Plane Waves In Theory Of Micropolar Thermoelasticity With Diffusion

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***Abstract*—In this paper, the governing equations of micropolar thermoelasticity with diffusion are formulated in context of Lord-Shulman theory of generalized thermoelasticity. The plane wave solutions of these equations indicate the existence of six plane waves. The speed of these plane waves are computed for a particular material and plotted against the diffusion parameter, thermal parameter and frequency.**

Keywords—Micropolar,thermoelasticity, diffusion,plane waves.

I. INTRODUCTION

Linear elasticity describes the mechanical behavior of common solid materials, e.g., concrete, wood and coal. However, this theory does not apply to the behavior of many new synthetic materials of the elastomer and polymer type, e.g., polymethyl-methacrylate, polyethylene and polyvinyl chloride. Structures in modern engineering are made with the use of polycrystalline and fibrous materials. The micropolar theory of elasticity takes into account the granular character of the medium, where solids can undergo macrodeformations and microrotations. The motion in micropolar solids is completely characterized by the displacement vector and the microrotation vector, whereas in classical elasticity, the motion is characterized by the displacement vector only. A micropolar continuum is a collection of interconnected particles in the form of small rigid bodies which undergoes both translational and rotational motions. Granular media and multimolecular bodies are typical examples of this medium. Rigid chopped fibres, elastic solids with rigid granular inclusions, and other industrial materials such as liquid crystals are other important examples of this medium. Eringen and his co-workers [1-3] developed the theory of micropolar elasticity. This theory aroused much interest due to its possible applications in investigation of deformation properties of solids, where the classical theory becomes inadequate. For example, the micropolar theory has possible applications in investigating materials with bar-like molecules which exhibit microrotation effects and support body and surface couples. Parfitt and Eringen [4] studied plane waves in linear theory of micropolar elasticity. Eringen [5] extended the theory of micropolar elasticity for heat conducting elastic solids by including thermal effects. Lord-Shulman generalization of linear micropolar thermoelasticity was developed by Boschi and Iesan [6]. Dost and Tabarrok [7] developed a Green-Lindsay generalization of micropolar thermoelasticity. Ciarletta [8] developed a theory of micropolar thermoelasticity without energy dissipation. Chandrasekharaiah [9] formulated a theory of heat flux micropolar thermoelasticity. Diffusion is a spontaneous movement of the particles from region of high concentration to low concentration. The fields of temperatures and diffusion in solids can not be neglected in research on development of high technologies. At elevated and low temperatures, the processes of heat and mass transfer play the decisive role in many problems of satellites, returning space vehicles, and landing on water or land. Even oil companies are interested in the process of thermodiffusion for more efficient extraction of oil from oil deposits. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment.

Sherief et. al [10] developed the theory of generalized thermoelastic diffusion with one relaxation time, which allows the finite speed of propagation of waves. Singh [11, 12] studied the reflection of P and SV waves from free surface of an elastic solid with generalized thermodiffusion. Aouadi [13] developed the general equations of motion and constitutive equations based on the theory of Lord-Shulman with one relaxation time for a general homogeneous anisotropic medium with a microstructure, taking into account the effects of heat and diffusion. In the present paper we solve the governing equations for a linear, homogeneous and isotropic micropolar thermoelastic solid with diffusion for plane waves. The speeds of these plane waves are plotted against various diffusion and thermal parameters. Some particular cases are also discussed.

II. GOVERNING EQUATIONS

Following Aouadi [13] the basic equations for homogeneous isotropic generalized linear micropolar thermoelastic diffusion in the absence of body forces, body couples, heat and mass diffusive sources are:

Equations of motion

$$\sigma_{ji,j} = \rho \ddot{u}_i \quad (1)$$

$$\varepsilon_{ijk}\sigma_{jk} + \mu_{ji,j} = \rho J_{ij}\ddot{\phi}_j \quad (2)$$

Generalized Fourier's law of heat conduction

$$q_i + \tau_0\, \dot{q}_i = -K\theta_{,i} \quad (3)$$

Equation of mass flux vector

$$\eta_i + \tau \dot{\eta}_i = -D P_{,i} \qquad (4)$$

Entropy equation

$$\rho T_0 \dot{S} = -q_{i,i} \qquad (5)$$

Conservation of mass equation

$$\dot{C} = -\eta_{i,i} \qquad (6)$$

Constitutive equation

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + (2\mu + \kappa)e_{ij} + \kappa\varepsilon_{ijk}(r_k - \phi_k) - \beta_1\theta\delta_{ij} - \beta_2 C\delta_{ij} \qquad (7)$$

$$\mu_{ji} = \alpha\,\phi_{kk}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} \qquad (8)$$

$$\rho S = \beta_1 e_{kk} + \frac{\rho C_E}{T_0}\theta + aC \qquad (9)$$

$$P = bC - \beta_2 e_{kk} - a\theta \qquad (10)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (11)$$

where u_i is the displacement vector field, σ_{ij} is the force stress tensor, e_{ij} is the strain tensor, ϕ_i is the vector of internal rotations, μ_{ji} is the moment of couple stress tensor, ϕ_{ji} is the microcurvature tensor, ε_{ijk} is the alternating tensor, J_{ij} is the microrotation tensor, r_i is the rotation vector, ε_{ij} is the micro-strain tensor, η_i denotes the flow of the diffusing mass vector, q_i is the vector of heat flux, S is the entropy, T is the absolute temperature, P is the chemical potential per unit mass, and C is the concentration of the diffusive material in the elastic body, K is the coefficients of the thermal conductivity, D is the coefficients of diffusion, τ is diffusion relaxation time, τ_0 is thermal relaxation time , $\theta = T - T_0$ is the temperature of the medium in its natural state assumed to be such that $\left|\frac{\theta}{T_0}\right| << 1$, T is the absolute temperature of the medium, T_0 is the reference temperature of the body, λ and μ are Lame's constants ρ, C_E are, respectively, the density and specific heat at constant strain, a, b, $\alpha, \beta, \gamma, \kappa$ are constitutive coefficients, δ_{ij} is the Kronecker delta, $\beta_1 = (3\lambda + 2\mu + \kappa)\alpha_t$, $\beta_2 = (3\lambda + 2\mu + \kappa)\alpha_c$, Here α_t, α_c are the coefficients of linear thermal expansion and diffusion expansion respectively.

With the help of relations (7) and (8) in equations (1) and (2), we obtain

$$(\mu + \kappa)\nabla^2\vec{u} + (\lambda + \mu)\nabla(\nabla.\vec{u}) + \kappa\nabla\times\vec{\phi} - \beta_1\nabla\theta - \beta_2\nabla C = \rho\ddot{\vec{u}} \qquad (12)$$

$$\gamma\nabla^2\vec{\phi} - 2\kappa\vec{\phi} + (\alpha + \beta)\nabla(\nabla.\vec{\phi}) + \kappa\nabla\times\vec{u} = \rho j\ddot{\vec{\phi}} \qquad (13)$$

From equations (3), (5) and (9), we have

$$(1 + \tau_0\frac{\partial}{\partial t})[\ \beta_1 T_0\nabla.\dot{\vec{u}} + \rho C_E\dot{\theta} + aT_0\dot{C}\ ] = K\nabla^2\theta \qquad (14)$$

From equations (4) , (6) and (10), we have

$$D\beta_2\nabla^2\left(\nabla.\vec{u}\right) + Da\nabla^2\theta - Db\nabla^2 C + \dot{C} + \tau\ddot{C} = 0 \qquad (15)$$

By Helmholtz representation of vectors, the displacement and microrotation vectors are written in terms of scalar potentails q, ξ and vector potentails $\vec{U}$, $\vec{\Pi}$ as

$$\vec{u} = \nabla q + \nabla\times\vec{U}, \quad \nabla.\vec{U} = 0, \qquad (16)$$

$$\phi = \nabla\xi + \nabla\times\vec{\Pi}, \quad \nabla.\vec{\Pi} = 0, \qquad (17)$$

With the use of equations (16) and (17) into equations (12) to (15), we have

$$(c_1^2 + c_3^2)\nabla^2 q - \bar{\beta}_1\theta - \bar{\beta}_2 C - \ddot{q} = 0 \qquad (18)$$

$$(1 + \tau_0\frac{\partial}{\partial t})[\ \beta_1 T_0\nabla^2\dot{q} + \rho C_E\dot{\theta} + aT_0\dot{C}\ ] = K\nabla^2\theta \qquad (19)$$

$$D\beta_2\nabla^4 q + Da\nabla^2\theta - Db\nabla^2 C + \dot{C} + \tau\ddot{C} = 0, \qquad (20)$$

$$(c_2^2 + c_3^2)\nabla^2\vec{U} + c_3^2(\nabla\times\vec{\Pi}) - \ddot{U} = 0 \qquad (21)$$

$$c_4^2\ \nabla^2\vec{\Pi} - 2\omega_0^2\vec{\Pi} + \omega_0^2(\nabla\times\vec{U}) - \ddot{\Pi} = 0 \qquad (22)$$

$$(c_4^2 + c_5^2)\nabla^2\xi - 2\omega_0^2\ \xi - \ddot{\xi} = 0 \qquad (23)$$

where

$$c_1^2 = \frac{\lambda + 2\mu}{\rho},\ c_2^2 = \frac{\mu}{\rho},\ c_3^2 = \frac{\kappa}{\rho},\ c_4^2 = \frac{\gamma}{\rho j},$$

$$c_5^2 = \frac{\alpha + \beta}{\rho j},\ \omega_0^2 = \frac{\kappa}{\rho j},\ \bar{\beta}_1 = \frac{\beta_1}{\rho},\ \bar{\beta}_2 = \frac{\beta_2}{\rho}.$$

The equations (18) to (20) are coupled in scalar potentials q, θ, C and equations (21) and (22) are coupled in vector potentials $\vec{U}$, $\vec{\Pi}$. Equation (23) is uncoupled in the scalar potential ξ .

III. FORMULATION OF THE PROBLEM AND SOLUTION

We consider plane waves propagating in homogeneous isotropic micropolar thermoelastic half space with diffusion in the positive direction of a unit vector $\vec{n}$ as follows:

$$(q, \theta, C) = (\bar{q}, \bar{\theta}, \bar{C})\, e^{ik(\vec{n}.\vec{r} - Vt)} \qquad (24)$$

where $\bar{q}$, $\bar{\theta}$, $\bar{C}$ are the constants, V is the phase velocity, k is the wave number and $\vec{r}$ is the position vector.
Making use of equation (24) into equations (18) to 20), we have

$$\omega^2 [V^2 - (c_1^2 + c_3^2)]\, \bar{q} - \bar{\beta}_1 V^2 \bar{\theta} - \bar{\beta}_2 V^2 \bar{C} = 0 \quad (25)$$

$$\beta_1 T_0\, \omega^2\, \bar{q} + (\bar{K} - \rho C_E V^2)\bar{\theta} - a T_0 V^2 \bar{C} = 0 \quad (26)$$

$$D\beta_2 \omega^2 \bar{q} - Da V^2 \bar{\theta} + (DbV^2 - \tau^* V^4)\bar{C} = 0 \quad (27)$$

where $\tau^* = \tau + \frac{i}{\omega}$, and $\omega = kV$ is the circular frequency of the wave.
The non-trivial solution of equation (25) to (27) requires

$$V^6 - A_0 V^4 + A_1 V^2 + A_2 = 0 \quad (28)$$

where

$$A_0 = c_1^2 + c_3^2 + \bar{\bar{K}} + \varepsilon + \bar{D} + \bar{\bar{D}},$$

$$A_1 = (c_1^2 + c_3^2)(\bar{\bar{K}} + \bar{D} + \bar{\bar{D}}) + \bar{D}(\bar{\bar{K}} + \varepsilon + \frac{2\bar{\beta}\varepsilon a}{b} - \frac{\beta_2^2}{\rho b}),$$

$$A_2 = \bar{\bar{K}}\,\bar{D}[\frac{\beta_2^2}{\rho b} - (c_1^2 + c_3^2)],$$

$$\bar{\bar{K}} = \frac{\bar{K}}{\rho C_E},\ \varepsilon = \frac{\beta_1^2 T_0}{\rho^2 C_E},\ \bar{\bar{D}} = \frac{Da^2 T_0}{\rho C_E \tau^*},\ \bar{D} = \frac{Db}{\tau^*},$$

$$\bar{\beta} = \frac{\beta_2}{\beta_1}.$$

Here, the three complex roots $V_j^2, (j = 1,2,3)$ of equation (28) correspond to Coupled Longitudinal Displacement (CLD) wave , Coupled Thermal (CT) wave and Coupled Mass Diffusion (CMD) wave, respectively. If we write $V_j^{-1} = V_j^{*-1} - i\,\omega^{-1} q_j$, (j = 1, 2, 3), then clearly V^*_j and q_j are the speeds of propagation and the attenuation coefficients of the CLD, CT, and CMD waves.

Parfitt and Eringen [4] have shown that equations (21) and (22) represent "Coupled Transverse waves" propagating with velocity $V_{4,5}$ given by

$$V_{4,5}^2 = \frac{1}{2(1-x)}\left\{\begin{array}{l} c_2^2 + c_3^2 + c_4^2 - (c_2^2 + c_3^2/2)x \\ \pm[\{c_4^2 - c_2^2 - c_3^2 + (c_2^2 + c_3^2/2)x\}^2 \\ + 2c_3^2 c_4^2 x]^{1/2} \end{array}\right\} \quad (29)$$

where, $x = \frac{2\omega_0^2}{\omega^2}$, the wave with speed V_4 corresponds to Coupled Transverse Displacement (CTD) wave and the wave with speed V_5 corresponds to Coupled Transverse Microrotational (CTM) wave. These waves are not affected by diffusion and thermal parameters. The solution of equation (23) represents Longitudinal Microrotational wave propagating with velocity V_6 given by

$$V_6^2 = (c_4^2 + c_5^2) + 2\omega_0^2 / k^2 \quad (30)$$

The set of coupled transverse waves with speed V_4 and longitudinal microrotational wave with speed V_6 exist only when for $\omega > \sqrt{2}\,\omega_0$ below which they degenerate into distance decaying vibrations.

IV. PARTICULAR CASES

(i) Micropolar thermoelastic medium
In absence of diffusion parameters, i.e., if we consider $\beta_2 = b = a = D = 0$ in equation (28), we obtain the following velocity equation

$$V^4 - [c_1^2 + c_3^2 + \bar{\bar{K}} + \varepsilon]V^2 + \bar{\bar{K}}(c_1^2 + c_3^2) = 0, \quad (31)$$

which corresponds to speeds of CLD and CT waves in micropolar thermoelastic medium.
(ii) Thermoelastic medium with diffusion
In absence of microrotation parameters, i.e., if we consider $\kappa = 0; j = 0,\ c_3^2 = 0,\ c_4^2 = 0,\ c_5^2 = 0,\ \omega_0^2 = 0,$ in equation (28), we obtain the following velocity equation

$$V^6 - L_0 V^4 + L_1 V^2 + L_2 = 0, \quad (32)$$

where,

$$L_0 = c_1^2 + \bar{\bar{K}} + \varepsilon + \bar{D} + \bar{\bar{D}},$$

$$L_1 = c_1^2(\bar{\bar{K}} + \bar{D} + \bar{\bar{D}}) + \bar{D}(\bar{\bar{K}} + \varepsilon + \frac{2\bar{\beta}\varepsilon a}{b} - \frac{\beta_2^2}{\rho b}),$$

$$L_2 = \bar{\bar{K}}\,\bar{D}[\frac{\beta_2^2}{\rho b} - c_1^2].$$

The three roots of equation (32) corresponds to speeds of CLD, CT and CMD waves in a thermoelastic medium with diffusion.

IV. NUMERICAL RESULTS AND DISCUSSION

The waves with speed V_4, V_5 and V_6 are not affected by diffusion and thermal parameters. Equation (28) represents three plane waves with speed V_1, V_2 and V_3 which are affected by diffusion and thermal parameters. To observe the effects of diffusion and thermal parameters on these waves we solved the equation (28) numerically and obtain the real value of the speed of propagation of CLD, CT, and CMD waves for the following material constants of Aluminium-epoxy composite at T_0 = 27°C [14],

λ = 7.59 × 10^{11} dyne.cm^{-2},
μ = 1.89 × 10^{11} dyne.cm^{-2},
ρ = 2.7× 10^3 gm.cm^{-3},
C_E = 2.361 cal.gm^{-1}.°C^{-1},

κ = 0.0149 × 10^{11} dyne.cm^{-2},
K =0.492 cal. .cm^{-1}.s^{-1}.°C^{-1},
D = 0.5 **gs**.cm^{-3},
b = 0.05 cm^{5}.gm^{-1}.s^{-1},
a = 0.005 cm^{2}.s^{-2}. °C^{-1},
α_t = 0.005,
α_c = 0.05,
τ = 0.05s,
ω = 5 Hz,
τ_0 = 0.04s.

The speed of CLD, CT, and CMD waves are plotted against the diffusion parameter D, a, b, thermal conductivity K and frequency ω in Figure 1 to 5. The speed of CLD wave is 1.6118 × 10^4 cm.s^{-1} at D = 0. It increases slowly to1.63598 × 10^4 cm.s^{-1} at D = 1. The speed of CT wave is 0.00077 × 10^4 cm.s^{-1}at D = 0. It increases sharply to 0.43473 × 10^4 cm.s^{-1} at D = 1. The speed of CMD wave is 0.28871 × 10^4 cm.s^{-1} at D = 0. It increases to 0.36662 × 10^4 cm.s^{-1} at D = 1. These variations of speeds of CLD, CT and CMD waves are shown by black, blue and red lines respectively in Figure 1.

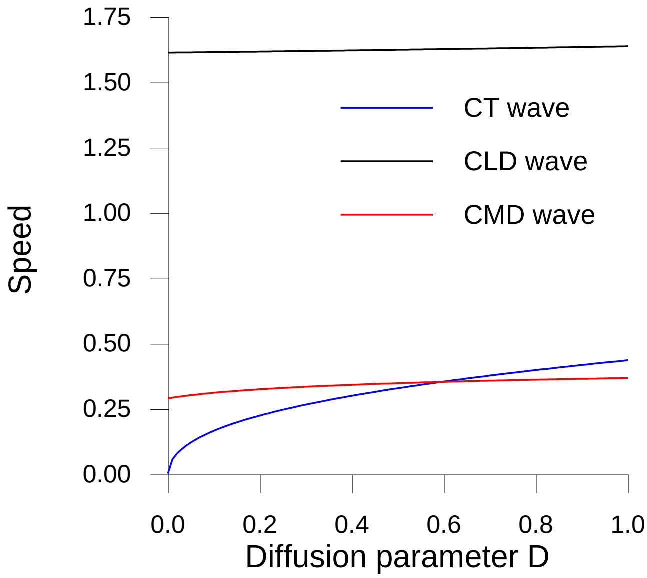

Figure 1. *Variations of the speeds of plane waves against diffusion parameter D*

The speed of CLD wave is 1.62313 × 10^4 cm.s^{-1} at b = 0. It decreases slowly to 1.61088 × 10^4 cm.s^{-1} at b = 1. The speed of CT wave is 0.38178 × 10^4 cm.s^{-1} at b = 0. It decreases to 0.03604 × 10^4 cm.s^{-1} at b = 0.220 and then increases sharply to 0.27090 × 10^4 cm.s^{-1} at b = 1. The speed of CMD wave is 0.33781 × 10^4 cm.s^{-1} at b = 0. It increases sharply to 0.89571 × 10^4 cm.s^{-1} at b = 1. These variations of speeds of CLD, CT and CMD waves are shown by black, blue and red lines respectively in Figure 2.

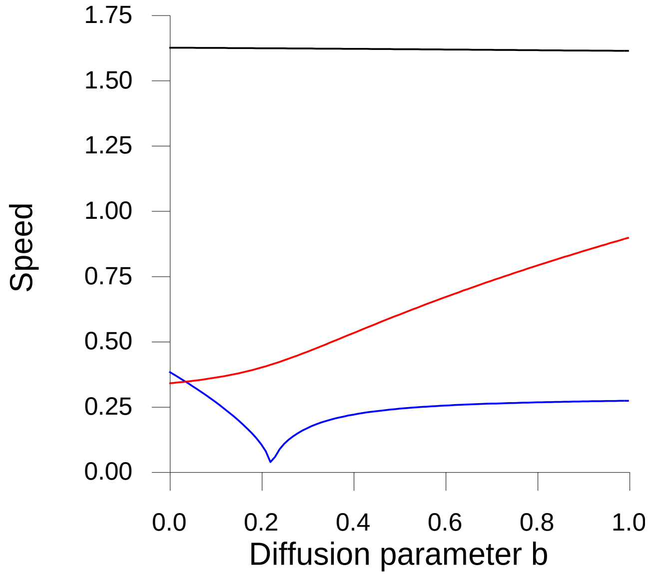

Figure 2. *Variations of the speeds of plane waves against diffusion parameter b*

The speed of CLD wave is 1.62305 × 10^4 cm.s^{-1} at a = 0. It decreases slowly to 1.60397 × 10^4 cm.s^{-1} at a = 0.5. The speed of CT wave is 0.33059 × 10^4 cm.s^{-1} at a = 0. It decreases to 0.11278 × 10^4 cm.s^{-1} at a = 0.5. The speed of CMD wave is 0.34362 × 10^4 cm.s^{-1} at a = 0. It increases to 1.01924 × 10^4 cm.s^{-1} at a = 0.5. These variations of speeds of CLD, CT and CMD waves are shown by black, blue and red lines respectively in Figure 3.

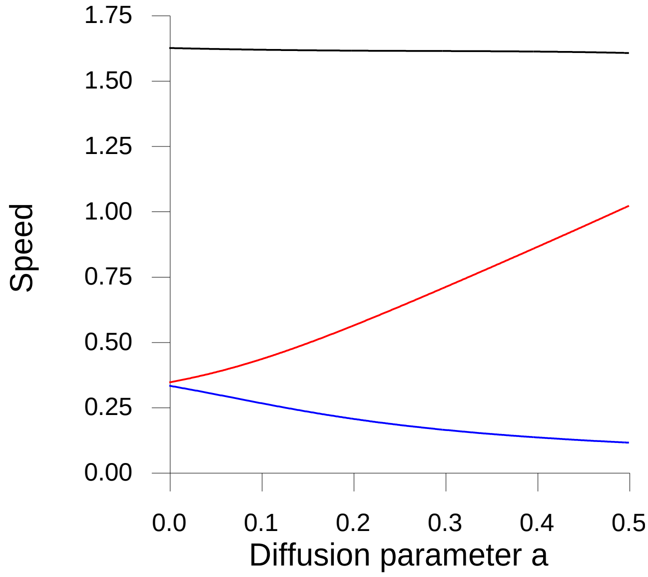

Figure 3. *Variations of speeds of plane waves against diffusion parameter a*

The speed of CLD wave is 1.61682 × 10^4 cm.s^{-1} at K = 0. It increases slowly to 1.62799 × 10^4 cm.s^{-1} at K = 1. The speed of CT wave is 0.26772 × 10^4 cm.s^{-1} at K = 0. It increases slowly to 0.34519 × 10^4 cm.s^{-1} at K = 1. The speed of CMD wave is 0.00049 × 10^4 cm.s^{-1} at K = 0. It increases sharply to 0.46774 × 10^4 cm.s^{-1} at K = 1. These variations of speeds of CLD, CT and CMD waves are shown by black, blue and red lines respectively in Figure 4.

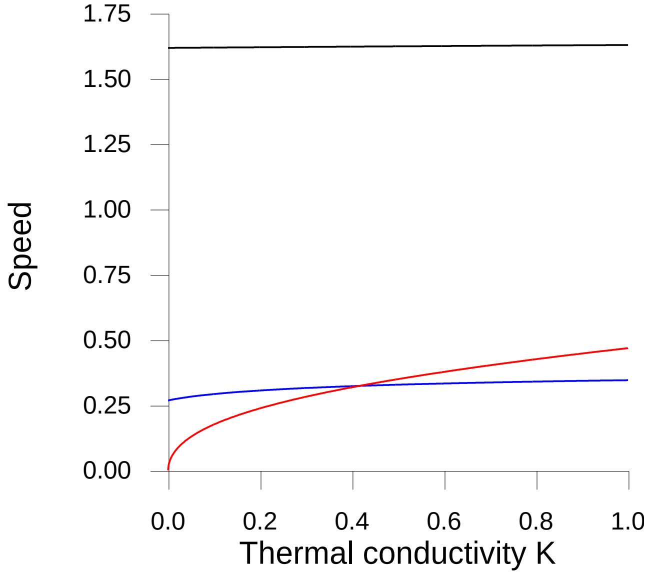

Figure 4. *Variations of the speeds of plane waves against thermal conductivity K*

The speed of CLD wave is 1.60798 × 10^4 cm.s^{-1} at ω = 0.1 Hz. It increases slowly to 1.71883 × 10^4 cm.s^{-1} at ω = 20 Hz. The speed of CT wave is 0.04726 × 10^4 cm.s^{-1} at ω = 0.1 Hz. It increases to 0.54911 × 10^4 cm.s^{-1} at ω = 20 Hz.. The speed of CMD wave is 0.04982 m × 10^4 cm.s^{-1} at ω = 0.1Hz. It increases to 0.59531 × 10^4 cm.s^{-1} at ω = 20 Hz. These variations of speeds of CLD, CT and CMD waves are shown by black, blue and red lines respectively in Figure 5.

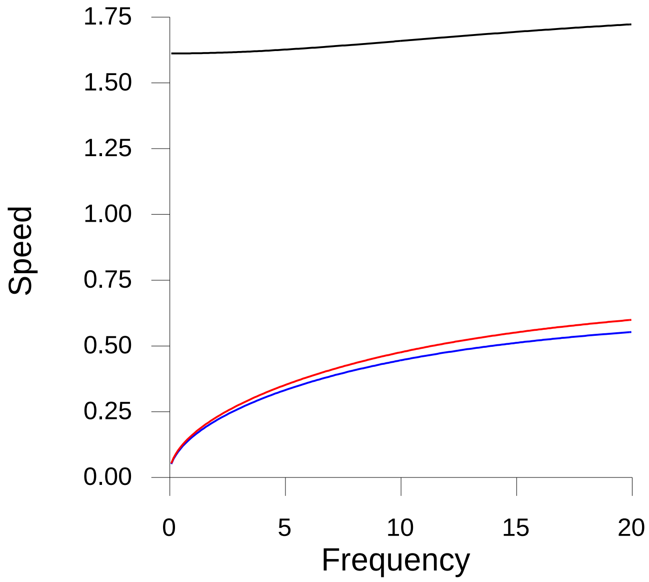

Figure 5. *Variations of the speeds of plane waves against frequency ω*

V. CONCLUSIONS

The plane wave propagation in a homogeneous, isotropic and linear micropolar thermoelastic half space with diffusion is studied. It is found that there exist six plane waves namely Coupled Longitudinal Displacement (CLD) wave, Coupled Thermal (CT) wave, Coupled Mass Diffusion (CMD) wave, Coupled Transverse Displacement (CTD) wave, Coupled Transverse Microrotational (CTM) wave and Longitudinal Microrotational wave. Out of these six waves three waves CLD, CT and CMD with wave speeds V_1, V_2 and V_3 are influenced by diffusion parameter, thermal parameter, frequency, thermal relaxation time and other material constants. The numerical results for a particular example of material show the significant effect of diffusion parameter, thermal parameter and frequency on the speeds of plane waves.

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