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Vibration and Tracking Control of A Single Link Flexible Manipulator Using LQR and Command Shaping

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***Abstract*—Residual vibrations and transient deflections are detrimental to the optimal performance of flexible manipulators. This paper presents a linear quadratic regulator (LQR), combined with three command shapers, for position control and residual vibration suppression of a single link flexible manipulator. Conventional (time delay) zero vibration (ZV) and zero vibration derivative-derivative (ZVDD) input shaping filters are designed using the damping ratio and natural frequency of the flexible manipulator. While an output-based input shaping filter, in which model parameters are not required, is designed using the system's output. In each case, the filters are incorporated with LQR for position tracking. Simulation results show the output-based filter performed better than the time delay filters in suppressing residual vibration.**

Keywords—Single link flexible manipulator; output-based filter; input shaping; LQR; residual vibration; ZV; ZVDD.

I. INTRODUCTION

Flexible manipulators are machines which are basically used for transferring objects from one point to another. It is used in aerospace, space exploration, nuclear plants and automotive industries for various tasks such as assembling, spray painting and welding [1]–[4]. Flexible manipulators have several advantages over rigid body manipulators, these include light weight design, high mobility, high speed, less energy consumption, operation safety and less cost [5]. However, due to the flexibility of these manipulators and rigid hub motion of the motor, there are high levels residual vibrations and transient deflection when transferring objects from one point to another. This causes difficulty in positioning of payload hence, low productivity and less efficiency in operation of the system. This is a big challenge in control and robotics engineering [6]–[8]. A lot of techniques were presented by various researchers on how the problems stated above can be solved; these include open and close loop control techniques.

In open loop control, vibration control of single link flexible manipulator using command shaping method was presented in [9], it is a feedforward control for eliminating residual vibration. Various input shaping filters were also designed. These techniques are cheap as no sensor is required [10], [11]. In [12], analysis to error of natural frequencies of ZV, zero vibration derivative (ZVD) and ZVDD was conducted using a flexible beam, their performance was also checked. Microcontroller based input shaping for vibration control of single link flexible manipulator was proposed in [2], embedded ZV, ZVD and ZVDD are designed and their performance are compared. Output based input shaping technique was proposed in [13], it is designed using the output signal of the target system, thus avoiding the problem of parameter uncertainty. Output based filter comparison with ZV and ZVD for vibration control of motor rotating plate was presented in [14]. Simulations and experimental results show the effectiveness of this technique.

Feedforward control alone is affected by peripheral disturbances [15], so there is need for input shaping filters to be incorporated with closed loop to improve the robustness of the system [16]. Passivity-based control of single link flexible manipulator was proposed in [17], linear strain feedback was used in this technique. Consequently residual vibration was eliminated and accurate position was achieved. Shape optimization techniques was presented in [18], using various optimization problems for comparison.

Hybrid input shaping and proportional integrative derivative (PID) for the control of flexible manipulator was proposed in [5] for position and vibration control. In [19], vibration and tracking control of flexible manipulator using hybrid fuzzy logic controller was presented. In this method a proportional derivative (PD-type) fuzzy logic control was used for control of rigid body motion while PID was used for vibration control of the system. PID and state feedback control of a single link flexible manipulator was presented in [20] and their performance was compared. In addition, pneumatic drive action vibration control of flexible manipulators using an adaptive interaction PD controller was presented in [21]. In this technique an adaptive PD controller was used for vibration and accurate positioning control. In [22] input tracking and vibration control of single link flexible manipulator using LQR and collocated PID was presented, vibration was suppressed and reference tracking was achieved. Input shaping and LQR for input tracking and vibration control of single link flexible manipulator was presented in [23], input tracking was achieved with near zero vibration. Modified Genetic algorithm for optimization of feedback gains of a PD controller was proposed in [24] for position and vibration control of single link flexible manipulator, fast converging and high accuracy was achieved.

Based on the above literature, this paper proposes LQR with output based input shaping filter for position and residual vibration suppression. Their performances are compared with LQR-ZV and LQR-ZVDD. The rest of the paper is organized as follows; section II presents system description and model of single link flexible manipulator. Section III presents control schemes. Section IV explains simulation results and performance of the filters. Section V presents the conclusion.

II. SYSTEM DESCRIPTION

This section presents system dynamics of single link flexible manipulator.

A. *System description*

A single link flexible manipulator system is described in Fig. 1 where; θ is the hub angle and α is the tip deflection. There are movable and non - movable coordinates, XOY is fixed while POQ is a mobile coordinates, τ is the torque applied at the hub. The manipulator is assumed to be torsion and stiff in vertical bending, effect of gravity is neglected, constant cross sectional area and properties of the materials are uniform. Hence the parameters of the system are: $T_m, B_{eq}, K_{stiff}, J_{link}$ and J_{eq} which are load torque, viscous damping coefficient, total stiffness of the model, moment of inertia of the link and equivalent moment of inertia of the model respectively. The nominal values are as recorded in table 1.

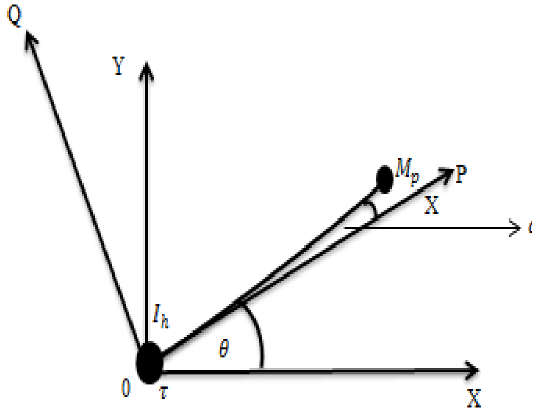

Fig. 1. *Schematic diagram of single link flexible manipulator*

TABLE I. SYSTEM PARAMETERS

Parameters	***Symbols***	***Values***	***Units***
Viscous damping coefficient	B_{eq}	$4X10^{-3}$	N/m ^{2}
Total stiffness of the model	K_{stiff}	$5.7X10^{-3}$	Nm/deg
Moment of inertia of the link	J_{link}	1.3978	kg/m^2
Equivalent moment of inertia of the model	J_{eq}	$2X10^{-3}$	kg/m^2

B. *Model of the system*

The model of single link flexible manipulator used in this work is as derived in [25]. The system is represented in state space as:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

In which:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 212.7091 & -19.6573 & 0 \\ 0 & -616.9655 & 19.6573 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 34.6024 \\ -34.6024 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

And

$$x = \begin{bmatrix} \theta & \alpha & \dot{\theta} & \dot{\alpha} \end{bmatrix}$$

III. CONTROL DESIGN

In this section, two types of input shaping technique and LQR are presented, that is conventional input shaping and output based input shaping. ZV, ZVDD and output based filters are designed to eliminate tip deflection while LQR is designed to control the hub angle position.

A. *Conventional Input shaping technique*

Conventional (time delay) input shaping is a feedforward technique which is very good in residual vibration suppression. The time instances and amplitudes of the pulses are calculated using damping ratio and natural frequency of the single link flexible manipulator system [26].

Hence, Fig. 2 shown the command shaping process of ZV shaper with two impulses and the parameters are obtained as:

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{1+k} & \frac{k}{1+k} \\ 0 & \tau_d \end{bmatrix} \tag{3}$$

In which:

$$\tau_d = \frac{\pi}{\omega\sqrt{(1-\zeta^2)}}$$

And

$$k = e^{\frac{-\pi\zeta}{\sqrt{(1-\zeta^2)}}}$$

Where,

ζ and τ_d are damping ratio and time delay respectively.

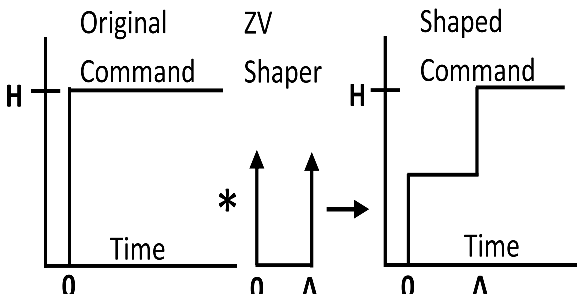

Fig. 2. *Input shaping process*

Furthermore, to improve the robustness of the input shaping to errors in frequency, zero vibration derivative derivatives (ZVDD) is designed and the algorithm is as obtained in [12]. ZVDD parameters are obtained as;

$$\begin{bmatrix} A_i \\ t_i \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+k)^3} & \frac{3k}{(1+k)^3} & \frac{3k^2}{(1+k)^3} & \frac{k^3}{(1+k)^3} \\ 0 & \tau_d & 2\tau_d & 3\tau_d \end{bmatrix} \tag{4}$$

Where k and τ_d are the same as in (3).

B. *Output based input shaping*

In this technique, the filter is designed using only the signal output of the target system. A reference system which is used in the design of the filter is designed based on the dynamic response of the system. The output of the system was measured using simulation and the filter gains were obtained by writing a program code in MATLAB.

To explain the basic principle of this technique, a second order system shown in (5) is considered as in [13].

$$G(s) = \frac{Kw_n^2}{s^2 + 2\xi w_n s + w_n^2} \tag{5}$$

Where;

k_m, ξ_m and w_n are static gain, damping ratio and natural frequency respectively.

Let the reference system be design in a form:

$$M(s) = \frac{k_m w_m^2}{s^2 + 2\xi_m w_m s + w_m^2} \tag{6}$$

The filter can be designed as:

$$F_o(s) = \frac{k_m w_m^2 s^2 + 2\xi w_n s + w_n^2}{Kw_n^2 s^2 + 2\xi_m w_m s + w_m^2} \tag{7}$$

Then based on zero-pole cancelation, product of $G(s)$ and $F_s(s)$ will gives $M(s)$, therefore, adequate static gain, damping ratio and bandwidth can be achieved by choosing k_m, ξ_m, w_m respectively [13], [27].

Thus,

$$F(s) = \frac{s^2 a_2 + a_1 s + a_0}{s^2 + 2\xi_m w_m s + w_m^2} \tag{8}$$

The aim is to obtain the filter gains (a_0, a_1, a_2) so that zeros of $F_s(s)$ will cancel the poles of $G(s)$,

Thus $F_s(s) = F_0(s)$ and poles of $G(s)$ are identical [27],[13].

The aim of the design is to find accurate filter coefficients so that the target system has zero or little vibration. A critically damped system is considered as a reference system which can be realized as:

$$G_r(s) = \frac{w^2{}_c}{(s + w_c)^2} \tag{9}$$

Where w_c is the bandwidth of the system and is selected based on the time response of the system. This system has little or zero vibration. A Cost function is used to minimize the difference between the output of the reference system and that of the target system.

Hence;

$$E(s) = w(t)\int_0^T (y(t) - y_r(t))d_t \tag{10}$$

Where:

$w(t), y(t)$ and $y_t(t)$ are the weighting factor, output of the target system and output of the reference system respectively. (10) is further elaborated as:

$$E(a_{1,}a_2,....a_n) = \int_0^T w(t)\,((\sum_{i=0}^{m} a_i y_i(t)) - y_r(t))^2 \tag{11}$$

Therefore $a_1, a_{2,} \dots a_m$ can be obtained from (12) and

$$\int_0^T w(t)\, y_k(t)\, ((\sum_{i=0}^{m} a_i y_i(t)) - y_r(t))dt = 0 \qquad (12)$$

Hence, simplifying (12) as:

$$S_{\alpha,\beta} = \int_0^T w(t)\, y_\alpha(t)\, y_\beta(t),$$
$$\alpha = 0,1,2,3\dots m \qquad (13)$$
$$\beta = 0,1,2,3,\dots m$$
$$\alpha + \beta \neq 0$$

$$S_{\alpha,r} = \int_0^T w(t)\, y_\alpha(t)\, y_r(t), \qquad (14)$$
$$\alpha = 0,1,2,3\dots m$$

Therefore, simplifying (12), (13) and (14) will give:

$$\sum_{i=0}^{m} a_k S_{k,i} - S_{k,r} = 0, \qquad (15)$$
$$K = 0,1,2,3\dots.m$$

In this paper, the single link flexible manipulator system is type-1, hence a_0 is zero. Also, since only the hub angle and tip deflection are considered, the order of the system is reduced to four. Hence, the reference system was designed by considering the dynamic response of the system. By selecting $\mathrm{w_c} = 10$ and using (9), the reference system is as:

$$G_r(s) = \frac{10000}{s^4 + 40s^3 + 600s^2 + 4000s + 10000} \qquad (16)$$

Therefore, the filter gains are calculated by in MATLAB using the following relation:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \begin{bmatrix} S_{1r} \\ S_{2r} \\ S_{3r} \\ S_{4r} \end{bmatrix} \qquad (17)$$

Hence, the filter gains are obtained as;

$$a_1 = 5680.5, a_2 = 484.7298, a_3 = 14.4578 \ and \ a_4 = 0.7952$$

Substituting in (8), the filter is obtained as;

$$F(s) = \frac{0.7952s^4 + 14.4578s^3 + 484.7298s^2 + 5680.5s}{s^4 + 40s^3 + 600s^2 + 4000s + 10000}$$

(18)

C. *LQR control*

In addition to feedforward control, LQR is employed for the control of hub angle so that both vibration and position of single link flexible manipulator are optimally controlled. LQR is a full state feedback control that is usually used in industries for mechanical system control. However, the requirement of sensors for each state makes the control system expensive. In this control technique, a control law is selected $u = \psi(x)$ to regulate the state x and minimize the performance index:

$$J = \int_0^\infty x^T(t)Qx(t) + u^T(t)Ru(t) \qquad (19)$$

Where: $Q^T = Q \geq 0 \ and \ R^T = R > 0$

R and Q are control and state penalty matrices. They are tuned to obtain the gains matrix *K*= [1 0.01 0.005 0.01] for the ZV and ZVDD while *K*= [0.0024 -9.9973 0.03 -0.9987] for the output-based shaper. However, R has to be relatively smaller than Q for good control action and deviation of x from zero will not be heavily penalized [23]. Fig. 3 is a complete block diagram showing the input shaping filter incorporated with LQR.

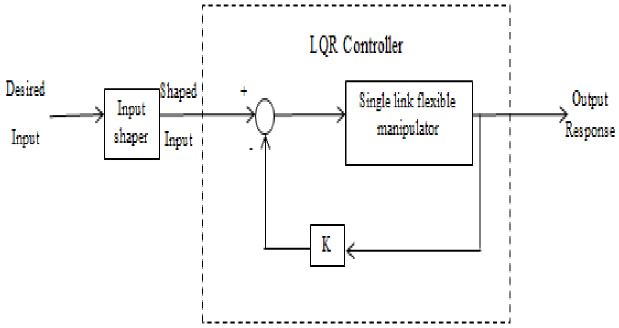

Fig. 3. *LQR and input shaping control*

IV. RESULTS AND DISCUSSION

In this section, the control actions of the filters with LQR using 90^o reference will be discussed. The damping ratio and natural frequency of the system is used in calculating the time delay filters' parameters shown in Table 2, while only the signal output of the target system is used in designing the output-based filter. As shown in Fig. 4 and Fig. 5, the output based filter with LQR shows a good performance in both tracking and vibration control. ZV and ZVDD with LQR also performed well but their performance depend on the derivative order of the filter. Higher derivatives perform better in vibration suppression but have a slower response. The higher the order of derivative the better the vibration suppression but the lower the speed response of the system, also the performance can be further justified with angular velocity and tip velocity as shown in Fig. 6 and Fig. 7. Based on the simulation results shown below, the LQR with output-based filter performance is better in suppression of residual vibration.

TABLE II. SHAPERS' PARA METERS

Shaper	***ZV***	***ZVDD***
A1	0.81357	0.53849
A2	0.18643	0.37019
A3	-	0.08483
A4	-	0.00648
t1 (sec)	0	0
t2 (sec)	0.13554	0.13554
t3 (sec)	-	0.27108
t4 (sec)	-	0.40662

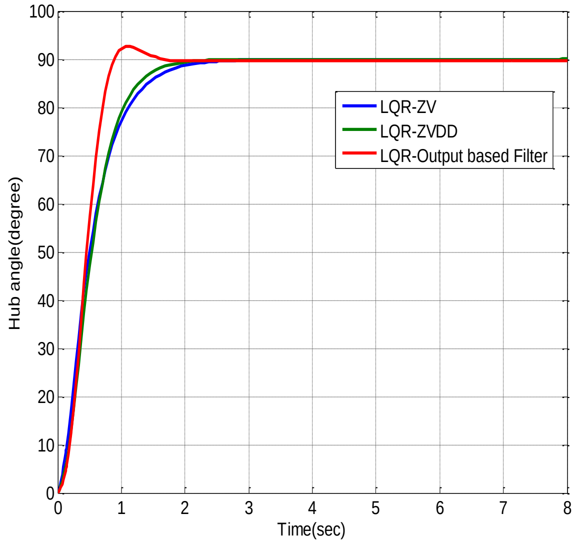

Fig. 4. *Hub angle*

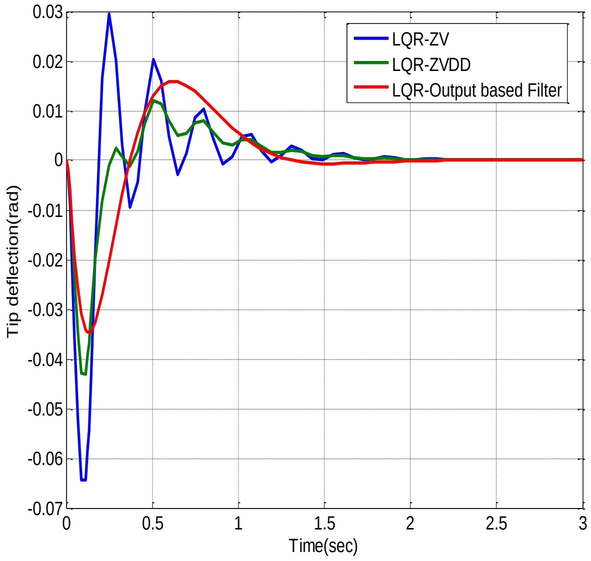

Fig. 5. *Tip deflection*

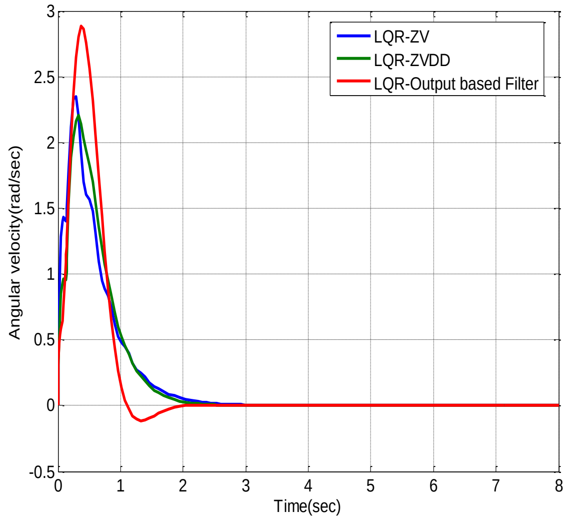

Fig. 6. *Angular velocity*

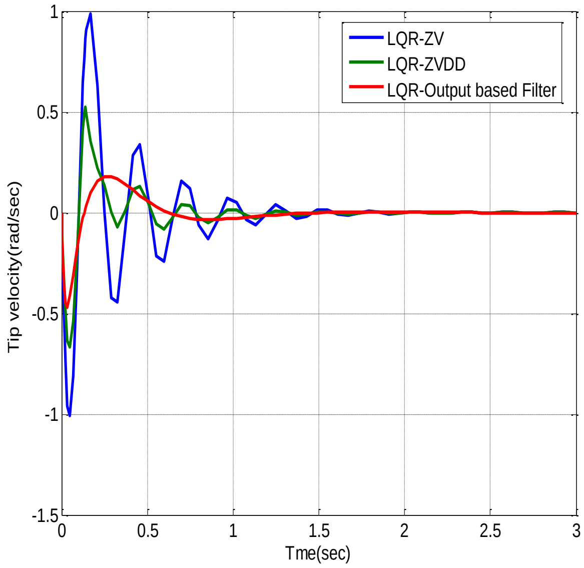

Fig. 7. *Tip velocity*

V. CONCLUSION

Performances of the time delay input shapers with LQR and output-based input shaper with LQR is presented and they are assessed based on vibration suppression and set point tracking. Time delay filters with higher order derivatives performed better in vibration suppression and increased robustness. However, increasing the order of the derivative leads to an increased delay in the system. The output-based input shaper with LQR performed remarkably in both vibration control and reference tracking with near zero tip deflection.

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