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A Comparative Analysis Of The Dispersion Curves In The Semi-Classical Theory Of Laser And Electromagnetically Induced Transparency

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***Abstract*— A comparative analysis of the dispersion curves in the region of zero detuning for semi classical theory of laser, electromagnetically induced transparency (EIT) and the anomalous dispersion curves in optics has been made. There is a certain element of similarity between the dispersion curves of semi classical theory of laser and the EIT in the vicinity of resonance. Our work reveals that the idea of EIT can be traced back to the dispersion curves of the semi classical theory of laser**

Keywords—Semi classical theory of Laser; EIT; Dispersion

I. INTRODUCTION

The semiclassical theory of laser [1,2] has explained a large variety of laser behaviours particularly in gaseous phase. One of the well known phenomena predicted by this theory is the Lamb-dip, which has also been observed experimentally. Fully quantum theoretical treatment of the semiclassical theory was also given by Scully and Lamb in 1967 [3]. Lamb dip plays an important role in the development of laser as it gives rise to a new branch of high resolution spectroscopy, which is known as Doppler free spectroscopy. One must not forget Haken's contribution in the field of laser [4-6] and also parallel works of Louisell [7]. The present work, which we prefer to call a short review, is primarily concerned with EIT [8-13]. We have indicated in our work that the idea of EIT was hidden in the semiclassical theory of laser. It is noteworthy that in the works related to EIT no mention of the semiclassical theory of laser, specifically of the dispersion curves, has been made.

II. DISPERSION CURVES IN THE SEMICLASSICAL THEORY OF LASER

We have observed that in the semiclassical theory of laser [2] two basic equations have been initially worked out for representing the condition of laser oscillation inside a laser cavity and for homogeneously broadened medium, with two level atoms. These equations are

$$\dot{E}_n + \frac{1}{2}\frac{\nu}{Q_n}E_n = -\frac{1}{2}\frac{\nu}{\varepsilon_0}I_m\left(\mathcal{P}_n\right) \quad (1)$$

and

$$\nu_n + \dot{\phi}_n = \Omega_n - \frac{1}{2}\frac{\nu}{\varepsilon_0}E_n^{-1}R_e\left(\mathcal{P}_n\right) \quad (2)$$

Where, E_n is the electric field amplitude of mode *n*, ν is the laser oscillation frequency in radian/sec, Q_n is the cavity quality factor for n^{th} mode, $\mathcal{P}_n$is the slowly varying complex polarization for mode *n*, $\nu_n$is the laser frequency for mode *n*, $\phi_n(t)$ is the slowly varying phase of the *n*th laser mode, andε_0 is the permittivity of vacuum = $8.85\times10^{-12}F/m$. In (1) and (2) the real part of $\mathcal{P}_n$is in phase with the electric field and it results in the dispersion due to the medium. The imaginary part is in quadrature with the electric field and results in gain or loss. In terms of Fourier components of susceptibility the polarization becomes

$$\mathcal{P}_n = \varepsilon_0\chi_n E_n = \varepsilon_0\left(\chi_n' + i\chi_n''\right)E_n$$

Substituting this into basic equations one can obtain the mode amplitude and frequency determining equations. Specifically the frequency determining equation is

$$\nu_n + \dot{\phi}_n = \Omega_n - \frac{\nu\chi_n'}{2} \quad (3)$$

This frequency determining equation implies that the oscillation frequency of the nth modeν_n is shifted from the passive cavity frequency Ω_n by an amount $-\nu\chi_n'/2$ .The frequency determining equation also predicts a pulling of the oscillation frequency ν_n from the passive cavity frequency towards the line centre, according to the equation

$$\nu_n + \dot{\phi}_n = \Omega_n + \sigma_n - \rho_n I_n \quad (4)$$

Where

$$\sigma_n = \frac{(\omega-\nu_n)}{\gamma}\mathcal{L}(\omega-\nu_n)F_1 \quad (5)$$

$$\rho_n = \frac{(\omega-\nu_n)}{\gamma}\mathcal{L}^2(\omega-\nu_n)F_3 \quad (6)$$

$\omega = \omega_a - \omega_b$ is atomic line centre frequency in laser medium.

$F_1 = \frac{1}{2}\nu\wp^2\left[\varepsilon_0\hbar\gamma\right]^{-1}\bar{N}$ and $F_3 = \frac{3}{2}\frac{\gamma_{ab}}{\gamma}F_1$

$\bar{N}$ is related to the relative excitation and known as average population inversion density. The relative excitation is given by

$$\Pi = \frac{\bar{N}}{\bar{N}_T};\ \text{and}\ \bar{N}_T = \frac{\varepsilon_0\hbar\gamma}{\gamma^2 Q_n} \qquad (7)$$

Where $\bar{N}_T$ is the value of the population inversion *N* at threshold. From (4) we have, using the values of σ_n and ρ_n

$$\nu_n + \dot{\phi}_n = \Omega_n + \sigma_n - \rho_n I_n$$

and finally

$$\nu_n = \frac{\Omega_n + \omega\mathscr{S}}{1+\mathscr{S}};\ I_n = \frac{a_n}{\beta_n} \qquad (8)$$

where the stabilizing factor is given by

$$\mathscr{S} = \frac{F_1}{\gamma}\mathcal{Z}(\omega-\nu_n)\left[1-1+\frac{\nu/2Q_n}{\mathcal{Z}(\omega-\nu_n)F_1}\right] = \frac{\nu}{2Q_n\gamma}.$$

Thus (8) can be expressed as

$$\nu_n = \frac{\Omega_n + \omega\nu/2Q_n\gamma}{1+\nu/2Q_n\gamma} = \frac{\Omega_n\gamma + \omega(\nu/2Q_n)}{\gamma + (\nu/2Q_n)} \qquad (9)$$

The index of refraction as determined by (9) is

$$\eta(\nu_n) = \frac{\Omega_n}{\nu_n} = 1 + \frac{1}{\gamma}(\nu/2Q_n)(1-\omega/\Omega_n)$$

$$\Rightarrow \eta(\nu_n) - 1 = \frac{1}{\gamma}(\nu/2Q_n)(1-\omega/\Omega_n) \qquad (10)$$

Another way to arrive at equation of the form of (10) is as follows

We have near threshold

$$\nu_n = \Omega_n + \sigma_n - \rho_n\frac{a_n}{\beta_n}$$
$$= \Omega_n + \sigma_n$$
$$= \Omega_n + \frac{\omega-\nu_n}{\gamma}\mathcal{L}(\omega-\nu_n)F_1$$
$$= \Omega_n + \frac{\omega-\nu_n}{\gamma}\mathcal{L}(\omega-\nu_n)\frac{1}{2}\nu\gamma^2(\varepsilon_0\hbar\gamma)^{-1}\bar{N}$$
$$= \Omega_n + \frac{\omega-\nu_n}{\gamma}\mathcal{L}(\omega-\nu_n)\frac{1}{2}\nu\gamma^2(\varepsilon_0\hbar\gamma)^{-1}\frac{\varepsilon_0\hbar\gamma}{\gamma^2 Q_n}\Pi$$
$$= \Omega_n + \frac{\omega-\nu_n}{\gamma}\mathcal{L}(\omega-\nu_n)\frac{1}{2}\nu\gamma^2(\varepsilon_0\hbar\gamma)^{-1}\Pi\bar{N}_T$$

which finally gives

$$\eta(\nu_n - \omega) - 1 = \frac{\nu_n - \omega}{\gamma}\left[\frac{\gamma^2}{\gamma^2 + (\nu_n-\omega)^2}\right]\Pi \quad (11)$$

The dispersion curve representing (11) is drawn in Fig. 1(a). It may be noted that for an absorbing medium, $\Pi < 1$.

III. ELECTROMAGNETICALLY INDUCED TRANSPARENCY

In this section we describe briefly the salient features of the phenomenon electromagnetically induced transparency (EIT), which is the consequence of coherent trapping and resultant dark state. One can make optical resonant transition transparent to laser radiation often with most of the atoms remaining in the ground state. EIT is now a technique for eliminating the effect of the medium on a propagating beam of electromagnetic radiation. The technique may be used to create large populations of coherently driven uniformly passed atoms thereby making possible new types of photonic devices. It can be shown how in a three level atom quantum interference is introduced by driving the upper levels with a strong coherent field. Under appropriate conditions the medium becomes transparent [zero absorption] for a probe field. We do not reproduce the detailed calculations here which may be found in reference [8], but illustrate with the help of Fig.2, how in a so called Λ - configuration EIT takes place.

The level $|a\rangle$ and $|b\rangle$ are coupled by a probe field of amplitude $\mathcal{E}$ and frequency ν. The main objective is to study the absorption and dispersion of the frequency ν. In this configuration the upper level $|a\rangle$ is coupled to the middle level $|c\rangle$ by a strong coherent (laser) field of frequency ν_μ which has complex Rabi frequency

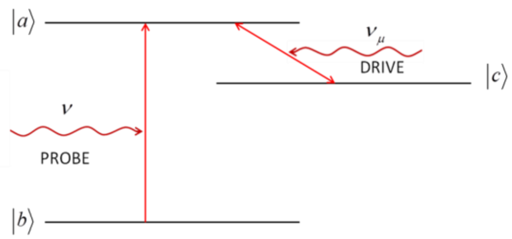

Fig.1.*Three level (Λ - configuration) atom for electromagnetically induced transparency*

$\Omega_\mu \exp(-i\phi_\mu)$. The off diagonal decay rates for ρ_{ab}, ρ_{ac} and ρ_{cb} are denoted by γ_1, γ_2 and γ_3 respectively. The real and imaginary parts of the complex susceptibility worked out for the three level atomic configurations are given by the following expressions

$$\chi' = \frac{N_a|\wp_{ab}|^2\Delta}{\varepsilon_0\hbar Z}\left[\gamma_3(\gamma_1+\gamma_3) + \left(\Delta^2 - \gamma_1\gamma_3 - \Omega_\mu^2/4\right)\right]$$
(12)

$$\chi'' = \frac{N_a |\wp_{ab}|^2}{\varepsilon_0 \hbar Z}\left[\Delta^2(\gamma_1+\gamma_3)-\gamma_3\left(\Delta^2-\gamma_1\gamma_3-\Omega_\mu^2/4\right)\right]$$
(13)

Where N_a is the number of atom per unit volume or atom number density and

$$Z = \left(\Delta^2-\gamma_1\gamma_3-\Omega_\mu^2/4\right)+\Delta^2(\gamma_1+\gamma_3)^2 \quad (14)$$

Equations (12) and (13) are the basic equations for investigating the dispersion and absorption respectively of the medium.

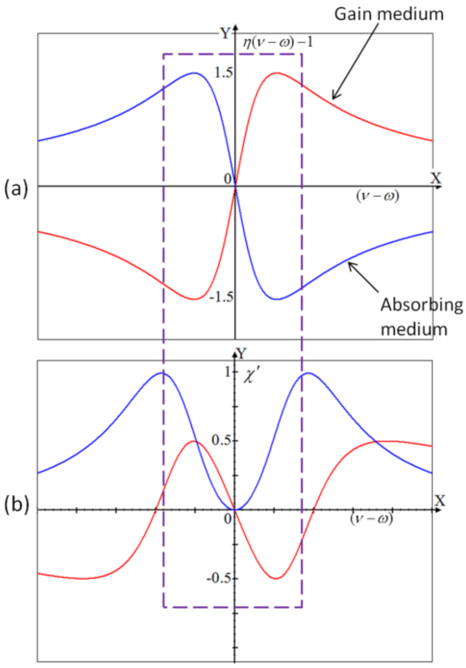

Fig.2 (a & b): *Comparison between two dispersion curves. In (b) $\chi' \to red$ and $\chi'' \to blue$ .Near resonance $(\nu-\omega=0)$ both the dispersion curves show that absorption is zero.*

χ' and χ'' are plotted versus the value of detuning Δ in units of atomic decay γ_1 for $\Omega_\mu = 2\gamma_1$, and $\gamma_1 >> \gamma_3 (\gamma_3 = 10^{-4}\gamma_1)$ .This choice is arbitrary but is justified. As may be inferred from Fig 1b, for zero detuning, $\chi' = 0$ and $\chi'' = 0$. This means that absorption is zero where the index of refraction is unity. Thus the medium becomes transparent under the action of the strong coherent field. This is what is known as electromagnetically induced transparency (EIT). As shown in Fig. 1a, the curves representing the index of refraction versus detuning in the semiclassical theory of laser exhibit identical feature. We observe that the index of refraction as given by (11) is unity when the detuning is zero and similar to Fig. 1b, the absorption is zero. In Fig. 1(a,b) we illustrate with the help of a rectangular block enclosing both the dispersion curves showing how the two dispersion curves, χ', χ'' *vs* Δ in EIT and the refractive index versus detuning in the semiclassical theory of laser look alike. We would like to emphasize that the dispersion relation as given by (11) is the result of the frequency determining equation (3) which predicts a pulling of the oscillation frequency ν_n by an amount Ω_n by an amount $-\nu\chi_n'/2$. However the dispersion curves in EIT is a consequence of atomic coherence and quantum interference.

From the considerations set forth above, it would follow that except in the region away from zero detuning the medium would be transparent to an electromagnetic radiation under suitable experimental conditions. We reasonably conclude that the idea of electromagnetically induced transparency is hidden in the dispersion curve of the semiclassical theory of laser as formulated by Lamb [1]. Why the phenomenon as described above in Fig. 1(a, b) manifests itself is not difficult to understand. In both the problems (EIT and semiclassical theory of laser) the topics of primary interest are absorption and dispersion.

In this connection we would like to bring up the treatment of the phenomenon of absorption, dispersion and refractive index by Feynman [14]. The physics of problem discussed by Feynman indicates within certain degree of accuracy the physics of EIT and dispersion in semiclassical theory of laser. According to Feynman the dispersion equation for a particular kind of atom with several resonant frequencies is given by

$$n = 1+\frac{q_e^2}{2\varepsilon_0 m}\sum_i \frac{N_k}{\omega_k^2-\omega^2+i\gamma_k\omega} \quad (15)$$

Where n is the refractive index, q_e is the electronic charge, N_k is the number of electrons per unit volume, ω is the angular frequency of the radiation, ω_k is the natural frequency of the electron with the damping factor γ_k . The index of refraction described by this formula varies with the frequency roughly like the curve in Fig 3.

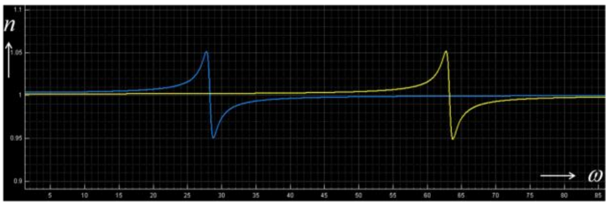

Fig.3. *The index of refraction as a function of frequency*

This curve is well known as anomalous dispersion curves in optics. We observe in Fig. 3 that so long as ω is not close to one of the resonant frequencies, the slope of the curve is positive. Such a positive slope represents normal dispersion. Very near the resonant frequencies however, there is a small range of ω's for which the slope is negative. Such a negative slope often refers to “anomalous” dispersion. As regards the phenomenon of absorption we would like to point out that when absorption of light is very small, this is to be expected from (15) that the imaginary part of the denominator $i\gamma_k\omega$ is much smaller than the term $\omega_k^2 - \omega^2$. But if the light frequency ω is close to ω_k than the resonant term ($\omega_k^2 - \omega^2$) can become small compared with $i\gamma_k\omega$ and the index of refraction becomes almost imaginary. The absorption of light becomes the dominant effect. It is the effect that gives the dark lines in in the spectrum of light which we receive from the sun. In general this is the explanation of the phenomenon of absorption of light which manifests itself in the form of absorption peaks or dark lines in the spectrum, as continuous radiation passes through a transparent or semi-transparent medium like solid, liquid or gases. Thus we have observed that imaginary part of the index of refraction means absorption and related to the term resonance between light frequency and natural frequency, that is, ω and ω_k . At resonance $\omega_k^2 - \omega^2$ = 0. It is to be noted that $\omega_k - \omega$ corresponds to the detuning Δ in our analysis of the dispersion curves in the semiclassical theory of laser or the dispersion curves in the phenomenon of electromagnetically induced transparency (EIT).

IV. SUMMARY

Summing up, we may say that there is some element of similarity in the dispersion curves as illustrated in Fig.1a, 1b and in Fig.2. Electromagnetically induced transparency is relatively a new topic and results as a result of destructive interference. The common elements of similarity exist in the region of resonance.

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