Nonlinear Torsional Vibration Analysis Of Variable Inertia Reciprocating Engines

Hameed D. Lafta, Akram H. Shather
Dept. of production engineering & metallurgy, Dept. of communication engineering
Technical college of engineering/ Sulaimani polytechnic university
Sulaimani, Iraq
hameed.lafta@spu.edu.iq,dr_akram75@yahoo.com

***Abstract*—The effective inertia of reciprocating machines is varied considerably over a cycle of a crankshaft. Due to this, the system expected a nonlinear linear torsional vibration, and also a harmonic excitation torque is produced. Consequently, large amplitudes of harmonic excitation torque, lead to the failure of the crankshaft, especially in heavy reciprocating machines. Neglecting of variation in inertia, lead to linear torsional vibration analysis, and it may not adequate one. In this paper, a mathematical expression for the variation in the inertia of the connecting rod, in terms of the crankshaft angle, is derived based on Fourier analysis. The nonlinear torsional vibration governing differential equation is obtained by using Lagrange's equation. The torsional vibration displacement considering variable inertia of the reciprocating parts is examined by developing a computer program using MATLAB/Simulink. The simulation results indicate the greater effect of variable inertia on the torsional vibration displacement and the system may depict large amplitudes with increasing engine speed. Also, the range of the critical speed is changed. This attributed to producing the harmonic excitation torque due to considering the variable inertia term. Which in turn give a rise to the phenomenon of secondary harmonic resonance. The present model is compared with equivalent inertia model. The results show irregular periodic motion and different trend of the frequencies and amplitudes. The present research is aimed to enhancement of the nonlinear torsional vibration analysis of reciprocating machines.**

Keywords—nonlinear; torsional vibration; variable inertia; reciprocating machines

I. INTRODUCTION

In the previous time, the issues concerning the analysis of the torsional vibration of reciprocating engines or machines were carried out with the effect of the variable inertia of reciprocating parts are often disregarded or simplified which would be result in linear torsional vibration analysis. But, during the last decay, the mathematical modeling and experimental study give an attention for the phenomenon arising due to the variable inertia of the moving part in reciprocating machines which emphasize the nonlinear torsional vibration nature of these machines. It was also verified that, the secondary phenomenon is responsible for many structural failure. Pasricha and Hashim [1] studied the effect of reciprocating mass on torsional vibration of diesel engine using an equivalent mass for reciprocating parts and taken into account the variation of inertia of the system. Their results show the region of instabilities at different speeds of engine rotation. Xiang J H, and Liao R D. [2] studied the variable inertia torsional vibration of crankshaft. Their study is based on the instantaneous kinetic energy method. They shown that the displacement and angular velocity of reciprocating components are vary with crankshaft motion, and resulting in generation of non constant inertia for the crankshaft assembly. Amin [3] proposed a new comprehensive model and solution method using analytical formulations to study both steady state and transient response of complex reciprocating trains. The analytical results recommended torsional reliable trains, to overcome the shaft torsional oscillations which may results in failure of rotating components of reciprocating trains due to secondary resonance. Ying, H et.al [4] presented and derived a general expression for the non-constant inertia of crank shaft assembly based on the instantaneous kinetic energy equivalence method. The natural frequency and mode shapes of crankshaft assembly are investigate using Eigen vector method. Their results indicated that when the non-constant inertia taken into account, the additional excitation torque due to non-constant inertia activities the 2nd order rolling vibration. Also, it is found that, the additional damping torque resulting from the non-constant inertia is the main nonlinear factor. Joshi, N.K., and Pravin, V.K. [5] analyzed torsional vibrations of typical marine propulsion. Their linear analysis constitutes of modal analysis, order analysis, harmonic analysis, stress analysis, corresponding to critical speeds of the system. Also, the non-constant inertia effect on the critical speeds and mode shapes was carry out using a developed computer program. They found that the range of critical speed over which hazardous vibration stress was widened due to the consideration of non-constant inertia. Cheng et.al [6] analyzed the crank shaft torsional vibration using a calculation method based on a flexible multi-body dynamics. Their developed model was simulated by ADAMS software. The results showed that before transformation the natural frequency of the crankshaft was closed to the excitation frequency, thus caused the vibration amplitude of crankshaft too large.

Nerubenko, [7] implemented a new design concept in hybrid power train to overcome the problem of torsional resonance vibrations which may be always represented the main problem for hybrid automobile power train. The new design approach based on optimal insertion in power train structure the device based on George Nerubenko US patent 7464800, having the control system with instantaneous frequencies tuner and variable damping device adjusted for all operational frequencies. Tests results and mathematical simulation illustrate the effectiveness of the new concept approach. Wei et.al [8] analyzed torsional vibration of motorized wheel vehicle by using a lumped mass method. In their work the inertia force of the reciprocating parts are included and the differential equations of non-linear torsional vibration were obtained. Their experimental and simulation results indicate that the torsional vibration is strongly nonlinear in their nature and resonance phenomenon occurs under the combined effect of non-linear parameters and external excitation. Joshi, N.K., and Pravin, V.K. [9] investigated experimentally the effect of torsional excitation of variable inertia effects in multi cylinder reciprocating engine. Their experimental results indicated that the nonlinear coupling was observed and the phenomenon of secondary resonance was approved, which in turn emphasized on the nonlinear nature of the torsional vibration of reciprocating multi cylinder engine.

In the present paper an attempt was made to consider the effect of the variation of the inertia of the reciprocating parts on the torsional vibration of reciprocating machines. The study is based on a development of closed form expression for the inertia of the reciprocating parts by using Fourier analysis. A mathematical model defining the variation on the mass moment of inertia of the reciprocating parts over a cycle of crankshaft was derived. The governing nonlinear differential equation of the torsional vibration of the system was found by using the energy method. The effect of the inertia ratio parameters on torsional vibration amplitudes and the variation on the frequency ratio over a crankshaft cycle are examined.

II. MATHEMATICAL FORMULATION

A. *Kinematical analysis of reciprocating machines*

The geometrical arrangement of a crankshaft-connecting rod-reciprocating mass of reciprocating engines or compressor is shown graphically in Fig.1.

Referring to Fig.1, the radial distance, r_{co} , between the center of gravity of the connecting rod and the center of rotation of the crankshaft may given by :-

$$r_{co} = \sqrt{(a\sin\alpha)^2 + (x - a\cos\alpha)^2} \tag{1}$$

Where

a: distance from center of gravity of connecting rod to the small end.
x: distance between crankshaft center and reciprocating mass.
α : angular position of the connecting rod with respect to the linear displacement of the mass M_r.

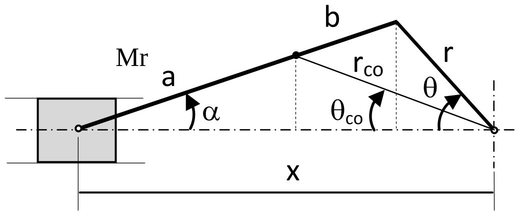

Fig. 1. Geometrical arrangement of a crankshaft-connecting rod-reciprocating mass

$$\sin\alpha = n\sin\theta$$
$$\cos\alpha = \sqrt{1 - n^2\sin^2\theta}$$
$$x = (a+b)\cos\alpha + r_c\cos\theta \tag{2}$$

Where n is the ratio of crankshaft radius, r_c, to the connecting rod length, l, and b is the distance from center of gravity of connecting rod to the big end .

Substituting of (2) in (1), then the radial distance r_{co} can be expressed in terms of the crankshaft angular displacement θ by:-

$$r_{co} = \sqrt{(an\sin\theta)^2 + (b\sqrt{1-(n\sin\theta)^2} + r_c\cos\theta)^2} \tag{3}$$

Also, it can be seen that from Fig.1, the angular displacement, θ_{co}, of the radial distance, r_{co}, measured from the horizontal line (the same datum of the crankshaft) is given by:-

$$\sin\theta_{co} = \frac{an\sin\theta}{r_{co}} \tag{4}$$

The derived expressions of both the radial distance r_{co} and the angular displacement θ_{co} can be used in the subsequent analysis to determine the kinetic energy of the connecting rod. But, because both expressions are periodic functions in terms of the crankshaft angle θ, their derivatives with the time producing complicated terms, and cannot be easily manipulated to obtain the final form of the system torsional vibration equations.

Fortunately, any periodic function can be represented by Fourier series expansion as an infinite sum of sins and cosines terms [10]. Consequently, the radial distance r_{co} and the angular displacement θ_{co} that are given in (3) and (4) can be reproduced by using Fourier analysis. Thus, a mathematical closed form solution of the system nonlinear torsional vibration displacement can be derived by using energy method.

B. *Fourier Series Analysis*

In order to apply a Fourier series presentation in the present work, the following reciprocating machine parameters are adopted, as shown in Table I below :-

TABLE I. RECIPROCATING MACHINE PARAMETERS

Parts	Mass (kg)	Mass moment of inertia (kg.m^2)	Dimensions (mm)
Crankshaft	-	I_c =0.203*10^{-3}	r_c = 37
Connecting rod	0.417	I_{cog} = 0.663*10^{-3}	a=92.18, b=28.6
Reciprocating mass	0.283	-	-

The Fourier series representation of any periodic function is given by:-

$$y(\theta) = \frac{a_o}{2} + \sum_{n=1}^{\infty}(a_n \cos n\theta + b_n \sin n\theta) \quad (5)$$

In many cases the integrals of the coefficients a_o, a_n, b_n, cannot be evaluated easily when the function y (θ) does not have simple form. Accordingly, if graphical representations of r_{co} and θ_{co} are available, then, the coefficients can be evaluated numerically by using a numerical integration procedure like the trapezoidal or Simpson's rule [10]. Thus, the coefficients are given by:-

$$a_o = \frac{2}{N}\sum_{i=1}^{N} y_i \quad (6)$$

$$a_n = \frac{2}{N}\sum_{i=1}^{N} y_i \cos\frac{2n\pi\theta_i}{\Theta} \quad (7)$$

$$b_n = \frac{2}{N}\sum_{i=1}^{N} y_i \sin\frac{2n\pi\theta_i}{\Theta} \quad (8)$$

Where N is an even number of equidistance of points over the period Θ.

The numerical evaluation of the coefficients a_o, a_n, and b_n results in obtaining the reproduced forms of both r_{co} and θ_{co}, as shown below:-

$$r_{co} = 0.037 + 0.0286\cos\theta = 0.0286(1.29 + \cos\theta) \quad (9)$$

$$\theta_{co} = 1.2\sin\theta - 0.6\sin 2\theta \quad (10)$$

A comparison between the actual function given by (3) and (4), and the approximate one given by (9) and (10) of both r_{co} and θ_{co}, are shown in Figs 2, and 3 respectively. The results indicate that the Fourier series expansion provides an acceptable mathematical expression for the radial distance r_{co} and the angular displacement, θ_{co}.

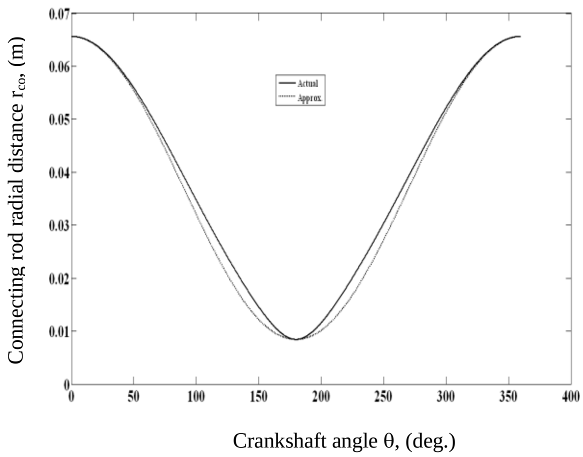

Fig. 2. Connecting rod radial distance, r_{co}, versus crankshaft angle θ.

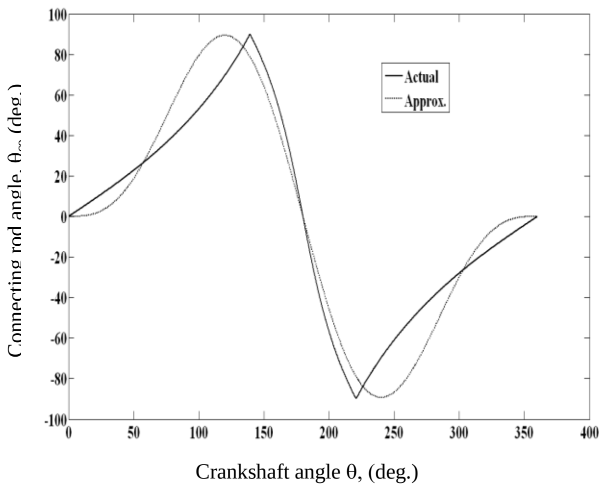

Fig. 3. Connecting rod angle, θ_{co}, versus crankshaft angle θ.

These expressions can be used in the subsequent analysis to study the nonlinear torsional vibration of reciprocating machines with taken into consideration the variation of the mass moment of inertia of the reciprocating parts (connecting rod and reciprocating mass).

III. NONLINEAR TORSIONAL VIBRATION ANALYSIS

A. *Energy method*

The system adopted in the present steady is schematically shown in Fig. 4. This system includes a flywheel (F) driven by reciprocating mass (Mr)-connecting rod (co)-crankshaft (c) engine with assuming that the gas pressure is neglected and the motion of the reciprocating mass is a simple harmonic motion.

The total kinetic energy T of the system is given by:-

$$T = \frac{1}{2}I_f \dot{\theta}_f^2 + \frac{1}{2}I_c \dot{\theta}^2 + \frac{1}{2}I_{co}\dot{\theta}_{co}^2 + \frac{1}{2}M_r r_c^2 \sin^2\theta\,\dot{\theta}^2 \quad (11)$$

Where the suffixes f, c, co, and r refer to the flywheel, crankshaft, connecting rod, and reciprocating mass, and I and θ represent their corresponding values of the mass moment of inertia and angular displacement.

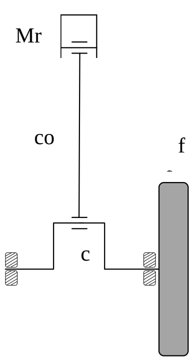

Fig. 4. Arrangement of reciprocating engine model.

Using the definition of the radial distance r_{co}, the mass moment of inertia of the connecting rod is given by:-

$$I_{co} = I_{cog} + M_{co} r_{co}^2 \tag{12}$$

Where Icog and Mco are represent the mass moment of inertia and mass of the connecting rod respectively. Substituting of (9), (10) after differentiation for $\dot{\theta}_{co}$, and (12) in (11), then, the kinetic energy of the system is given by:-

$$T = \frac{1}{2} I_f \dot{\theta}_f^2 + \frac{1}{2} I_c \dot{\theta}^2 + \frac{1}{2}(I_{cog} + M_{co}(0.0286(1.29+\cos\theta))^2) + (1.2(\cos\theta - \cos 2\theta))^2 \dot{\theta}^2 + \frac{1}{2} M_r r_c^2 \sin^2\theta\dot{\theta}^2 \tag{13}$$

The potential energy U of the system is given by:-

$$U = \frac{1}{2} k_t (\theta - \theta_f)^2 \tag{14}$$

Where k_t represents the torsional stiffness constant of the crankshaft.

The corresponding Lagrange's equation in terms of the coordinate θ is presented by:-

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial U}{\partial \theta} = 0 \tag{15}$$

Substituting of (13) and (14), in (15), results in:-

$$I_c\ddot{\theta} + \frac{1}{2} M_r r_c^2 (1 - \cos 2\theta)\ddot{\theta} + \frac{1}{2} M_r r_c^2 \sin 2\theta\dot{\theta}^2 + \begin{pmatrix} 1.44\left(I_{cog} + 0.00081796 M_{co}(1.29+\cos\theta)^2\right)\times \\ (\cos\theta - \cos 2\theta)^2 \end{pmatrix}\ddot{\theta} + \left[\begin{matrix} 1.44\times 1.6641\left(I_{cog} + 0.00081796 M_{co}\right)\begin{pmatrix} -2\cos\theta\sin\theta + \\ 2\sin\theta\cos 2\theta + \\ 4\cos\theta\sin 2\theta - \\ 4\cos 2\theta\sin 2\theta \end{pmatrix} + \\ 0.0011778 M_{co}\left(\begin{matrix} 2.58\begin{pmatrix} -3\sin\theta\cos^2\theta + \\ 4\cos 2\theta\cos\theta\sin\theta - \\ 4\sin 2\theta\cos^2\theta - \\ 4\cos\theta\sin\cos 2\theta - \\ \sin\theta\cos^2 2\theta \end{pmatrix} + \\ \begin{pmatrix} 6\cos^2\theta\cos 2\theta\sin\theta - 4\cos^3\theta\sin\theta + \\ 8\cos^3\theta\sin 2\theta - 2\cos\theta\sin\cos^2 2\theta - \\ 4\cos^2\theta\cos 2\theta \end{pmatrix} \end{matrix}\right) \end{matrix}\right]\dot{\theta}^2 + k_t(\theta - \theta_f) = 0 \tag{16}$$

The following relations are used in the subsequent analysis, and defined such that:-

$$\begin{aligned} &\beta = \theta - \theta_f \\ &\theta = \omega t + \beta \\ &\tau = \omega t \\ &\frac{d()}{dt} = \omega\frac{d()}{d\tau} = \omega()' \\ &\omega_n = \sqrt{\frac{k_t}{I_{eq}}} \\ &r_f = \frac{\omega}{\omega_n} \\ &I_{eq} = I_c + \tfrac{1}{2} M_r r_c{}^2 + 1.6641 I_{cog} + 2.6241 M_{co} \\ &\varepsilon_1 = \frac{\tfrac{1}{2} M_r r_c{}^2}{I_{eq}} \\ &\varepsilon_2 = \frac{2.3963 I_{cog}}{I_{eq}} \\ &\varepsilon_3 = \frac{0.0011778 M_{co}}{I_{eq}} \end{aligned} \tag{17}$$

Where:-

I_{eq}: equivalent mass moment of inertia of the system.

r_f: frequency ratio.

β: torsional vibration displacement.

ω: frequency of the crankshaft.

ω_n: natural frequency of the system

ε_1: inertia ratio of the crankshaft.

ε_2 and ε_3: inertia ratio of the connecting rod corresponding to mass moment of inertia and mass respectively.

Substituting of (17), and neglecting higher order and product derivative terms, then (16), becomes:-

$$
\begin{aligned}
&\left[1-\varepsilon_1\cos 2\tau+\varepsilon_2\begin{pmatrix}0.5\cos 4\tau+0.5\cos 2\tau-\\ 2\cos\tau\cos 2\tau\end{pmatrix}+\varepsilon_3\begin{pmatrix}2.08\cos\tau-0.74795\cos 2\tau+1.0425\cos 2\tau-\\ 3.0382\cos\tau\cos 2\tau+2.08\cos\tau\cos 4\tau+\\ 0.5\cos 8\tau\end{pmatrix}\right]\beta''\\
&+2\left[\varepsilon_1\sin 2\tau+\varepsilon_2\begin{pmatrix}-\sin 2\tau-\sin 4\tau+\\ 2\sin 4\tau\cos 2\tau+4\cos\tau\sin 2\tau\end{pmatrix}+\varepsilon_3\begin{pmatrix}1.29\sin\tau-7.8241\sin 2\tau-3.7041\sin 4\tau+\\ 3.3282\sin\tau\cos 2\tau+6.7864\cos\tau\sin 2\tau+\\ 1.29\sin\tau\cos 4\tau+0.75\cos\tau\cos 4\tau+\\ \sin 4\tau\cos\tau-0.75\cos 2\tau\sin 4\tau\end{pmatrix}\right]\beta'\\
&+\left[2\varepsilon_1\cos 2\tau+\varepsilon_2\begin{pmatrix}2\cos 2\tau-\cos 4\tau-\\ 8\sin\tau\sin 2\tau+10\cos\tau\cos 2\tau\end{pmatrix}+\varepsilon_3\begin{pmatrix}1.29\cos\tau-8.9918\cos 2\tau-\\ 14.8164\cos 4\tau+16.901\cos\tau\cos 2\tau-\\ 13.4428\sin\tau\sin 2\tau-6.16\sin\tau\sin 4\tau-\\ 0.71\cos\tau\cos 4\tau-0.75\sin\tau\cos 4\tau-\\ 3\sin\tau\sin 4\tau+4\cos\tau\cos 4\tau+\\ 1.5\sin 2\tau\sin 4\tau\end{pmatrix}+\frac{1}{r_f^2}\right]\beta\\
&=-\varepsilon_1\sin 2\tau-\varepsilon_2\begin{pmatrix}-\sin 2\tau-\sin 4\tau+\\ 2\sin 4\tau\cos 2\tau+4\cos\tau\sin 2\tau\end{pmatrix}+\varepsilon_3\begin{pmatrix}1.29\sin\tau-7.8241\sin 2\tau-3.7041\sin 4\tau+\\ 3.3282\sin\tau\cos 2\tau+6.7864\cos\tau\sin 2\tau+\\ 1.29\sin\tau\cos 4\tau+0.75\cos\tau\cos 4\tau+\\ \sin 4\tau\cos\tau-0.75\cos 2\tau\sin 4\tau\end{pmatrix}
\end{aligned}
\tag{18}
$$

The above equation represents the equation of nonlinear torsional vibration of reciprocating machines including the terms of variable inertia of the reciprocating parts.

IV. SIMULATION RESULTS

The torsional vibration displacement including the variable inertia of reciprocating machines was developed by using MATLAB/Simulink. With appropriate initial conditions the time domain torsional vibration displacement is shown in Fig.5, with frequency ratio of r_f =0.0833. The response emphasized on the fact of the nonlinear nature of the torsional vibration when the variation of the inertia of reciprocating parts is considered which in turn makes the system response unsteady. Also, with increasing the frequency ratio to r_f = 0.111, as shown in Fig. 6, the response is being more unstable.

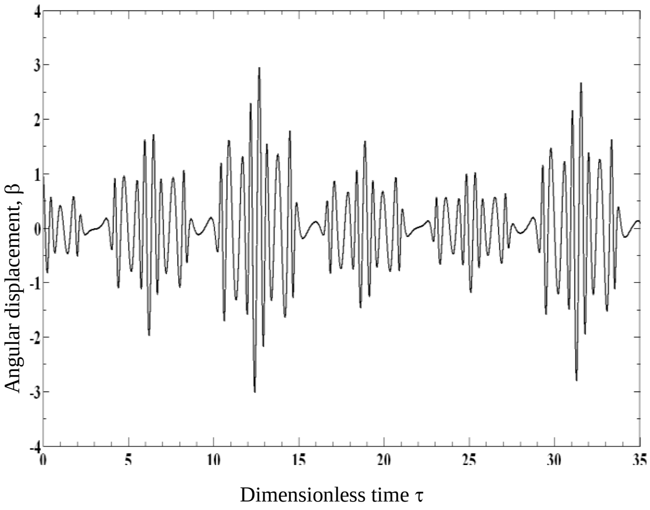

Fig. 5. Torsional vibration dispalcement versus dimensioless time at frequency ratio, r_f = 0.111.

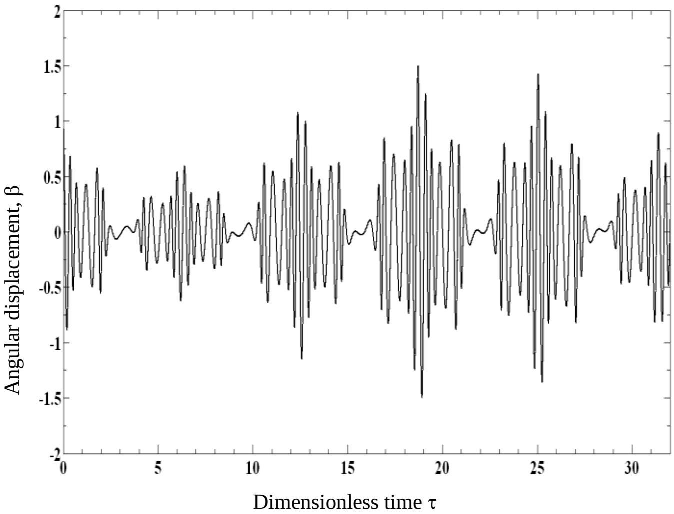

Fig. 6. Torsional vibration dispalcement versus dimensioless time at frequency ratio, r_f = 0.0833.

This reflects the strong effect of variation in the inertia of reciprocating parts, which in turn, increasing the sensitivity of the system for the increase in speed engine. As well as, the variation in the inertia of the system result in changing the critical range of engine speed. This may be attributed to the rising of what is called secondary harmonic resonance phenomenon as a direct effect of the cyclic variation of the inertia of reciprocating parts [4, 8, 9].

The variation of the engine reciprocating parts mass moment of inertia and system natural frequency over crankshaft angle is shown in Figs.7 and 8 respectively. The result of these two figures indicates the frequency coupling between the system natural frequency and the engine speed [9]. Also, it can be seen that, the cyclic variation in effective inertia of the engine resulting in cyclic variation of frequencies and corresponding amplitudes [4].

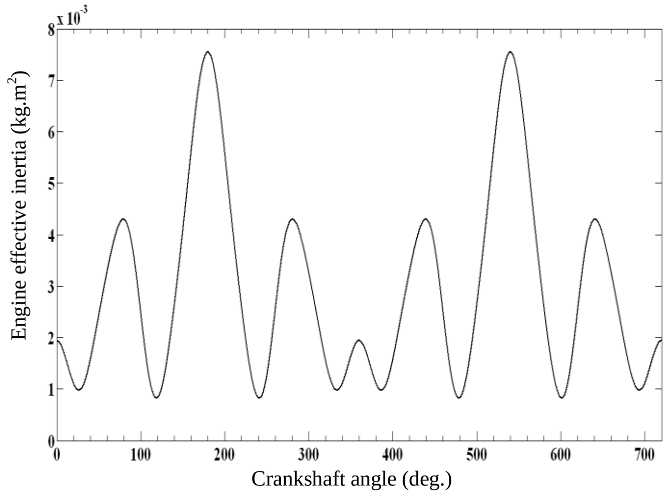

Fig. 7. Engine effective mass moment of inertia versus crankshaft angle θ, at frequency ratio, $r_f = 0.0833$.

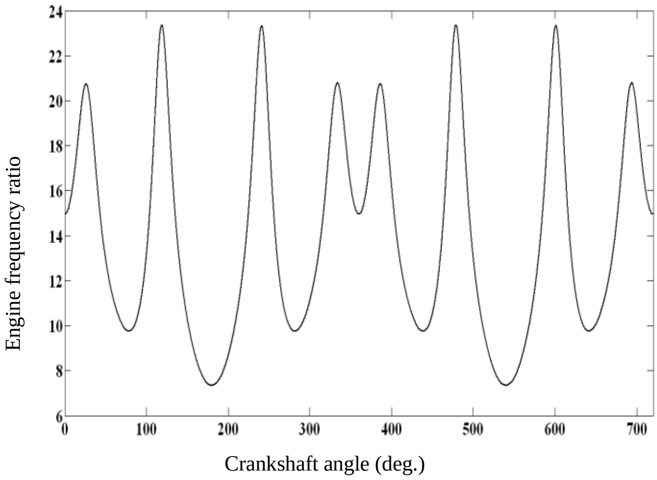

Fig. 8. Engine frequency ratio versus crankshaft angle θ

The variation of the harmonic excitation torque over a crankshaft angle is shown in Fig. 9. It can be seen that, the considering of the variation of the inertia of the reciprocating parts in the analysis results in introducing of a nonlinear harmonic excitation torque [4, 9].

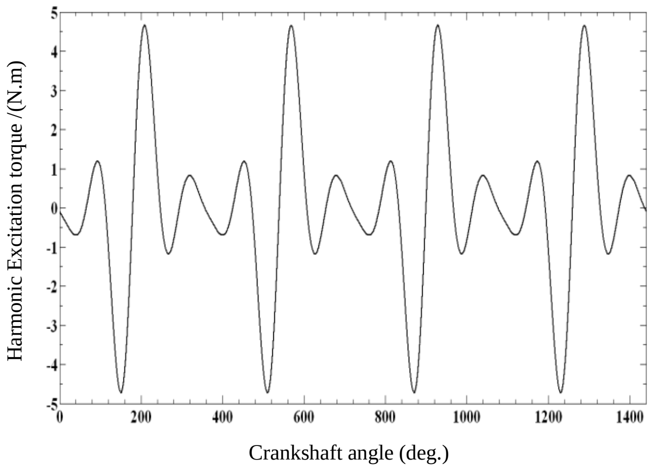

Fig. 9. Harmoinc excitation torque/(N.m) versus crankshaft angle θ at frequency ratio, $r_f = 0.0833$.

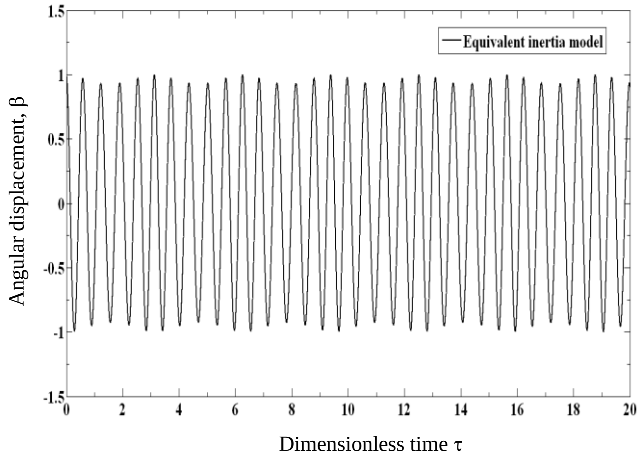

Fig. 10. Torsional vibration dispalcement versus dimensionless time, τ, of equivalent ratio model at frequency ratio, $r_f = 0.1$.

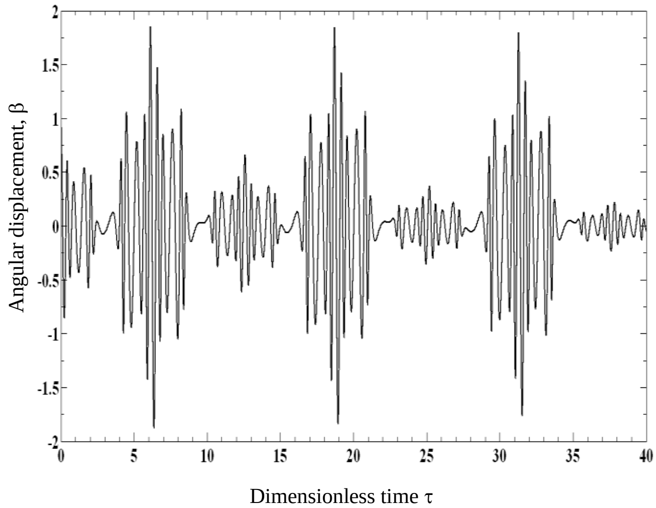

Fig. 11. Torsional vibration dispalcement versus dimensionless time τ, at frequency ratio, $r_f = 0.1$ (present model)

Fig.10 and Fig.11 show a comparison between the torsional vibration displacements with considering variable inertia of reciprocating parts in the present model with the equivalent inertia model followed by researchers such as [1]. It can be seen that the torsional vibration displacement of the variable inertia model is larger than that deduced by equivalent inertia model for the same engine speed. This is attributed to the complicated nature of the harmonic excitation torque introduced by the variable inertia of reciprocating parts [4, 8, 9].

Fig.12 shows the variation of the torsional displacement with dimensionless time for different frequency ratios (engine speed). It is shown that the torsional vibration displacement is greatly affected by the engine speed and may be lead to a serious damage of the system rotating parts when in it is being near the harmonic resonance condition [5]. Also, with increasing frequency ratio, the torsional vibration trend of the system is being more closely to chaotic motion and irregular periodic oscillations (non harmonic).

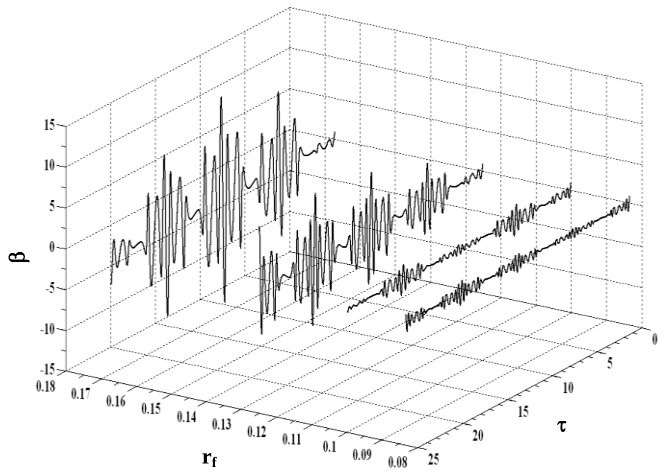

Fig. 12. Torsional vibration dispalcement versus dimensionless time,τ, at different frequency ratio, r_f

The effect of inertia of reciprocating parts on torsional vibration displacement of reciprocating machines can be examined over a range of percentage inertia ratio of connecting rod (ε_3), as shown in Fig. 13. It can be seen that with percentage increase of 70% in ε_3, the increase in β_{max} is with 3 orders of magnitude. This indicates that, with increasing the inertia ratio of connecting rod, the system torsional vibration is no longer within expectable values. Also, the attention should be given for the connecting rod part of reciprocating masses than other parts, because its inertia variation represents the main dominant part of the effective inertia of the system. Consequently, for heavy engines, such as marine engine, when the connecting rod inertia ratio of appreciable values, the torsional vibration analysis should be carried out with considering the variation in the inertia of reciprocating parts [4].

The results obtained in the present work give an indication that the proposed analytical solution based on Fourier analysis provided acceptable results and can be followed to examine the nonlinear torsional vibration of variable inertia reciprocating machines.

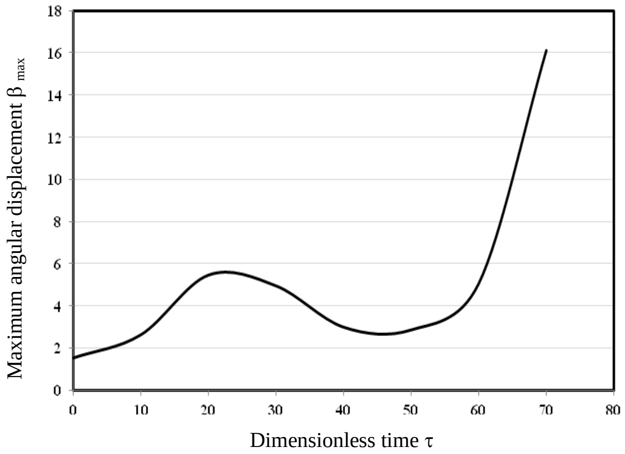

Fig. 13. Maximum Torsional vibration amlpitude versus percentage increase in ε_3 at frequency ratio r_f =0.0833.

V. Conclusions

The variation in the inertia of reciprocating parts is greatly effecting the torsional vibration displacement of the system, and with increasing the engine speed the torsional vibration trend of the system is being more closely to chaotic motion and irregular periodic oscillations (non harmonic) is excepted.

The introducing of the variable inertia of reciprocating parts in analysis produces the harmonic excitation torque, and gives a rising for the secondary harmonic excitation phenomenon. This in turn induced cyclic variation of the system frequencies and corresponding amplitudes over a cycle of crankshaft rotation.

A comparison with equivalent inertia model show large torsional vibration amplitude deduced by the present model. This is attributed to the complicated nature of the harmonic excitation torque introduced by the variable inertia of reciprocating parts.

With increasing the inertia ratio of connecting rod, the system torsional vibration is no longer within expectable values, and an attention should be given for the connecting rod part of reciprocating masses than other parts. Because its inertia variation part represents the main dominant part of the effective inertia of the system. Consequently, for heavy engines, such as marine engine, when the connecting rod inertia ratio of appreciable values, the torsional vibration analysis should be carried out with considering the variation in the inertia of reciprocating parts.

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