

# Numerical Studies On Wave Run Up, Compared With Experimental Results

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**Abstract:** In this paper, we have studied wave run-up and its various properties in different types of waves. Wave run-up is one of the most important topics in coastal engineering. It is a complicated phenomenon in the branch of fluid mechanics. There is no proper numerical study about the wave run-up. When the wave enters the surf zone then it breaks and it dissipates energy, as a result the water level increased. The maximum uprush of water above the still water level is defined as wave run-up. When the run-up height is very high, it is overtopping the coastal boundary and damages the properties such as forest, fields and local area. For these reason different types natural disasters such as Tsunami, Cyclone, Storm surge, land-slide etc. are occurred. It is necessary to know about the run-up height in various situations to reduce its ferocity. We will discuss about how to control the wave run up numerically as well as mathematically.

**Keywords—Wave run-up; Dissipates energy; Surf zone; Tsunami, Cyclone; Coastal Area**

## I. INTRODUCTION

As the scientific view, coastal engineering is a most important branch of fluid mechanics as well as wave run up is one of the most important topics in coastal engineering. We know that wave run-up is defined as the maximum vertical extent of wave uprush on a beach or structure above the still water level. In the design of coastal structure it is an important factor to expose the wave attack. Sea dikes are generally designed in such a way that little or no wave run-up overtops the structure. In tsunami, storm surge and other disasters wave run-up is very high. In that situation beach land and coastal areas are damaged drastically. To reduce this natural disaster, it is important to know about wave run-up in various situations. The readers are referred to [1] where the author studied wave run-up on composite slopes and proposed a graphic approach to estimate the run-up height on a sloping beach. The investigation on run-up

height empirically was done by [2]. Battjes [3] studied the computation of wave set-up, long shore currents, run-up and over topping due to wind generated waves. Synolakis [4] measured the run-up for solitary waves. Li and Raichlen [5] measured the wave run-up for breaking and non-breaking solitary wave. But the detailed characteristics of the wave breaking process, such as the shape of plunging jet and the subsequent splash-up, cannot be described by the numerical model. Hubbard and Dodd [6] measured the wave run-up numerically. From the above discussion, it is clear that wave run-up is an important issue for coastal engineering. So, the objective of this study is to investigate the wave run-up and its various properties numerically with experimental set up. The paper focuses the laboratory investigation of wave run-up. The laboratory measurements of wave run-up by several authors have been discussed and compared with the empirical formula. The wave run-up on smooth and rough slope has also been discussed.

## II. PHYSICAL CONFIGURATION OF WAVE RUN-UP

Wave run-up is one of the most important factors affecting the design of coastal structures exposed to wave attack. Generally revetments and sea dikes are designed in such a way that no wave run-up overtops the structure. Knowledge of wave run-up is of course important for a variety of concerns, including the determination of the optimum crest elevation of structure or the location of a beach setback line for construction. When waves move in shallow water, they become unstable and break. After breaking, they dissipate their energy and produce air bubbles. As a result it is assumed that the entrainment of air bubbles has significant effect to increase water level. The maximum uprush of water above the still water level is defined as the wave run-up. It is difficult to measure the wave run-up experimentally.

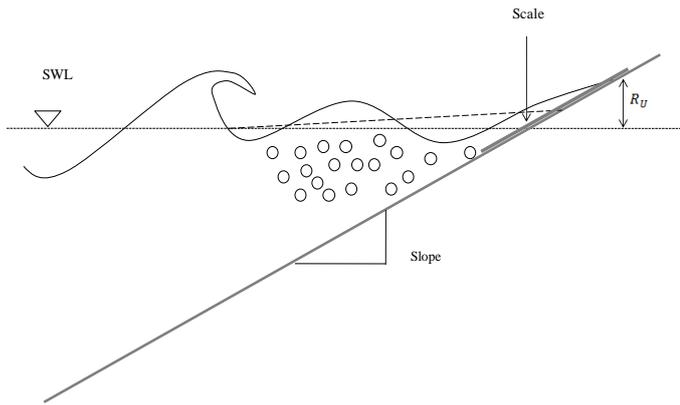


Fig. 1: sketch of wave run-up.

The measurement of the wave run-up of a sloping beach, it is important to keep knowledge about run-up, the determination of the optimum crest elevation, the construction of the beach, the angle of the beach slope. In the surf zone, a portion of the incident wave oscillatory motion will be converted by the wave breaking process to forward translation of the water mass. This results in the formation of a run-up the face of a beach as shown in figure 1.

As waves move into shallow water, they become unstable and break. After breaking a large number of air bubbles have been created and as a result the water level increases, which has significant effect on wave run-up. Wave run-up on the sloping beach cannot be developed theoretically if the waves do not break. Possibility due to the complexity of the air water mixture they were unable to obtain theoretical study for wave run-up in the surf zone. Hunt [7] investigated the wave run-up height empirically. Saville [1] proposed a graphic approach to estimate the run-up height on a sloping beach. Bowden et al. [8] examined the wave run-up height in details with a series of experiments. Chanson et al. [9] measured the wave run-up at downstream of nappe impact. Battjes [10] proposed an empirical formula, which depends only on beach slope and wave period. Hoque and Aoki [11] measured the wave run-up on a sloping beach including the effects of air bubble. They investigated the significance of increased water level due to entrained air bubble on wave run-up. Shankar and Jayaratne [12] measured wave run-up on smooth and rough slope. They investigated the wave run-up and compared with the results given in the Shore Protection Manual (1984), Automated Coastal Engineering System (1992) and results of other investigators.

### III. EXPERIMENTAL SET-UP

#### A. For slopping beach

To measure the wave run-up experimentally Hoque and Aoki [11] was set-up a wave channel of 20 m long, 0.80 m wide and 0.60 m deep filled with fresh water to a depth of 0.30 m which is shown in the figure (2). The flume was horizontal and the two sidewalls along the breaking zone of the flume were made of glass panels supported by a metal frame. A sloping bottom (1V: 20H) was installed at 9.65 m distant from a wave generator. The wave generator was a piston type with a vertical flat plate moving horizontally in sinusoidal motion. Water depth and surface elevation were measured using a pointer gauge and two capacitance wave gauges respectively. The wave gauges were calibrated every time by raising and lowering the water level in the wave flume and the relationship between the wave amplitude and the output voltage was linear. Two wave gauges were positioned at 5 m and 5.30 m from the wave maker to get the information of the incident wave height. A scale was set up along the beach slope.

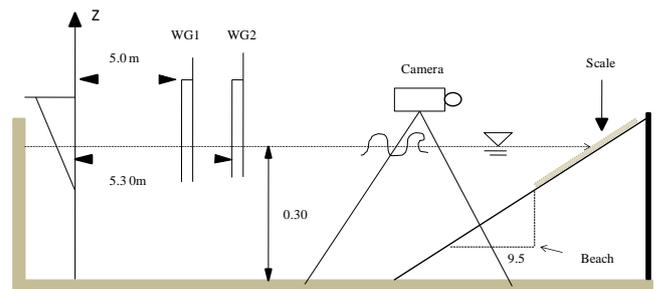


Fig. 2: Experimental setup for wave run-up.

A high speed digital camera was set on top of the wave flume with a supplementary support that used to observe the wave breaking process. The video recorder started after the wave maker. Because of the inertia of the wave maker, some of the initial waves were not fully developed and no breaking was observed. The wave breaking process was observed with a video camera and the images were recorded on video recorder. The video recording and subsequent digital image analyses were used to estimate the wave run-up in breaking waves. By the camera approximately 20-30 waves were recorded for each wave steepness and were taken the average wave run-up.

#### B. Experimental condition

A summary of the experiments run in the laboratory was given in below table, which showed the input wave conditions and some of the results

**Table 1:**

The data of the laboratory experiment of wave run-up.

Wave period T sec	Wave length $L_o$ (m)	Wave height $H_o$ (m)	Obs. run-up $R_u/H_o$
0.95	1.27	0.13	0.318
0.95	1.27	0.08	0.494
0.95	1.27	0.11	0.379
1.50	2.34	0.08	0.638
1.50	2.34	0.13	0.499
1.50	2.34	0.15	0.473
2.0	3.25	0.09	0.883
2.0	3.25	0.12	0.744
2.0	3.25	0.15	0.638
3.0	5.0	0.07	1.430
3.0	5.0	0.15	1.250
3.0	5.0	0.19	1.010

**C. Observation of entrained air**

The visualized entrained air bubbles during the breaking process were observed. Air bubbles were entrained at the plunging point, not exactly at the breaking point. Similar phenomena also occurred in a spilling breaker. After a wave had broken as a spilling or plunging breaker a transition occurred. In spilling breaker, the surface roller was grown and air bubbles were kept constant in the roller from breaking point to some distance and then entrained into water. They observed that the plunging jet generated a splash up of water, which continued the breaking process and created large coherent vortices that could reach the bottom. At the plunging point, some air bubbles propagated inward direction with wave propagation near the free surface and some were spread backward direction near the bottom, which may be due to the water stagnation and advection. As was typical for plunging waves, a large air tube was produced from the initial impact at breaking and hit undisturbed water at a second plunge point, with the cycle starting again with another splash up.

**IV. EXPERIMENTAL RESULTS**

**A. Wave run-up sloping beach**

The empirical theory of wave run-up height  $R_u$  is developed by Hunt [2] based on experimental data is defined by

$$\frac{R_u}{H_o} = \frac{\tan \theta}{\sqrt{\frac{H_o}{L_o}}} \tag{1}$$

where  $\theta$  is the slope angle (i.e.  $\tan \theta = m$ ),  $H_o$  represent the deep water wave height and  $L_o$  is the deep water wave length. Equation (1) is derived for breaking waves. As in(1) indicates that the dimensionless wave run-up depends primarily on the wave steepness and the beach slope.

From figure (3) it is clearly observed that the circular shape points for experimental data as well as the dotted line denotes the data from Hunt's formulae. For all the figures, the experimental data are broadly scattered over a wide range of  $H_o/L_o$ . It is also found that a higher value of  $H_o/L_o$  leads to a reduction in the value of  $R_u/H_o$ . The values of  $R_u/H_o$  are decreased with respect to the values  $\left(\frac{1}{7.5}\right)$  to  $\left(\frac{1}{10.5}\right)$  of slopes that are shown in figures(3),(4), (5) and (6). However, for the certain values of  $H_o/L_o$  and  $R_u/H_o$ , If we alter the magnitude of the slope then the run-up of the wave will be changed. As can be observed from the following figures, for small values of slope the run-up of the wave will be increased. As a result, the wave overtopping the dikes will be inundated the coastal areas. On the other hand, the run-up of the wave will be decreased for large values of slope. Consequently, it is not possible to enter water in the coastal areas.

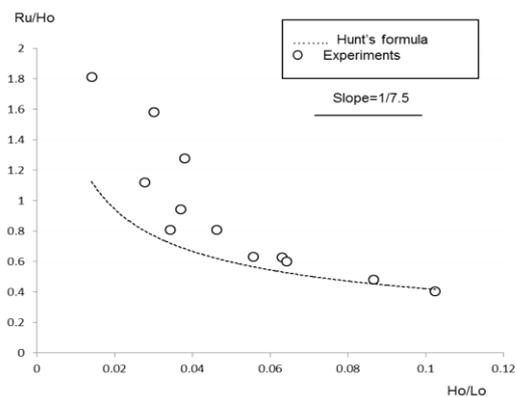


Fig. 3: Wave run-up height as a function of wave steepness for

$$\text{slope} = \left(\frac{1}{7.5}\right).$$

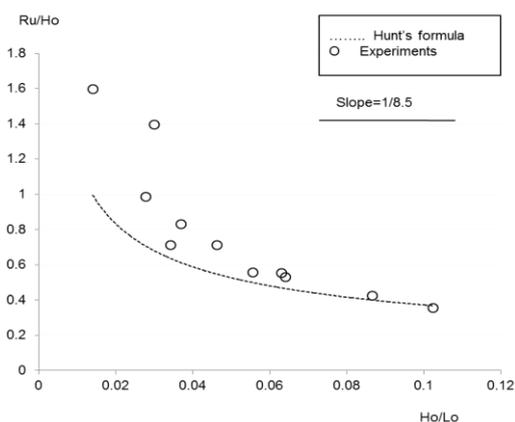


Fig. 4: Wave run-up height as a function of wave steepness for

$$\text{slope} = \left(\frac{1}{8.5}\right).$$

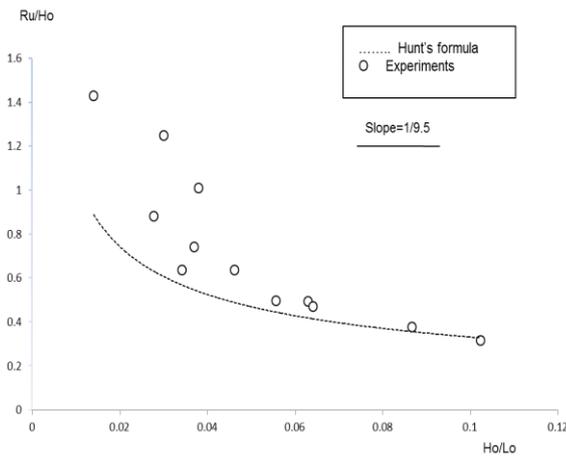


Fig. 5: Wave run-up height as a function of wave steepness for

$$\text{slope} = \left( \frac{1}{9.5} \right)$$

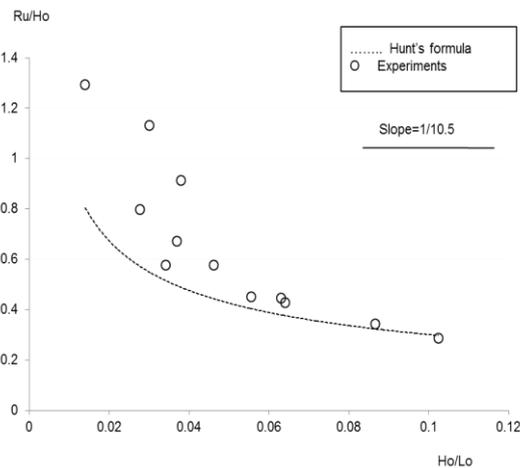


Fig. 6: Wave run-up height as a function of wave steepness for

$$\text{slope} = \left( \frac{1}{10.5} \right)$$

### B. Wave run-up on smooth and permeable slope

Initially, the experimental run-up data were processed to compare with other theories and formula to check the validity of results. The previous methods, given in the Shore Protection Manual (1984), show approximately 15% over-prediction of experimental run-up heights for smooth impermeable breakwater models figure(7). Similarly, there is 15% over-estimation of SPM wave run-up values (Hudson and Jackson, 1962) from measured run-up levels for rough permeable breakwater model of 20 mm graded gravel and 1.2 mm wire mesh. It was assumed that the tested permeable slope consisted of riprap with a gradient of 1:2. The deviations between present experimental results and the results of SPM arise due to uncertainty in interpolation of values between designs charts provided in figures 8-10.

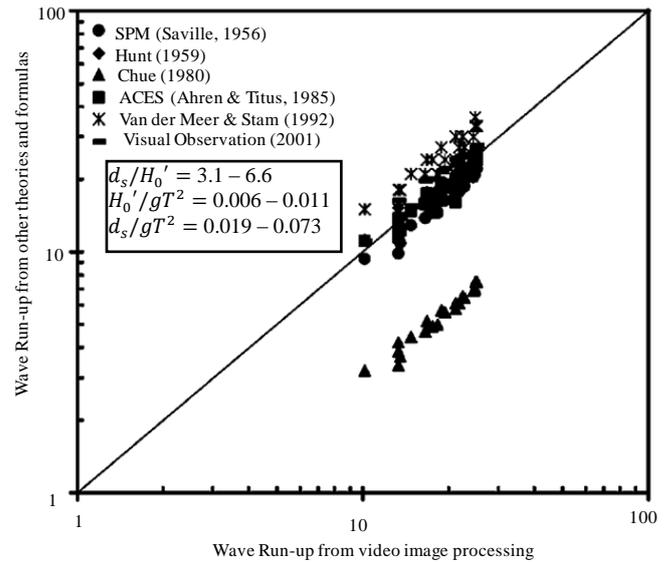


Fig. 7: Comparative analysis of experimental results of wave run-up on smooth impermeable breakwater model (1:2) with available results.

ACES (1992) show a good concurrence and 15% over-estimation with the measured run-up heights for smooth impermeable and rough permeable breakwater models, respectively figures(8) and (10). It recommends the following general equation (Ahrens and Titus, 1985), characterized by the surf similarity parameter ( $\xi$ ) according to the wave-structure regimes, for the wave run-up on smooth slopes.

$$R = CH_i \begin{cases} C_p = 1.002\xi; \xi \leq 2 & (2) \end{cases}$$

$$R = CH_i \begin{cases} C_r = \left( \frac{3.5 - \xi}{1.5} \right) C_p + \left( \frac{\xi - 2}{1.5} \right) C_{nb}; 2 \leq \xi \leq 3.5 & (3) \end{cases}$$

$$R = CH_i \begin{cases} C_{nb} = 1.181 \left( \frac{\pi}{2\theta} \right)^{0.375} e^{[3.187 \left( \frac{2\xi}{H_i} \right)^2]}; \xi \leq 3.5 & (4) \end{cases}$$

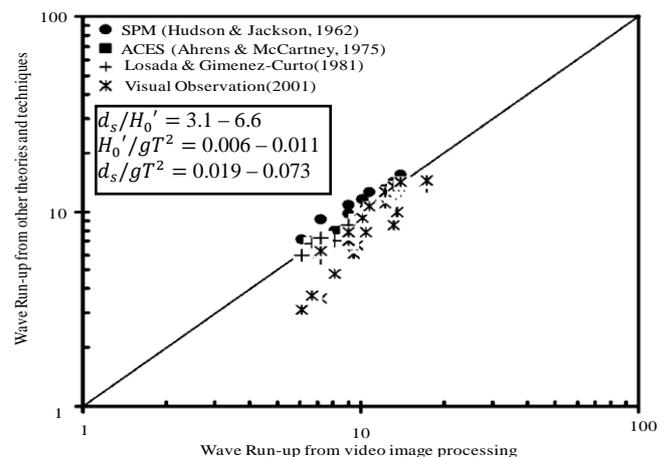


Fig.8: Comparative analysis of experimental results of wave run-up on rough permeable breakwater model with available results (Slope 1:2, GS=20 mm +1.2 mm WWM, n=51.3%).

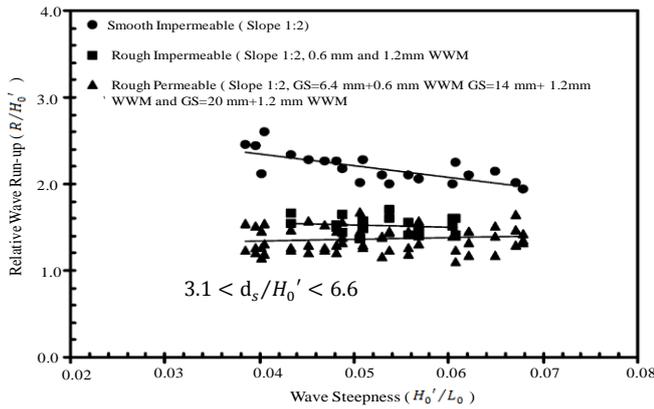


Fig. 9: Relative wave run-up with wave steepness for various smooth and rough breakwater models.

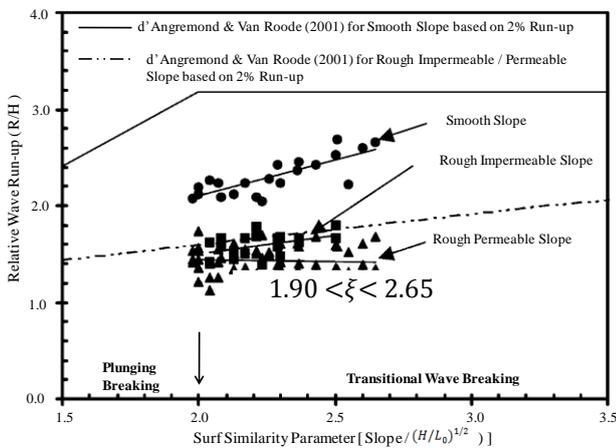


Fig. 10: Wave run-up against surf similarity parameter for various breakwater models.

Moreover, ACES (Ahrens and McCartney, 1975) presents an empirical method based on the non-linear function of the surf similarity parameter ( $\xi$ ) for estimating the wave run-up on structures protected by various types of primary armor units.

$$R = H_i \left( \frac{a\xi}{1 + b\xi} \right) \quad (5)$$

Where a ,b = empirical coefficients depend on the types of armor units.

For this comparison, 0.956 and 0.398 were used for a and b respectively by assuming the armor units to be graded riprap. There is a better representation of measured run-up values with the results given by the Hunt formula [7] which was derived for wave run-up on smooth slopes.

$$\frac{R}{H} = \frac{\tan \theta}{\sqrt{L_0}} \quad (6)$$

Most seawalls and breakwaters have steep slopes around 1:1.5 to 1:2.5, where the Hunt formula has limitations. Chue (1980) adapted and combined a number of standard prediction formulas to produce a single equation of wider applicability for wave run-up.

$$\frac{R}{H_0} = 1.8 \left[ 1 - 3.111 \left( \frac{H}{L_0} \right) \right] \xi_0 \left\{ 1 - e^{-\sqrt{\frac{\pi}{2\alpha}} \frac{1}{\xi}} \right\} \quad (7)$$

$$\text{where } \xi_0 = \frac{\tan \theta}{\left( \frac{H}{L_0} \right)^{0.4}}$$

The use of the exponent 0.4 was justified as “found to fit remarkably well” but no supporting reference was given. A significant deviation (70% under-prediction) in the results based on equation (7) can be observed in figure(7). The large deviation could possibly be due to the highly empirical nature of the proposed equation. Van der Meer and Stam (1992) and De Waal and Van der Meer (1992) identified two regions of wave breaking on a smooth sloping structure and relationships were derived for 2% run-up level

$$\frac{R_{u2\%}}{H_s} = \begin{cases} 1.5\xi; & \xi \leq 2 \\ 3\xi; & \xi \geq 2 \end{cases} \quad (8)$$

Equation (9) over-predict the measured run-up heights by approximately 30% for the smooth impermeable slope. Losada and Gimenez-Curto (1981) presented the following formula to calculate the wave run-up on rough slopes.

$$\frac{R_u}{H} = A \left( 1 - e^{-B\xi} \right) \quad (10)$$

Figure (8) depicts 15% under-prediction of wave run-up,  $R_u$ , computed using equation (10) with the experimental results. This under-prediction could possibly be due to the empirical nature of the proposed equation.

#### V. Nomenclature:

Symbol	Meaning
$R$	Wave run-up (m)
$\xi$	Surf similarity parameter (-)
$H_i$	Incident wave height (m)
$C_p$	Plunging breaking coefficient (-)
$\theta$	Breakwater slope (radians)
$C_t$	Transitional breaking coefficient (-)
$\eta_c$	Crest height of the wave above SWL
$C_{nb}$	Non-breaking coefficient
$C$	Corresponding breaking coefficient (-)
$\xi_0$	Surf similarity parameter(-)
$H_0$	Deep water wave height (m)
$L_0$	Deep water wave length (m)
$\alpha$	Breakwater slope (degrees)
$H$	Incident wave height (m)
$H_s$	Significant wave height (m)
$R_{u2\%}$	2% run-up level (m)
$A, B$	Experimental coefficients (1.322 & 0.966)

#### VI. Conclusion

This paper focused the investigation of wave run-up on various slope. From the investigation of the experiment conducted by Hoque and Aoki (2001) the wave run-up had been performed and compared with that of empirical formula. The results of wave run-up

obtained from the experimental data could correlate with empirical formula, although a little inconsistency was obtained between the experimental results and empirical formula. This inconsistency could be ascribed to inadequate data point and error during data collection. Also from the investigation of the experiment conducted by Shankar and Jayaratne [12], there were some instances of under-prediction and over-prediction, it could be concluded that the experimental results of wave run-up on smooth and rough slopes of coastal structures correlate well with the results of the Shore Protection Manual (1984), Automated Coastal Engineering System (1992) and the results of other investigators. During wave breaking we observed that the wave run-up has various properties on various slopes which are important in Coastal Engineering.

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