would normally be regressed against porosity, or against 1/porosity, such that a continuous Q_V -curve can then be obtained from porosity log.



Figure 9: The Thomas-Stieber Crossplot for Shale Typing 2. Recognition of other shale distributions from a Density/Neutron Xplot.

7. Conclusion

Evaluation of shaly sands can be very complex. Furthermore, all logging responses and interpretation techniques are influenced by the shale. The resistivity of shaly sand is lower than that of clean sand with the same porosity and hydrocarbon saturation.

We have described how to accurately determine the hydrocarbon saturation in dispersed shaly sands using the Waxman-Smits equation/model. Unlike the Archie's equation which wrongly attribute the low resistivity to a high water saturation, the Waxman-Smits equation correctly attributes it to the influence of clay, avoiding overestimation of S_W in the process. Application of Waxman-Smits, therefore, results in a correct prediction of hydrocarbon saturation. Also, unlike the Archie's equation, the Waxman-Smit equation contains additional term $B.Q_V$. The B term depends on salinity and temperature, while the Q_V term strongly depends on rock type/lithology.

As a result, the application of Waxman-Smits equation/model yield more hydrocarbon than the application of Archie's equation, if the term $R_W.B.Q_V$ is much higher than 0.1 (10%). This is seen if the formation water is relatively fresh, and/or if Q_V is high, and/or at high temperatures. The maximum absolute increase in hydrocarbon saturation is 0.25, fraction of pore volume.

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