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Static analysis of isotropic, orthotropic and functionally graded material beams

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***Abstract*—This paper presents static analysis of isotropic, orthotropic and Functionally Graded Materials (FGMs) beams using a Finite Element Method (FEM). Ansys Workbench15 has been used to build up several models to simulate different types of beams with different boundary conditions, all beams have been subjected to both of uniformly distributed and transversal point loads within the experience of Timoshenko Beam Theory and First order Shear Deformation Theory. The material properties are assumed to be temperature-independent, and are graded in the thickness direction according to a simple power law distribution of the volume fractions of the constituents. All results have been compared with previously published researches and a good agreement has been achieved.**

Keywords—isotropic; orthotropic; FGM; beams; Ansys; finite element; modeling.

I. INTRODUCTION

Isotropic and orthotropic beams with different loading and different boundary conditions have been studied in details from researches all over the world. New beam materials and/or ingredients were the main points of interest; all boundary conditions have been discussed carefully with all known loading elements.

Orthotropic beams with different fiber orientation angles have been used widely in many military and civilian applications, new technologies have been applied to produce new structural beams and plates with different kinds of fibers and matrices in composite structure.

The concept of functionally graded materials (FGMs) was first introduced in 1984 by a group of materials scientists in Japan as a mean of preparing thermal barrier materials. Since then, FGMs have attracted many of researcher's interest to produce a heat-shielding material, then they have investigated different models to produce a structure made of functionally graded materials to resist both of mechanical and thermal loads.

In 1989 FUH-Gwo YUAN et al [1] have derived a new finite element model includes separate rotational degrees of freedom for each lamina, and it can be used for long and short beams, this laminated finite element model gives good results for both stresses and deflections when compared with other solutions.

In 1993 Lidstrom [2] have used the total potential energy formulation to analyze equilibrium for a moderate deflection 3-D beam element, the condensed two-node element reduced the size of the problem, compared with the three-node element, but increased the computing time. The condensed two-node system was less numerically stable than the three-node system. Because of this fact, it was not possible to evaluate the third and fourth-order differentials of the strain energy function, and thus not possible to determine the types of criticality

In 1997 Reddy et al [3] have presented the state-space concept in conjunction with the Jordan canonical form to solve the governing equations for the bending of cross-ply laminated composite beams. They have used the classical, first-order, second-order and third-order theories have been used in the analysis. Exact solutions have been developed for symmetric and anti-symmetric cross-ply beams with arbitrary boundary conditions subjected to arbitrary loadings. Several sets of numerical results are presented to show the deflected curve of the beam, the effect of shear deformation, the number of layers and the orthotropicity ratio on the static response of composite beams.

In 2002 Chakaborty et al [4] have presented a new refined locking free first order shear deformable finite element to study the application of the element to handle different types of structural discontinuities such as ply- drops, multiply connected beams with rigid joints, lap joints and the beams with delamination. Results from the analysis show that the formulated element predicts response that compares very well with the available results concerning the same problem.

In 2010 Usik Lee et al [5] they have developed a spectral element model for an axially loaded bending–shear–torsion coupled composite laminated beam which is represented by the Timoshenko beam model based on the first-order shear deformation theory. The high accuracy of the spectral element model is then numerically verified by comparing with exact theoretical solutions or the solutions obtained by conventional finite element method.

In 2013 El Shafei [6] has developed a new finite element model to analyze the response of isotropic and orthotropic beams with different boundary conditions, the assumed field displacements equations of the beams are represented by a first order shear deformation theory and the Timoshenko beam theory. The equations of motion of the beams are derived using Hamilton's principle. The shear correction factor is used to improve the obtained results. The obtained results of the proposed model are compared to the available results of other investigators, good agreement is generally obtained. His proposed model is valid to decrease the error due to un-accurately modeling of the curvature present in the actual material under bending which known as shear Locking.

In 2003 Chakraborty et al [7] has developed a new beam element to study the thermo-elastic behavior of functionally graded beam structures. The element is based on the first-order shear deformation theory and it accounts for varying elastic and thermal properties along its thickness, the stiffness matrix has super-convergent property and the element is free of shear locking. Both exponential and power-law variations of material property distribution are used to examine different stress variations. Static models show that it is an effective way to smoothen stress jumps in bi-material beams.

In 2009 Mseut Simsek [8] has investigated the static analysis of a functionally graded (FG) simply-supported beam subjected to a uniformly distributed load by using Ritz method within the framework of Timoshenko and the higher order shear deformation beam theories. The effect of various material distributions on the displacements and the stresses of the beam are examined. By choosing a suitable power-law exponent, the material properties of the FG beam can be tailored to meet the desired goals of minimizing stresses and displacements in a beam-type structure.

In 2011 Carlos A. Almeidaa [9] et al have presented a geometric nonlinear analysis formulation for beams of functionally graded cross-sections by means of a Total Lagrangian formulation. The influence of material gradation on the numerical response is investigated in detail. The behavior of beams of graded cross-sections is compared with homogeneous material beams. The developed nonlinear techniques may be applicable to extend the existing body of work on linear formulations for functionally graded beams to the geometric nonlinear range and will contribute to connect research with actual industrial applications.

In 2012 A.R. Daneshmehr et al [10] have investigated an elasto-plastic FGM simply supported Euler-Bernoulli beam with rectangular section subjected to uniformly distributed transverse loading by variation method. Material properties define by power law. The analytical solution illustrates stress response of the beam and the required moment to have fully plastic beam is determined.

In 2013 Raghuvir Mehta et al [11] have used a finite element model for both static and dynamic behavior of functionally graded material beams, according to the power law, they have investigated all results and compared their results with a steel beam.

In the present work, several models have been built up using Ansys Workbench15 to simulate static analysis of isotropic, orthotropic and functionally graded material beams with different slenderness ratio, different boundary conditions and subjected to different types of loads.

II. BEAMS MODELING

A. *Isotropic beams*

Isotropic beams are the beams with independent material properties of direction, such materials have only two independent variables and they are usually expressed as the Young's modulus (**E**) and the Poisson's ratio (**υ**), however the alternative elastic constants such as the bulk modulus (K) and/or the shear modulus (**G**) can also be used. For isotropic materials, (**G**) and (**K**) can be calculated from (**E**) and (**υ**) by a set of equations and vice-versa.

BEAM188 is suitable for analyzing slender to moderately stubby/thick beam structures (see **Error! Reference source not found.**). The element is based on Timoshenko beam theory which includes shear-deformation effects. The element provides options for unrestrained warping and restrained warping of cross-sections (see **Error! Reference source not found.**).

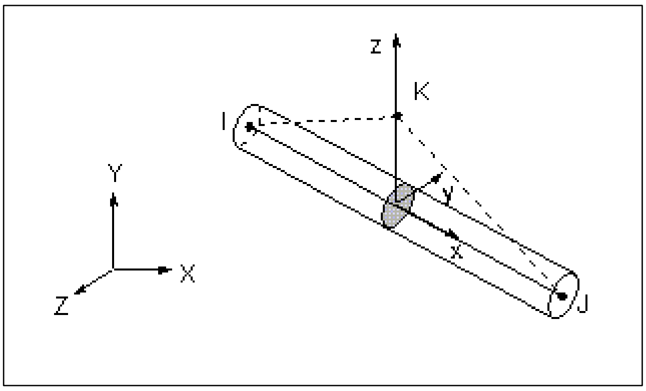

Fig. 1 Beam188 geometry

B. *Orthotropic beams*

Orthotropic beams are beams with unidirectional composite lamina has three mutually orthogonal planes of material property symmetry at least two orthogonal planes of symmetry [12], where the material properties are independent of direction within each plane, such materials require nine independent variables in their constitutive matrices. The nine variables are usually expressed as three Young's moduli ($\boldsymbol{E_x}, \boldsymbol{E_y}, \boldsymbol{E_z}$), three Poisson's ratios ($\boldsymbol{\upsilon_{xy}}, \boldsymbol{\upsilon_{yz}}, \boldsymbol{\upsilon_{zx}}$) and three shear moduli ($\mathbf{G}_{xy}, \mathbf{G}_{yz}, \mathbf{G}_{zx}$).

SHELL181 is suitable for analyzing thin to moderately-thick shell structures. It is a four-node element with six degrees of freedom at each node as seen in Fig. 2, translations in the x, y, and z directions, and rotations about the x, y, and z-axes. (If the membrane option is used, the element has translational degrees of freedom only). The degenerate triangular option should only be used as filler elements in mesh generation.

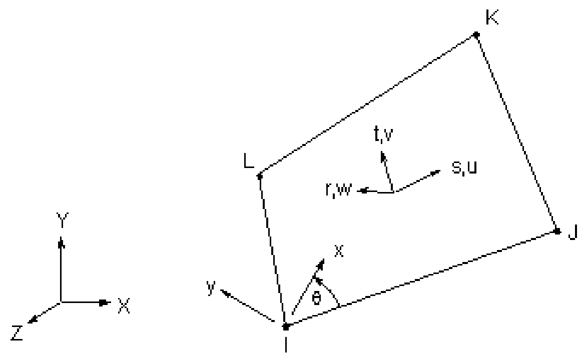

Fig. 2 Shell181 geometry

C. Functionally graded material beams

Functionally Graded Materials are orthographic materials usually made from a mixture of ceramics and metal. In FGM, the volume fraction of fundamental materials are gradually varying layer by layer, so that their properties of materials demonstrate a smooth and continuous change from one layer to immediate interference layer.

The same element for orthotropic beams has been used to simulate FGM beams (see *Fig. 2*).

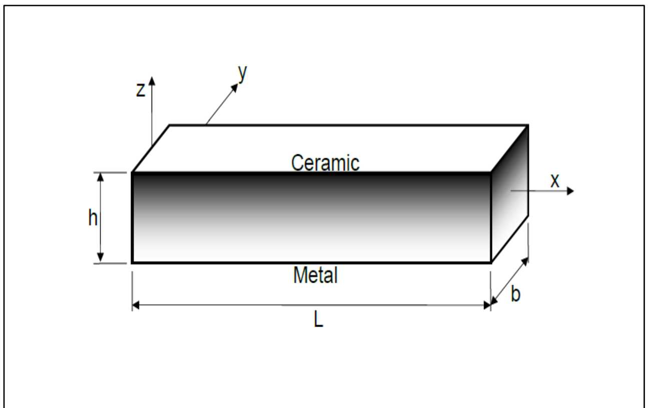

Fig. 3 Geometry of FGM beam

Material properties of each layer have been determined and calculated using different approaches, most researchers have used the power-law approach, exponential approach, and sigmoid approach to describe the volume fraction of the functionally graded materials [13] .

The volume fraction of the P-FGM is assumed to obey a power-law approach:

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \quad (1)$$

$$V_m = 1 - V_c \quad (2)$$

Where,

z… is the distance from mid-surface.

n… is the power law index (positive real number).

For n = 0 volume fraction of metal becomes one and homogeneous beam consisting only ceramic is obtained. When value of n is increased, content of metal in FGM increases. The effective material properties MP_{eff} are evaluated using the relation,

$$MP_{eff}(z) = MP_m V_m(z) + MP_c V_c(z) \quad (3)$$

Where,

MP_m and MP_c stand for material properties of metals and ceramics respectively.

III. Numerical Example and Validation

In this section, numerical examples have been executed to identify and clarify that using the previously mentioned elements in Ansys Workbench15 are suitable for investigating the static behavior of isotropic, orthotropic and FGM beams with different slenderness ratios and different boundary conditions.

A. Isotropic beams

Convergence study is important to identify the number of elements should be used in order to build the beam model in Ansys Workbench15. Several models have been built with different number of elements for the same material and geometrical properties as listed in Table 1 with the same boundary conditions.

Table 1 Material and geometric properties for an aluminum beam.

Property	Value	Unit
Modulus of Elasticity (E)	68.9	GPa
Shear Modulus of Elasticity (G)	27.6	GPa
Poisson's Ratio (υ)	0.25	----
Density (ρ)	2769	Kg/m^3
Length (L)	0.1524	m
Width (b)	0.0254	m
Height (h)	0.01524	m

1) Convergence study

The beam is subjected to a uniform distributed load of intensity 1 N/m. The obtained results are shown in Fig. 4 which presents the effect of number of element on the normalized transversal tip deflection of a cantilever beam, with length to height ratio (L/h=10), the normalized deflection can be expressed in the following equation:

$$\overline{w} = \frac{w\,10^2}{E\,I_{yy}\,L^2} \qquad (4)$$

Where;

w is the tip deflection obtained from Ansys Workbench15.

$\overline{w}$ is the normalized tip deflection according to Eq. (4).

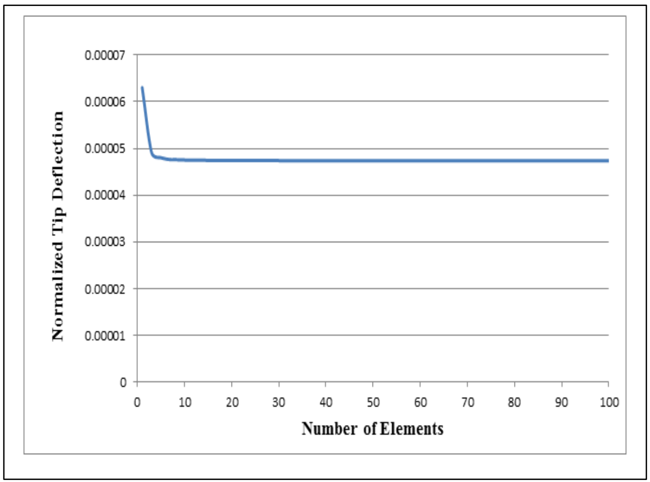

Fig. 4 Normalized transversal tip deflection with number of elements

It can be seen from Fig. 4 that the tip deflection starts to converge with number of elements equals to 20, which has a very good agreement with [6].

2) Static analysis

In this section, static deflection has been checked for a cantilever beam with different slenderness ratios (L/h) with the same applied force. Beam188 has been used to compare the results and to clarify that Beam188 element is the optimum element to simulate isotropic thick and thin beams.

A model has been built for an isotropic cantilever beam for different slenderness ratios with the following material properties: E= 29000, b= 1, υ=0.3 (all units obey SI standard) subjected to a tip load P= 100 N as given in [6] . Taking the number of elements equals to 35 for all different L/h ratio to ensure the converge of the static deflection value, taking the 35 elements does not affect the running time because of the simplicity of the used structure with just one material used.

Fig. 5 shows a graduated static deflection on the used cantilever beam with the mentioned force. It's clear that the maximum static deflection is located in the beam tip and its value equals to 32.837 m which is very accurate compared with different theories mentioned in **Error! Reference source not found.** for (L/h = 160/12).

The rest of obtained results are listed in Table 2 and compared with other results for the same slenderness ratios and the same applied force.

Analyzing the obtained results listed in Table 2 for different slenderness ratios leads to the fact that the presented model with Ansys Workbench15 using the specified element (Beam188) gives very accurate results and very close to the analytical solution using Timoshenko Beam Theory and also to the presented model in [6].

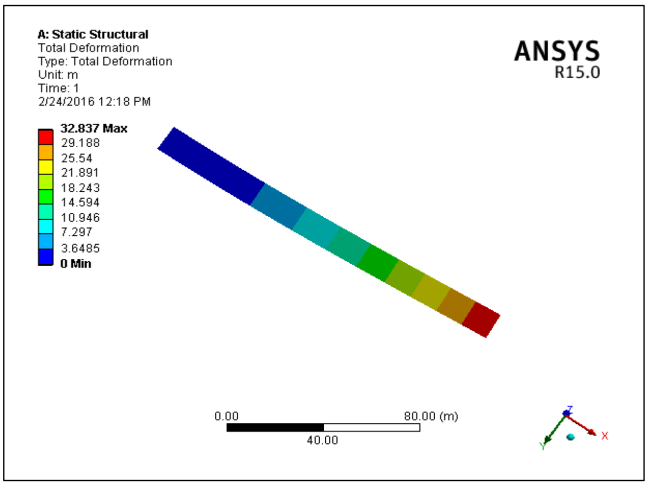

Fig. 5 Static Deflection of cantilever beam for L=160 and h=12 using Beam188 element

TABLE 2 TRANSVERSE TIP DEFLECTION OF THE ISOTROPIC CANTILEVER BEAM

L	h	[23]	CBT	TBT	FE	Present Work
160	12	32.831	32.694	32.838	32.823	32.837
80		4.157	4.0868	4.1586	4.1567	4.1578
40		0.546	0.5109	0.5467	0.5458	0.54634
12		0.0245	0.0138	0.0246	0.0239	0.02444
160	1	56485	56497	56498	56444	56498
80		7061.3	7062.1	7062.9	7056.3	7062.9
40		882.98	882.75	883.18	883.18	883.18
12		23.958	23.834	23.963	23.963	23.962

B. Orthotropic beams

Convergence study is important to identify the number of elements should be used in order to build the beam model in Ansys Workbench15 with the ACP module. Several models have been built with different number of elements for the same material and geometrical properties with the same boundary conditions.

1) Convergence study

In this section, the convergence has been checked for orthotropic beams with material and geometric properties given in Table 3.

TABLE 3 MATERIAL AND GEOMETRIC PROPERTIES OF THE COMPOSITE BEAM

Property	Value	Unit
E1	100	GPa
E2	4	GPa
G12, G13	2	GPa
G23	800	MPa
ρ	1	Kg/m^3
υ12	0.25	---
L/h	10	---

The mentioned cantilever beam has been checked for different fiber orientation angles [0/90/0/90], [45/-45/45/-45] and [30/50/30/50], the beam has been subjected to a uniform distributed load f1 of intensity 1 N/m.

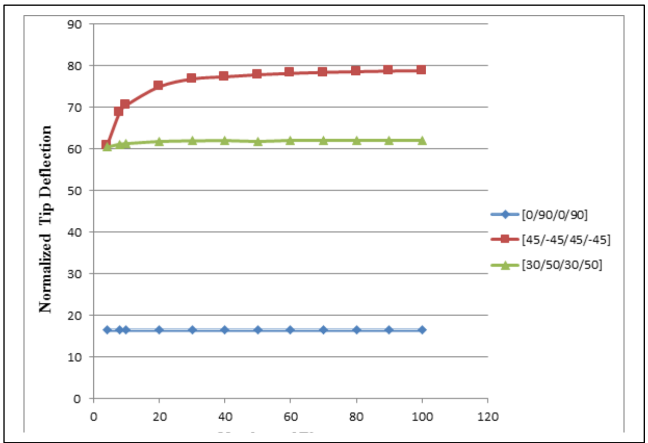

Fig. 6 Normalized transversal tip deflection with number of elements of a cantilever composite beam

Fig. 6 shows the effect of number of elements on the normalized transversal tip deflection, it is obvious that the normalized tip deflection starts to converge at reasonable numbers of elements for the different fiber orientation angles. And it's clearly shown that the [0/90/0/90] orthotropic beam starts to converge before the [45/-45/45/-45] and the [30/50/30/50] beams.

The normalized transversal tip deflection is given by the following relation:

$$\overline{w} = \frac{w A E_2 h^2 10^2}{f_1 L^4} \qquad \text{Eq. (5)}$$

Where,

w is the actual transversal deflection.

$\overline{w}$ is the normalized tip deflection.

It can be seen that the model starts to converge at a reasonable number of elements for the different fiber orientation angles.

2) Static analysis

To check the validity of the orthotropic beam, three layers symmetric cross-ply [0/90/0] and two layers anti-symmetric cross-ply [0/90] have been checked different boundary conditions (Clamped – Clamped, Clamped – Free).

All laminas are assumed to have the same thickness and made of the same materials, material properties are the same as listed in Table 3, different length to height ratio are used to validate the model; number of elements has taken equal to 100 for all beams. The mid-span deflection has been selected in order to compare the results with different theories.

Fig. 7 shows the graduated static deflection of a [0/90/0] clamped-clamped orthotropic beam. The upper left corner of the previous figure shows the maximum value of static deflection which is equal to 2.3579 E-6 m, using Eq. (4) the resultant non-dimensional mid-span deflection will be 0.15045 as listed in **Error! Reference source not found.**(4). Applying the same sequence gives us the mid-span deflection for different boundary condition for [0/90/0] orthotropic beams, the presented model has been compared with different theories and also with the presented model in [6].

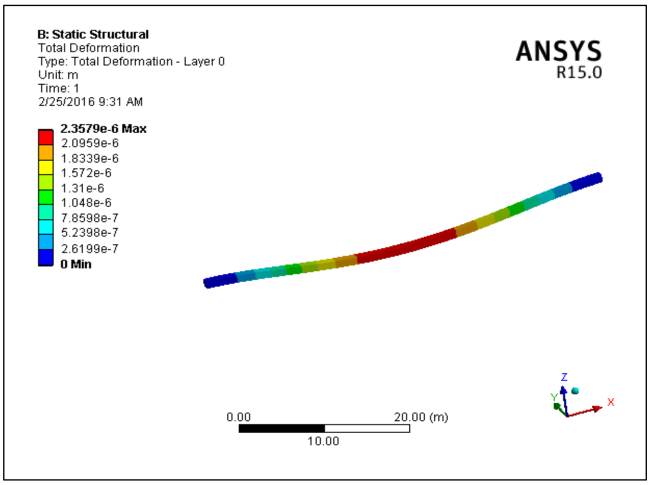

Fig. 7 Static deflection of clamped-clamped orthotropic beam with symmetric cross-ply [0-90-0] with L/h=50

Tables 4, 5, 6 and 7 show the difference between the present work and other different theories and also with the model presented in [6]. For low slenderness ratios (L/h = 5 and 10), the present work results have low accurate values compared with different theories, but results are close to the presented model in [6], for high slenderness ration (L/h = 50) the presented model has very accurate values with different boundary conditions and the results are very close to FSDT.

TABLE 4 NORMALIZED MID-SPAN DEFLECTION OF A CLAMPED-CLAMPED [0/90/0] ORTHOTROPIC BEAM

	L/h=5	L/h=10	L/h=50
[6]	1.8540	0.5694	0.1584
HOBT	1.537	0.532	0.147
SOBT	1.379	0.442	0.142
FOBT	1.629	0.504	0.144
RFSDT	1.629	0.504	0.144
CBT	0.129	0.129	0.129
Present Work	1.9037	0.5764	0.15045

TABLE 5 NORMALIZED MID-SPAN DEFLECTION OF A CLAMPED-FREE [0/90/0] ORTHOTROPIC BEAM

	L/h=5	L/h=10	L/h=50
[6]	6.2792	2.4221	1.1878
HOBT	6.824	3.455	2.251
SOBT	5.948	3.135	2.235
FOBT	6.698	3.323	2.243
RFSDT	6.693	3.321	2.242
CBT	2.198	2.198	2.198
Present Work	6.8125	2.7422	2.2653

TABLE 6 NORMALIZED MID-SPAN DEFLECTION OF A CLAMPED-CLAMPED [0/90] ORTHOTROPIC BEAM

	L/h=5	L/h=10	L/h=50
[6]	2.3750	1.0909	0.6794
HOBT	1.922	1.005	0.679
SOBT	2.124	1.032	0.679
FOBT	2.379	1.093	0.684
RFSDT	2.381	1.094	0.686
CBT	0.664	0.664	0.664
Present Work	2.39968	1.09692	0.68

TABLE 7 NORMALIZED MID-SPAN DEFLECTION OF A CLAMPED-FREE [0/90] ORTHOTROPIC BEAM

	L/h=5	L/h=10	L/h=50
[6]	17.128	13.194	11.935
HOBT	15.279	12.343	11.337
SOBT	15.695	12.400	11.338
FOBT	16.436	12.579	11.345
RFSDT	16.496	12.607	11.413
CBT	11.293	11.293	11.293
Present Work	16.6311	12.642	11.3568

C. *Functionally graded material beams*

Using Power-law Approach, it is possible to obtain an insight into the variation of the material properties across the thickness of the beam for different power law indices.

Convergence study is essential to determine the number of elements for the analyzed beams before investigating their static behavior.

1) *Convergence study*

In this section, the convergence has been checked for a functionally graded material beam with the material and geometric properties listed in

TABLE 8 MATERIAL AND GEOMETRICAL PROPERTIES FOR ALUMINUM AND ZIRCONIA

		Aluminum	Zirconia
Material Properties	Young's Modulus (E) [GPa]	70	200
	Poisson's	0.3	0.3

	Ratio (ν)		
Geometrical Properties	Length [m]	0.4, 1.6	0.4, 1.6
	Width [m]	0.1	0.1
	Height [m]	0.1	0.1

Convergence study for FGM beam made of Aluminum and Zirconia with simply supported boundary condition, L/h = 4 and power law exponent equals to 1 and subjected to a line pressure equals to 1 N/m.

The non-dimensional deflection is given by the relation:

$$\bar{\bar{w}} = \frac{5qL^4}{384E_{Al}I} \qquad \text{Eq. (6)}$$

Where,

$\bar{\bar{w}}$…. ..is the non-dimensional transverse deflection.

E_{Al} ….is Young's modulus of Aluminum.

q ……is the static load.

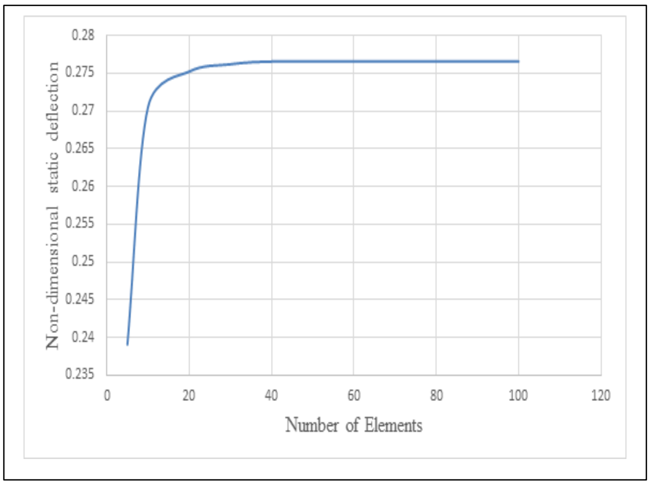

Fig. 8 Non-dimensional Deflection of a simply supported FGM beam with n=1 with different number of elements

Fig. 8 shows that Convergence of the simply supported functionally graded material beam with power-law index n = 1 is achieved with 40 elements.

2) *Static analysis*

In numerical results, the static responses of a 100 layers FGM beam are investigated. Functionally graded material (FGM) of the beam is composed of Aluminum and Zirconia with material and geometrical properties as shown in

Table 8, and its properties changes through the thickness of the beam according to the power-law. The bottom surface of the FG beam is pure Zirconia, whereas the top surface of the beam is pure Aluminum. The beam has been built with 100 layers to be sure that the number of layers will not affect the convergence study.

The width and the thickness of the beam are kept constant, a simply supported beam has been subjected to a uniformly distributed load of intensity 1 N/m.

The non-dimensional static deflection has been analyzed according to Eq.

TABLE 9 MAXIMUM NON-DIMENSIONAL TRANSVERSE DEFLECTION OFA SIMPLY SUPPORTED BEAM SUBJECTED TO A LINE PRESSURE OF INTENSITY 1 N/M FOR DIFFERENT VALUES OF POWER-LAW EXPONENT

Power-law Exponent	**Theory**	**Maximum Non-dimensional Transverse Deflection**	
		L/h = 4	L/h = 16
n=0 (Full Metal)	TBT	1.13002	1.00812
	HOSDT	1.15578	1.00975
	Present Work	1.15699	1.01014
n=0.2	TBT	0.84906	0.75595
	HOSDT	0.87145	0.75737
	Present Work	0.83846	0.72352
n=0.5	TBT	0.71482	0.63953
	HOSDT	0.73264	0.64065
	Present Work	0.703535	0.60981
n=1	TBT	0.62936	0.56615
	HOSDT	0.64271	0.56699
	Present Work	0.61465	0.53719
n=2	TBT	0.56165	0.50718
	HOSDT	0.57142	0.50780
	Present Work	0.55295	0.48792
n=5	TBT	0.49176	0.44391
	HOSDT	0.49978	0.44442
	Present Work	0.49823	0.44168
Full Ceramic	TBT	0.39550	0.35284
	HOSDT	0.40452	0.35341
	Present Work	0.40495	0.35354

Table 9 shows that the presented model of a FGM simply supported beam which has been subjected to a uniform line pressure of 1 N/m to the beam top surface gives very accurate results compared to the analytical results solved with the mentioned theories with all power law indices.

IV. Comparative Study

In this section, a comparison for the static behavior of isotropic, orthotropic and FGM beams has been investigated. Short beams with slenderness ratio equals to 4 and long beams with slenderness ratio equals to 16 have been subjected to both of line pressure and transversal point loads.

Aluminum has been used to simulate isotropic material beams, Epoxy_Carbon_UD_230GPa_Prepreg has been used to simulate orthotropic material beams, while Aluminum and Zirconia have been used together to simulate FGM beams, tables 10, 11 and 12 indicate the material and geometrical properties of each type respectively.

Table 10 Material and geometrical properties Aluminum beam

Property	Value	Unit
Modulus of Elasticity (E)	68.9	GPa
Poisson's Ratio (υ)	0.25	----
Density (ρ)	2769	Kg/m^3
Length (L_1)	0.4	m
Length (L_2)	1.6	m
Width (b)	0.1	m
Height (h)	0.1	m

Table 11 Material and geometrical properties for the Carbon-Epoxy orthotropic beam

Property	Value	Unit
Young's modulus of Elasticity in direction 1(E_1)	121	GPa
Young's modulus of elasticity in direction 2&3 (E_2 & E_3)	8.6	GPa
Shear modulus of elasticity (G12, G13	4.7	GPa
Shear modulus of elasticity (G23	3.1	GPa
Density (ρ)	1490	Kg/m^3
Poisson's ratio (υ12)	0.27	---
L_1	0.4	m
L_2	1.6	m
h	0.1	m
b	0.1	m

Table 12 Material and geometrical properties for the FGM beam

Property	Aluminum	Zirconia	Unit
	Value	*Value*	
Modulus of Elasticity (E)	70	200	GPa
Poisson's Ratio (υ)	0.3	0.3	----
Density (ρ)	2700	6511	Kg/m^3
Length (L_1)	0.4		m
Length (L_2)	1.6		m
Width (b)	0.1		m
Height (h)	0.1		m

A. *Model convergences:*

Convergence study has been executed for three different types of beams (Aluminum, orthotropic with [0/90/0/90] orientation and FGM with power-law index equals one). All beams have the same fixation (clamped-free) and the same applied load (uniformly distributed load of intensity 1 N/m). The functionally graded beam has been checked for 100 layers according to the power law exponent for n=1.

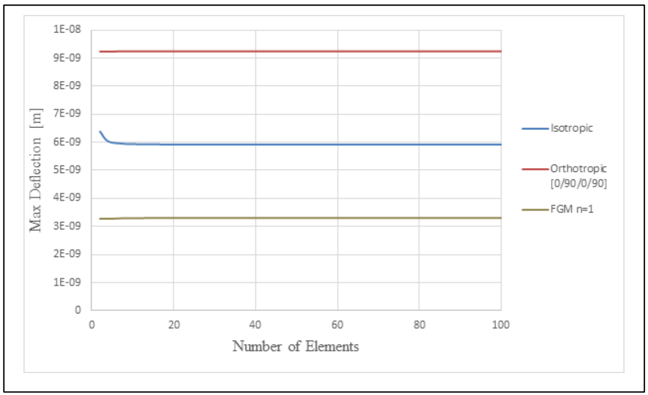

Fig. 9 Maximum deflection of a cantilever beam formed from isotropic, orthotropic and FGM with L/h=4 with different number of elements

graded beams starts to converge rapidly with low value of number of elements while the isotropic beam begins with a relative high value of maximum deflection (at NE = 2) and then begins to converge similarly as orthotropic and FGM beams.

B. *Static deflection*

Using different boundary conditions for the above mentioned beams and applying different typed of forces to analyze the static behavior with different orientation for the orthotropic beams and different power law exponent for the functionally graded beams. All beams have the same geometrical properties listed in Table 10

Table 11 and Table 12.

1) *Short beams static deflection (L/h=4)*

Beams with slenderness ration equals to 4 have been used to simulate short beams through this static analysis, all beams have been subjected to both of a transversal load with F = 10 N and a distributed load with q = 1 N/m. Three types of fixation have been used according to the common fixation types in real spacecraft, the first boundary condition is the clamped-free, the second one is clamped-clamped and the last one is the clamped-simply supported.

- Short beams subjected to a uniformly transversal distributed load (q=1 N/m):

In this section, short beams with slenderness ratio L/h = 4 have been subjected to a uniformly distributed load with intensity 1 N/m, and static deflection has been analyzed for different beams with different boundary conditions.

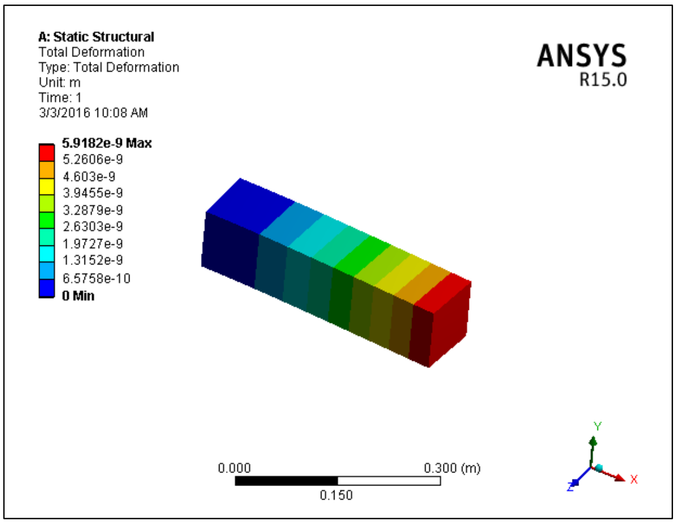

Fig. 10 clamped- free isotropic beam with L/h=4 subjected to line pressure (q=1 N/m)

Fig. 10 shows the graduated static deflection due to a uniformly distributed load with intensity 1 N/m of a cantilever isotropic beam. The upper left corner of the figure shows the maximum static deflection in [m] with red color, the dark blue color indicates the minimum static deflection parts.

Table 13 shows the maximum static deflection values of the above mentioned beams with different boundary conditions under the uniformly distributed load (q= 1 N/m).

TABLE 13 MAXIMUM TRANSVERSAL DEFLECTION FOR ISOTROPIC, ORTHOTROPIC AND FG BEAMS WITH L/H=4 SUBJECTED TO LINE PRESSURE (Q=1 N/M) WITH DIFFERENT BOUNDARY CONDITIONS

	C-F	C-C	C-S
Isotropic (Al)	5.918E-9	2.023E-10	3.405E-10
Orthotropic [0/90/0/90]	9.24E-09	9.64E-10	1.20E-09
Orthotropic [45/-45/45/-45]	2.51E-08	1.28E-09	1.62E-09
Orthotropic [30/50/30/50]	2.38E-08	9.83E-10	1.35E-09
FGM (n=0.5)	3.74E-09	1.29E-10	1.69E-10
FGM (n=1)	3.303E-9	1.106E-10	1.45E-10
FGM (n=5)	2.582E-9	8.403E-11	1.12E-10

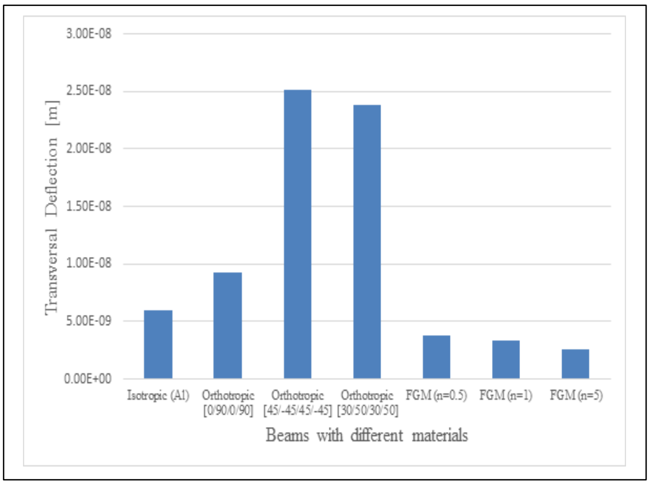

Fig. 11 Maximum transversal deflection of clamped-free beams with L/h=4 subjected to the same line pressure (q=1 N/m)

Fig. 11 shows that the orthotropic cantilever beam with orientation [45/-45/45/-45] has the biggest transverse deflection compared with other orthotropic, isotropic and FG beams. The same geometrical and material properties with same applied load, the orientation of fiber makes a big deflection difference between the different orthotropic beams. It shows also the big difference in deflection value between FG (with different power law index) beams and other beams, The FG beams have a valuable advantage when they r subjected to distributed load than both of isotropic and orthotropic beams.

With clamped-clamped beams the deflection difference is not that big as in cantilever beams, although the [45/-45/45/-45] still have the biggest transversal deflection compared with other orthotropic beams, all orthotropic beams have higher transversal deflection than isotropic and FG beams. FG beams still have the same advantage of lower deflection when they are subjected to distributed load in comparison with all other beams as shown in **Error! Reference source not found.**.

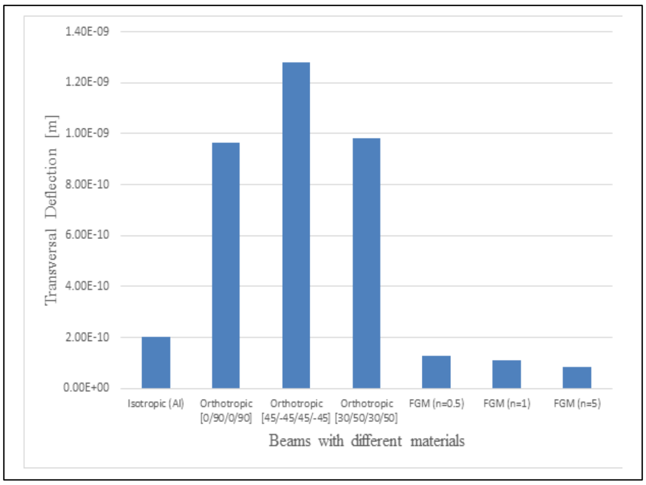

Fig. 12 Maximum transversal deflection of clamped-clamped beams with L/h=4 subjected to the same line pressure (q=1 N/m)

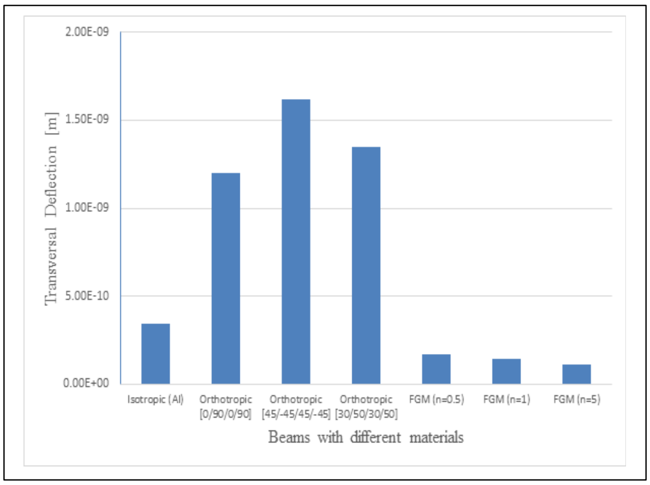

Fig. 13 Maximum transversal deflection of clamped-simply supported beams with L/h=4 subjected to the same line pressure (q=1 N/m)

Fig. 13 shows that the effect of the boundary condition changing at one end from clamped to simply supported has no big effect in the relative transversal deflection change between isotropic, orthotropic and FG beams, [45/-45/45/-45] orthotropic beam has the biggest value and the 5 power law index FG beam has the lowest one.

- Short beams subjected to a transversal point force (F =10 N):

In this section, short beams with slenderness ratio L/h = 4 have been subjected to a transversal point force with magnitude 10 N, and static deflection has been analyzed for different beams with different boundary conditions.

Fig. 14 show the graduated static deflection due to a point 10 N force of an orthotropic cantilever beam with orientation [30/50/30/50].

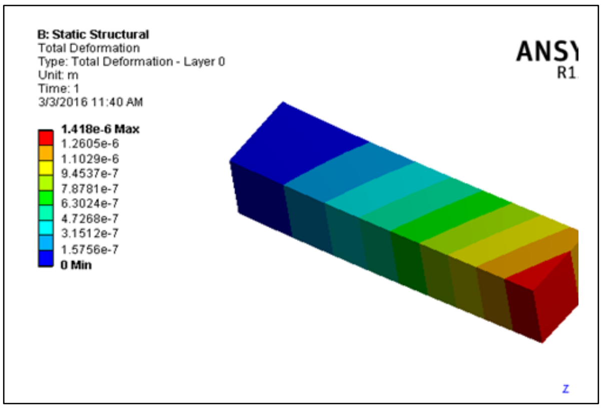

Fig. 14 clamped-free orthotropic beam [30/50/30/50] with L/h=4 subjected to a transversal force (F=10 N) at the tip

TABLE 14 MAXIMUM TRANSVERSAL DEFLECTION OF ISOTROPIC, ORTHOTROPIC AND FG BEAMS WITH L/H=4 SUBJECTED TO TRANSVERSAL FORCE (F=10 N) AT X=0.2 WITH DIFFERENT BOUNDARY CONDITIONS

	C-F **F at the tip**	**C-C**	**C-S**
Isotropic (Al)	3.89E-07	1.01E-8	1.51E-8
Orthotropic [0/90/0/90]	5.73E-07	2.41E-08	2.95E-08
Orthotropic [45/-45/45/-45]	1.49E-06	3.20E-08	4.06E-08
Orthotropic [30/50/30/50]	1.42E-06	2.45E-08	3.38E-08
FGM (n=0.5)	2.22E-07	3.22E-09	4.24E-09
FGM (n=1)	1.96E-07	2.77E-9	3.63E-9
FGM (n=5)	1.54E-07	2.09E-9	2.80E-9

Table 14 shows the maximum static deflection values of the above mentioned beams with different boundary due to a point 10 N force affecting on the tip for cantilever beams and at the mid-span for both of clamped-clamped and clamped-simply supported beams.

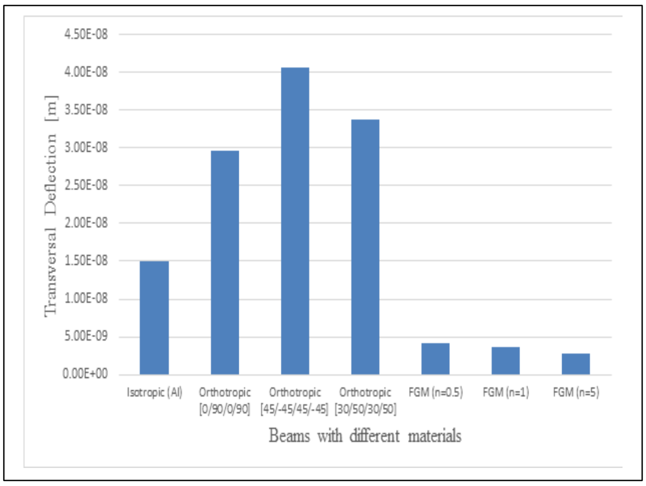

Fig. 15 Maximum transversal deflection of clamped-free beams with L/h=4 subjected to the same transversal force (F=10 N) at the tip

When all beams are subjected to a transversal point load (F=10 N), the [45/-45/45/-45] orthotropic beam keeps the 1st position in maximum transversal deflection compared with all beams and the relative big difference between both [45/-45/45/-45] and /30/50/30/50] orthotropic beams and other beams is still high. The [0/90/0/90] orthotropic beam has more resistance against deflection than other orthotropic beams, FG beams with different power law index still have the lowest transversal deflection than other beams and the 5 power law index still have the lowest value in comparison with all other beams as seen in Fig. 15.

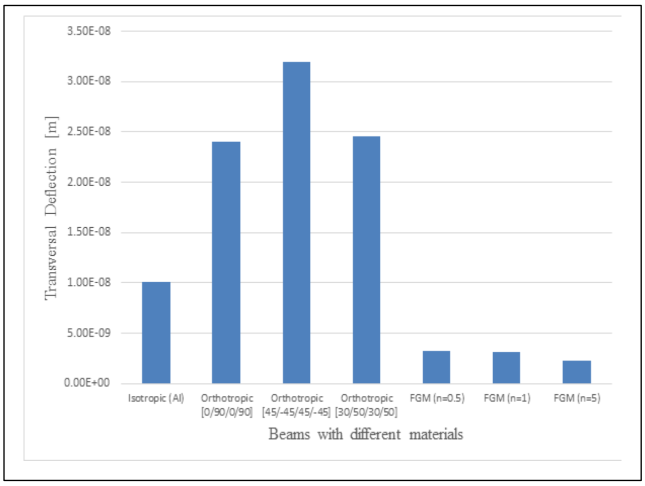

Fig. 16 Maximum transversal deflection of clamped-clamped beams with L/h=4 subjected to the same transversal force (F=10 N) at x=0.2 m

With clamped-clamped beams under the effect of a transversal point load, **Error! Reference source not found.** shows that the static deflection became lower than the cantilever one due to the boundary condition effect. The static deflection difference between the [0/90/0/90] beam and other orthotropic beams became lower. FG beams still have the lowest value of transversal deflection compared with other beams and the 5 power law index FG beam has the lowest value due to the high effect of ceramic modulus of elasticity (E) in its ingredients.

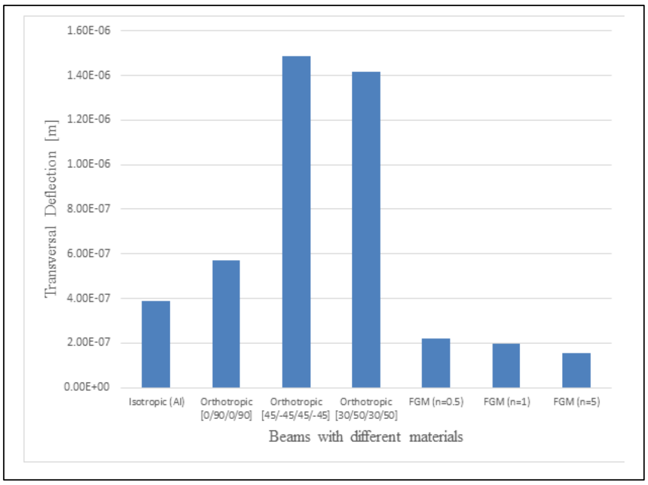

Fig. 17 Maximum transversal deflection of clamped-simply supported beams with L/h=4 subjected to the same transversal force (F=10 N) at x=0.2 m

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Changing the boundary condition from clamped-clamped to clamped-simply supported does not make a significant difference in the relative difference in transversal deflection of all beams, the [45/-45/45/-45] and the 5 power law index beams have the biggest and lowest values of static deflection respectively compared with all other isotropic, orthotropic and FG beams as seen clearly in Fig. 17.

2) Long beams static deflection (L/h=16)

Beams with slenderness ration equals to 16 have been used to simulate long beams through this static analysis, all beams have been subjected to both of a transversal load with F = 10 N and a distributed load with q = 1 N/m. Three types of fixation have been used according to the common fixation types in real spacecraft, the first boundary condition is the clamped-free, the second one is clamped-clamped and the last one is the clamped-simply supported.

- Long beams subjected to a uniformly transversal distributed load (q=1 N/m):

In this section, long beams with slenderness ratio L/h = 16 have been subjected to a uniformly distributed load with intensity 1 N/m, and static deflection has been analyzed for different beams with different boundary conditions.

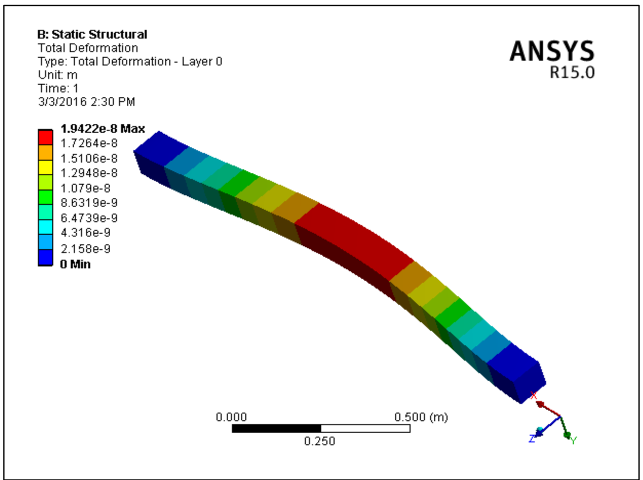

Fig. 18 A clamped- clamped FGM beam (n=0.5) with L/h=16 subjected to line pressure (q=1 N/m)

Fig. 18 *Fig. 18*shows the graduated static deflection of a clamped-clamped functionally graded material (n=0.5) beams, the upper left corner of the figure shows the maximum static deflection in [m] with red color, the dark blue color indicates the minimum static deflection.

Table 15 shows the maximum static deflection values of isotropic, orthotropic and functionally graded material long beams (L/h = 16)with different boundary conditions due to a uniformly distributed load with intensity (q=1N/m).

TABLE 15 MAXIMUM TRANSVERSAL DEFLECTION FOR ISOTROPIC, ORTHOTROPIC AND FG BEAMS WITH L/H=16 SUBJECTED TO LINE PRESSURE (Q=1 N/M) WITH DIFFERENT BOUNDARY CONDITIONS

	C-F	C-C	C-S
Isotropic (Al)	1.43E-6	3.11E-8	6.34E-8
Orthotropic [0/90/0/90]	1.81E-06	4.93E-08	8.90E-08
Orthotropic [45/-45/45/-45]	5.94E-06	1.33E-07	2.10E-07
Orthotropic [30/50/30/50]	5.34E-06	1.16E-07	1.91E-07
FGM (n=0.5)	8.96E-07	1.94E-08	2.92E-08
FGM (n=1)	7.94E-07	1.72E-08	2.55E-08
FGM (n=5)	6.24E-07	1.34E-08	2.03E-08

Fig. 19 shows that the transversal deflection of [45/-45/45/-45] and [30/50/30/50] orthotropic beams are relatively high with respect to other isotropic, orthotropic and FG cantilever beams with high slenderness ratio (L/h=16).

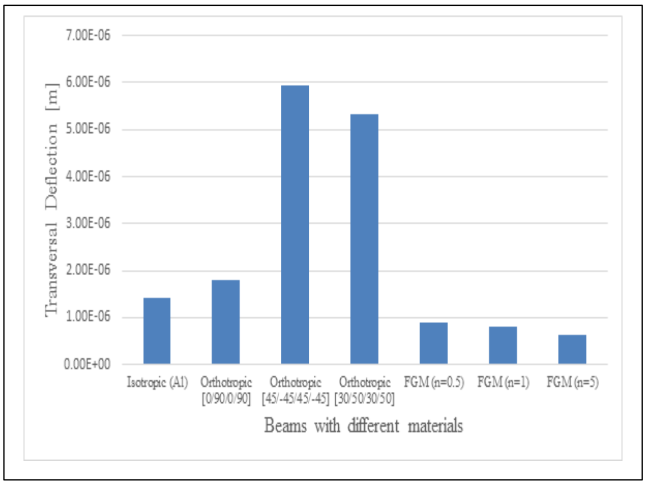

Fig. 19 Maximum transversal deflection of clamped-free beams with L/h=16 subjected to the same line pressure (q=1 N/m)

Fig. 19 shows that all the functionally graded material beams have low transversal deflection values compared with the other material beams, the 5 power law index FG beam has the lowest value of the static deflection, static deflection decreases with increasing the power index of functionally graded material beams, the isotropic beam has a relatively small value of static deflection compared with all orientation of the orthotropic beams with clamped-free fixation, the other

FG beams have lower value of static deflection compared with isotropic and orthotropic beams.

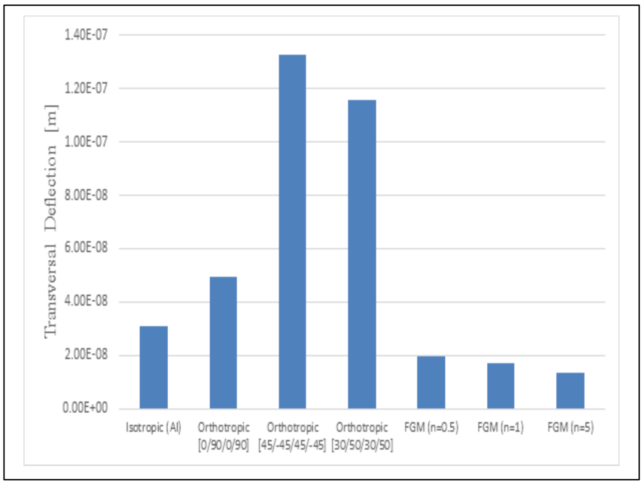

Fig. 20 Maximum transversal deflection of clamped-clamped beams with L/h=16 subjected to the same line pressure (q=1 N/m)

Changing the boundary condition from clamped-free to clamped-clamped with the same applied force and slenderness ratio as previous case lowers the value of static deflection, but keeps the difference of static deflection value between [45/-45/45/-45] and [30/50/30/50] orthotropic beams and other beams, also the all FG beams have lower static deflection value than other beams as seen in Fig. 20.

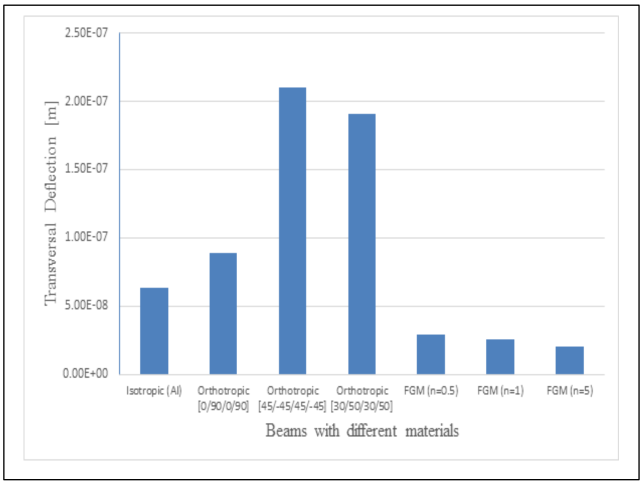

Fig. 21 Maximum transversal deflection of clamped-simply supported beams with L/h=16 subjected to the same line pressure (q=1 N/m)

Fig. 21 shows that all the compared clamped-simply supported beams have the same attitude towards static deflection as the same as clamped-clamped beams, only the values of the transversal deflection change lightly from the previous case, the 5 power law index FG beam still has the best resistance to the applied force and has the lowest value of static deflection.

- Long beams subjected to a transversal point force (F =10 N):

In this section, long beams with slenderness ratio L/h = 16 have been subjected to a transversal point force with magnitude 10 N, and static deflection has been analyzed for different beams with different boundary conditions.

Cantilever beams have been subjected to the point force at the tip, while clamped-clamped and clamped-simply supported beams have been subjected to the point force at the mid-span.

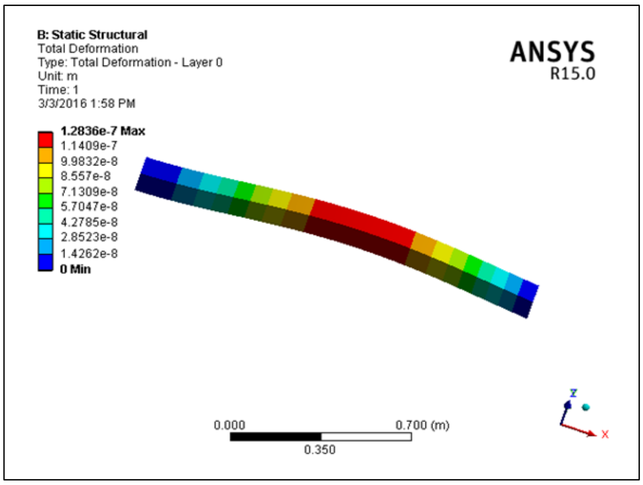

Fig. 22 A clamped-simply supported FGM beam (n=5) with L/h=16subjected to a transversal force (F=10 N) at x=0.8 m

Fig. 22 shows the graduated static deflection of a clamped-simply supported functionally graded material (n=5) beam, the upper left corner of the figure shows the maximum static deflection in [m] with red color, the dark blue color indicates the minimum static deflection parts.

TABLE 16 MAXIMUM TRANSVERSAL DEFLECTION FOR ISOTROPIC, ORTHOTROPIC AND FG BEAMS WITH L/H=16 SUBJECTED TO TRANSVERSAL FORCE (F=10 N) AT X=0.8 M WITH DIFFERENT BOUNDARY CONDITIONS

	C-F F at the tip	**C-C**	**C-S**
Isotropic (Al)	2.53E-05	3.89E-07	6.82E-07
Orthotropic [0/90/0/90]	2.70E-05	3.12E-07	5.62E-07
Orthotropic [45/-45/45/-45]	8.95E-05	8.41E-07	1.33E-06
Orthotropic [30/50/30/50]	8.03E-05	7.31E-07	1.21E-06
FGM (n=0.5)	1.34E-05	1.21E-07	1.82E-07

FGM (n=1)	1.19E-05	1.09E-07	1.61E-07
FGM (n=5)	9.37E-06	8.51E-08	1.28E-07

Table 16 shows the maximum static deflection values of isotropic, orthotropic and functionally graded material beams due to a transversal point load (F=10 N) affecting on the tip of cantilever beams and on the mid-span of both of clamped-clamped and clamped-simply supported beams with different boundary.

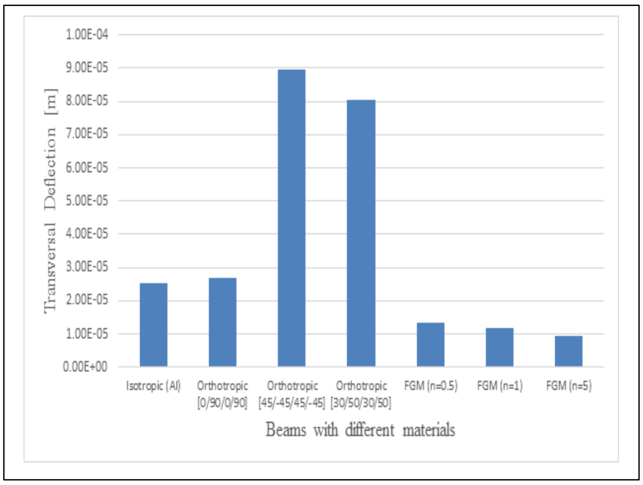

Fig. 23 Maximum transversal deflection of clamped-free beams with L/h=16 subjected to the same transversal force (F=10 N) at the tip

Applying a transversal force (F=10 N) at the tip of the different clamped-free beams with high slenderness ration (L/h = 16) as shown in Fig. 23 shows that the static deflection for all beams has increased significantly, and the difference between both of [45/-45/45/-45] and [30/50/30/50] and other isotropic, orthotropic and FG beams became high than the difference with low slenderness ratio (L/h =4). The 5 power law index FG beam has the lowest static deflection compared with all other beams.

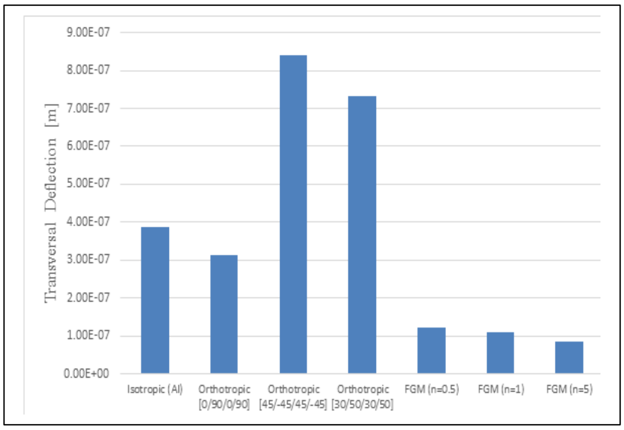

Fig. 24 Maximum transversal deflection of clamped-clamped beams with L/h=16 subjected to the same transversal force (F=10 N) at x=0.8 m

Applying the same force (F=10 N) with the clamped-clamped boundary condition for all beams as seen in Fig. 24 shows that the [45/-45/45/-45] and [0/90/0/90] orthotropic beams still have the highest value for static deflection, while the FG beams with different power law index values have the lowest values, and the 5 power law index FG beam is the best beam from the static deflection point of view compared with all mentioned beams.

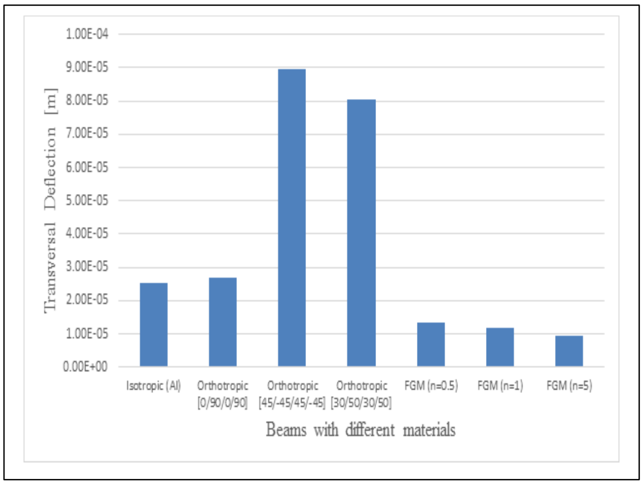

Fig. 25 Maximum transversal deflection of clamped-simply supported beams with L/h=16 subjected to the same transversal force (F=10 N) at x=0.8 m

Fig. 25 shows that all the compared clamped-simply supported beams have the same attitude towards static deflection as the clamped-clamped beams, only the values of the transversal deflection change slightly from the previous case, the 5 power law index FG beam still has the best resistance to the applied force and has the lowest value of static deflection. The functionally graded material beams have the lowest static deflection values compared with the other material beams and the value of static deflection decreases with increasing the value of the power index in the functionally graded material beams, as the effective material properties tends to take the values of the ceramic properties with increasing the power indices.

V. Summary and Conclusion

A detailed study for the static behavior of isotropic, orthotropic and FGM beams has been executed in this paper; isotropic beams with different slenderness ratios and with different boundary conditions have been investigated statically including convergence study. Orthotropic beams also have been studied statically with different number of laminas with different orientation including a convergence study. A detailed study for the functionally graded material beams has been executed for different boundary conditions, different slenderness ratios and different power law indices, taking into account the convergence study and the static behavior. A comparison between isotropic, orthotropic and functionally graded material beams has been executed from static analysis point of view, and the following conclusions have been drawn:

- Static analysis of isotropic beams using Beam188 element gives very accurate results and very close to the analytical solution using Timoshenko Beam Theory (TBT) which is a first order shear deformation theory.
- Static analysis of orthotropic beams using Shell181 element gives very accurate results and very close to the analytical solution using First order Shear Deformation Theory (FSDT) for relatively thin beams (high slenderness ratio) and good results for relatively thick beams (low slenderness ratio).
- Static analysis of the FGM beams using ACP module in Ansys Workbench15 gives accurate results with both thick and thin beams with different boundary conditions and for different power law indices with different types of loads (point load and uniformly distributed load), results becomes too close to the analytical solution with increasing the power law index.
- Convergence study of an isotropic, [0/90/0/90] orthotropic, and a FGM (n=1) beams shows that the converge starts rapidly for all beams, but the orthotropic one begins with a relatively high value of static deflection, all beams have been totally converged for the static deflection values at number of elements equals to 40.
- FGM beams with different power law indices have higher resistance to both of the uniformly distributed and the transversal point loads than isotropic and orthotropic beams with different types of fixation (clamped-free, clamped-clamped and clamped-simply supported) and for both of long and short beams.
- [45/-45/45/-45] and [30/50/30/50] orthotropic beams have the lowest resistance to both of the uniformly distributed and the transversal point loads among all compared (short and long) beams.
- Increasing the power index law in FGM beams leads to higher resistance to the different static loads.

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