Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 2 Issue 9, September - 2015

Simple Logistic and Bi-Logistic Growth used as forecasting models of greenhouse areas in Albanian agriculture

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***Abstract*— The principal purpose of this paper is to analyze and present a prediction of the greenhouse diffusion in Albanian agriculture using a trend analysis based on the widely known logistic curves. Technology forecasting in general, is useful for all intentional and systematic efforts to precede and understand the possible orientation, rate, features and impact of new technology. Even though imperfect, technology forecast facilitates better choices and decisions. An accurate forecast enables maximize profit and minimize loss from future situation. Any person, company, or country that can be affected by innovations so necessarily engages in forecasting technology.**

Keywords*— *Logistic growth model, dynamics of socio-technical systems, forecasting, greenhouse technology

I. Introduction

A lot of processes in biology, ecology, sociology, technology and other different areas follow S-shaped logistic growth. Scientists all around the world apply logistic growth model (LGM) for describe the diffusion of technologies, to foresee population variations, for micro and macro-economic studies and for many other objectives. This mathematical model is proved useful in describing a wide range of natural phenomena: in tumor growth [1], forecasting model in business and economic [2], estimation of livestock population [4], traffic models [6] etc.

Forecasters made the first notable prediction in technology when they applied exponential models to describe new technological and social evolution (e.g. Malthus, 1798). It was supposed logically that a new technology firstly would be chosen by one or by two others each, and so on in a model of exponential growth. In the end however, as in any real system, a bound on thorough selections would be achieved, convincing the researchers to the logistic growth model (LGM) to forecast technological and social change. At the end of 20th Centuary, researchers such as Lenz(1985), Martino (1972), and very productive Marchetti (1977, 1994, 1996) improve forecasting models and proved that the logistic growth was a excellent concept for forecasting technological change. The logistic indicate effectively universal application for modeling technology diffusion, likewise for modeling a lot of other individual and social behaviors. S-shaped logistic curves have been effectively applied in different area as demographics, biology, economics, ecology, psychology, engineering and many others. The application of the logistic model, e.g. to describe the population change (in biology, demographics) or the diffusion of the innovations and new products, likewise above all economic development, is very illustrative and attractive (mostly thanks to nice curve image. The popularity of the logistic map in the modeling of the variety of real phenomena happens in the middle of 20th century, and the corresponding publications are enormous. It is honorable to mention two great researchers Cesare Marchetti and Theodore Modis who have build the bases for the continuous increasing popularity of logistic model.

For many years, Technological Forecasting & Social Change has been a favorable and helpful support to present recent advances in studies on logistic equation.

The S-shaped logistic curve is frequently applied to describe and to forecast the evolution of social and economic processes. This is a appropriate model to explain the development of the presumed “Limited world”.

II. Materials And Methods

Loglet Lab [10] is a software package for analyzing logistic behavior in time-series data.

Processes of growth and diffusion often follow a logistic course. In some cases they behave as a series of logistic wavelets, or "loglets." In the easiest cases to recognize, a loglet appears as an S-shaped curve or a succession of many S-shaped curves. When loglets overlap in time, the overall logistic behavior of a system can be hard to discern and analyze. In niches or markets in which several populations or technologies compete, the growth and decline of each entry also often exhibit logistic behavior. This behavior depends on interactions among the competitors. Namely, if a technology's market share grows, it comes at the cost of shares of others. This process is well-described by the so-called "logistic substitution model." Again, discerning and quantifying the pattern can be hard. To advance and ease analyses of logistic behavior in time-series data, we use the "Loglet Lab" software package. Loglet Lab users can fit logistic curves to a single time-series and apply the logistic substitution model to multiple time-series. We can fit logistic curves to a single time-series data. Loglet analysis deals with the decomposition of growth process and disintegration into S – shaped logistic ingredient more or less similar to wavelet diffraction for signal processing and compression. The name "loglet" includes "logistic" and "wavelet". Loglet analysis includes three models: The first is the simple S-shaped logistic model. The second is "Bi-logistic" model is presented for the analysis of systems that experience two phases of logistic growth, either overlapping or sequentially. The third is the S-shaped logistic substitution model, which adjusts the model under the impact of competition within free market.

II.1. *Simple Growth Models*

The mathematical models applied in this paper are based on differential logistic equation. This equation applied to populations or technologies, they follow curves of increase or decrease during the time. Even though the populations and technological parameters vary in discrete values, we utilize the continuous model for simplicity in the cace of modeling large components. Montroll [11] shows the correspondence between physical and population "trajectories" suggesting "laws of social dynamics " influenced by Newton's Law of mechanics. The exponential law of growth is well known as the model of growth without limits. The rate of growth at time t is proportional to the population P(t) and it is defined as the derivative $\frac{dP(t)}{dt}$ then unlimited growth model presented as a differential equation

$$\frac{dP(t)}{dt} = rP(t)$$

. The particular solution of this differential equation is:

$$P(t) = a\, e^{rt}$$

where r is the parameter of growth and a is the initial population P(0). In despite of many populations increase exponetially for some time, there don't exist real sistem that can retain unlimited growth for a long time. Thus the parameters or bounderies are varies. Since the real systems aren't permanentely unlimited and can't sustain the exponential growth, the equation (1) should adapted with a bound or a carrying capacity that make it more natural in the sigmoidal shape. The best known modification of unlimited growth equation is logistic growth equation.The logistic growth start with P(t) and r of the exponential growth, and a new term is added $(1 - \frac{P(t)}{K})$ named "inhibitor factor" that slow the rate of growth while the bound K is approximated:

$$\frac{dP(t)}{dt} = \underbrace{rP(t)}_{\text{exponential factor}} \underbrace{\left(1 - \frac{P(t)}{K}\right)}_{\text{inhibitor factor}}$$

Notice that the inhibitor factor $(1 - \frac{P(t)}{K})$ is near to 1 if P(t) <<< K and tends to 0 when the population approaches the carrying capacity K, following a S-shaped (Sigmoidal) growth curve. The particular solution to the logistic differential equation is:

$$P(t) = \frac{KP_0}{P_0 + (K - P_0)e^{-rt}}$$

where P_0 is the population size at time t = 0.This function gives the familiar S-shaped curve. Note that three parameters r, K and P_0 specify exactly the trajectory: The growth rate parameter r determines the "width" or "slope" of the sigmoidal trajectory. The key property of logistic growth are: $\lim_{t\to\infty} P(t) = K$, the population will finally achieve its carrying capacity; The relative rate of growth decrease linearly when population increase; The size of population at the inflection point (the rate of growth is maximum) $P_{inf} = \frac{K}{2}$; $\lim_{K\to\infty} \frac{KP_0}{P_0 + (K - P_0)e^{-rt}} = e^{rt}$

It is more helpful to commute the parameters r and P_0 to two other variables that fix key time of growth process.

Δt: is the time needed for the logistic curve to growth from 10% to 90% of Carrying Capacity K (named characteristic duration).

t_m : is named midpoint i.e. the time t of the saturation point when the value of growth is half of saturation value K.

Using simple algebra, we have the relation between new and old parameters :

$$\Delta t = \frac{\ln 81}{r} \quad \text{and} \quad t_m = r \ln \frac{K - P_0}{P_0}$$

The three parameters K, Δt and t_m determine the parametrization of the logistic growth utilized as the fundamental block of support for Loglet analysis :

$$P(t) = \frac{K}{1 + e^{-\frac{\ln 81}{\Delta t}(t - t_m)}}$$

The parameter Δt is more helpful than r to describe the time-series data because the units are more convenient to estimate. The logistic curve is symmetric around the midpoint t_m.

Furthermore, the parameter Δt informes us about the duration of the cycle. But the full cycle: the period of growth from 1% to 99% of the saturation value K is equal $2\Delta t$.

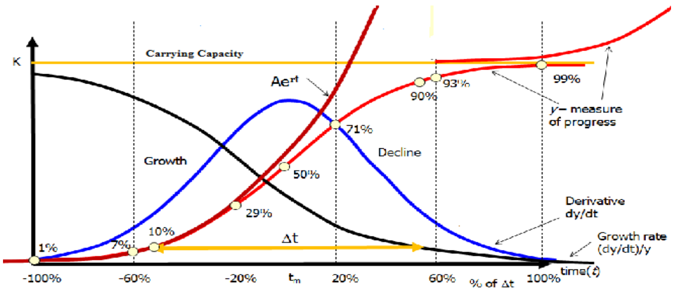

Fig.1. Characteristics of logistic growth

II.2. *Bi-logistical model of growth*

A lot of processes in nature and other areas follow logistic model of growth. Sometimes the simple S-shaped logistic curve gives a model correct of a system. Though the logistic growth model often is applied to processes non adequate. Practically all successful applications of the single growth model are in occurrences running in isolation. The saturation level (the capacity of environment) of a social process is limited by the present level of technology, which is always in progress. Generally, species can by chance adapt and spread their location. If the saturation level of a process varies during a period of logistic growth, another cycle of logistic growth with a different saturation level can superimpose on the first growth impulse. For instance mobile cellular telephone first replace fixed telephone and after it have another logistic growth process by itself. This system with two logistic growth pulses, existing simultaneously or in succession, it's named "Bi-logistical" by Meyer P. in [10]. The "Bi-logistical" is helpful in modeling many systems that include complicated growth processes not correctly modeled by the original logistic.

II.3. *The logistic substitution growth*

One of our interest of the paper is the analyse the rise, stability and decline of competitors replacing by one another. The technologies and products compete in the market, our study will focus on competition between different kind of greenhouse in Albanian agriculture. For this purpose we use the logistic substitution model that describes the market share of competitors. The period of life of one competitor can be divided into three specific stages: growth, saturation, and fall. We distinguish the particular moment of growth (development) depended on the emergence of new technologies. The growth and the fall follow logistic growth model that as we will note, impacts on the saturation stage.

The rules in base of the logistic substitution mode, as established by Marcheti [17], are :

- New technologies come into the business and increase following S-shaped logistic curve.

- Just one technology saturates the market at any given period of time.

- A technology during saturation doesn't follow a logistic path that is between the growth stage and the decline.

The technology in decline falls away firmly at logistic rule untouched by competition by new technologies.

The first rule suggests that the increase can be modeled with a logistic growth. As well the fourth suggests that the decline stage can be modeled with a logistic with a negative Δt. The second and the third imply us define saturation trajectory by competition from new technologies.

We see that for logistic substitution model, we use a logisic with just only two parametres Δt the carachteristic growth time and t_m the midpoint. The third parameter, carrying capacity k, is normalised at 1, or 100%. Without the emergence of a new technology, the old in growth stage would increase to a 100% of market share. In case of a new technology enter, its growth determines the old technology, inducing it to saturate and fall.

For each technology, it is obligatory to define the time interval that specify its logistic rate of growth(or decline). Between this interval of time is the information that will be used to calculate the logistic parameters that we will use to analyze the evolution of technologies.

II.4. Fisher-Pry transformation and visualization of the logistic growth

Generally, we represent logistic curve by just plotting on a positive coordinate system. We can change the variables to transform the logistic curve in a straight line. This view is called the Fisher-Pry Transform:

If we denote by $F(t) = \frac{P(t)}{K}$ and $FP(t) = \frac{F(t)}{1-F(t)}$ so we have:

$$\mathrm{Ln}\,(FP(t)) = \frac{\ln 81}{\Delta t}(t - t_m) \text{ therefore } P(t) = \frac{K}{1+e^{-\ln(FP(t))}}$$

Thus, if we graph FP(t) on a semi-log scale, the S-shaped logistic curve is transformed into linear. We notice that the interval of time in which the Fisher-Pry value is among 10^{-1} and 10^{1} is equal Δt, and the time of 10^{0} is the inflection point t_m.The right axis indicates the respective percent of saturation value 100%. Since the Fisher-Pry transform standardize every chart to the saturation point K, many logistic curves may be plotted on the chart to compare. Later, we will find out it helpful in case of analyzing more complex growth situations.

II.5. *The data*

The data about the greenhouses area of vegetables in Albania, for the period 1998-2012, are taken from the database of Albanian Institute of Statistics. The area of vegetables in two type of greenhouses: heating greenhouses (with glasses and with plastic) and solar greenhouses (with glasses and with plastic) are shown in table 1. Loglet Lab software is used to obtain the parameters of the model and the predicted values from logistic function.

III. Results And Discussion

The greenhouse technology is one of great innovation in agriculture. The data of table 1 indicate that the solar greenhouse area of vegetables and more specifically solar greenhouse with plastic are higher than the solar greenhouse area with glasses; and even higher than the heating greenhouse area. The total greenhouse area of vegetables is continuously increasing.

Table 1. Greenhouse area of Vegetables by type of Greenhouse

Year	Heating greenhouses			Solar greenhouses			Total Greenhouses
	With glasses	With plastic	Total	With glasses	With plastic	Total	
1998	30	0	30	98	181	279	309
1999	17	4	21	112	283	395	416
2000	15	2	17	114	331	445	462
2001	13	0	13	85	339	424	437
2002	12	1	13	70	426	496	509
2003	14	5	19	88	445	533	552
2004	18	10	28	79	553	632	660
2005	11	24	35	81	534	615	650
2006	19	29	48	65	562	627	675
2007	15	47	62	75	546	621	683
2008	18	57	75	65	564	629	704
2009	14	40	54	61	595	656	710
2010	16	41	57	80	691	771	828
2011	14	54	68	78	734	812	880
2012	14	26	40	69	831	900	940

Figure 2 indicates a decline of heating with glasses greenhouse area of vegetables from 1998 to 2002, followed by small fluctuations. Heating with plastic greenhouse area of vegetables has increased slowly from 1998 to 2004, and increased more from 2004 to 2008, then has happened big fluctuations of area and a decline.

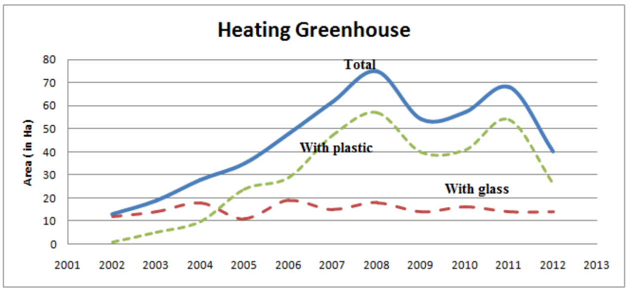

Fig. 2. Heating greenhouse area of vegetables

Figure 3 indicates a small decline of the solar with glasses greenhouse area of vegetables. Solar with plastic greenhouse area of vegetables shows a continuous increase during the study period.

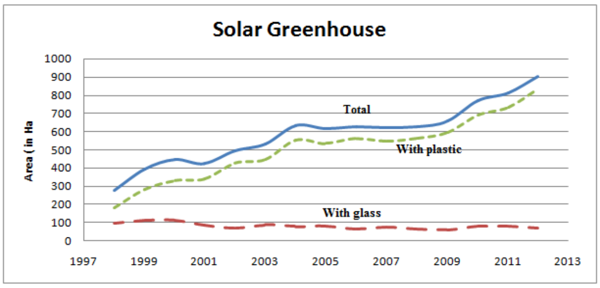

Fig. 3. Solar greenhouse area of vegetables

Solar Greenhouses area of vegetables continue to increase and indicates a S-shaped curve, whereas heating greenhouses area of vegetables is at low levels as indicated in figure 4.

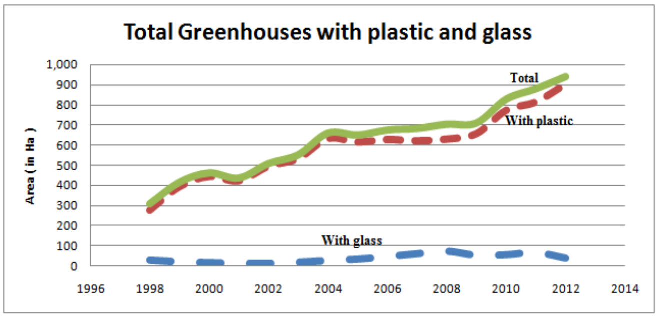

Fig. 4. Total greenhouse area of vegetables

It is clear that accurate forecasts are necessary for planning, making right decisions and implementing marketing strategies. Applying logistic model for forcasting needs the exact quantification of the growth process that is to find the value of the ceiling and the rate of the growth (control parameter). To obtain the values of r and K we use the Loglet Lab software.

Using Loglet Lab software and data base logistic curve is created for plastic greenhouse area. Figure 5 presents simple logistic fitted with data observed for the period of time 1998 – 2012 for area of plastic greenhouse total.

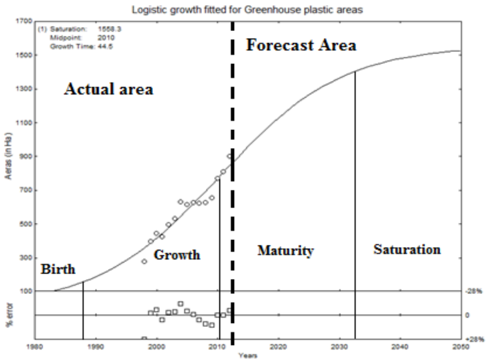

Fig.5. Single logistic curve fitted the plastic greenhouse area

The figure gives us the information about the fitted logistic growth of plastic greenhouse areas: the midpoint is 2010, the duration of growth time is 44.5, the decrease in the rate of change was at the year 2012 that indicate the growth curve passes an inflection point. Two periods 1988 – 2010 and 2010 - 2032 divide the logistic into growth and mature phases and the saturation value is 1558.3. We see also how well this single logistic curve fits the data observed, Loglet lab diplays the differences between actual data and those forecasted by fitted logistic curve. We see the graph of percentage errors and the maximum error is between $\pm$ 28% of the observed values.

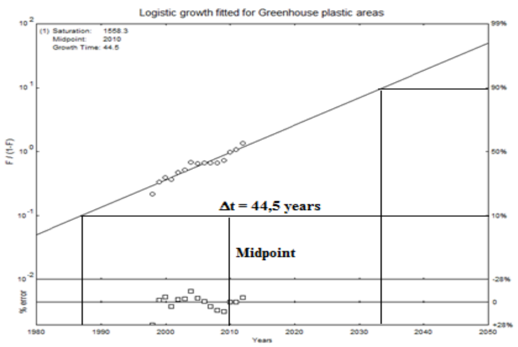

Fig.6. Fisher-Pry transformation of logistic fitted plastic greenhouse area

In order to improve the quality of forecast we will use Bi-logistical growth fitting curve for plastic greenhouse areas. Analyzing the results we can say if the sistem follows better a single logistical growth or bi-logistical growth.

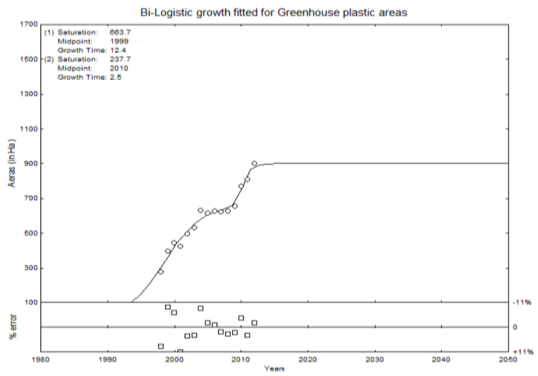

Fig. 7. Fitted bi-logistical growth curve of plastic greenhouse area

Immediately, we note that the bi-logistical curve fits better the growth process of greenhouse plastic areas. The percentage error is about 11% , so we can confirm that the growth process is compound with two growth pulses.

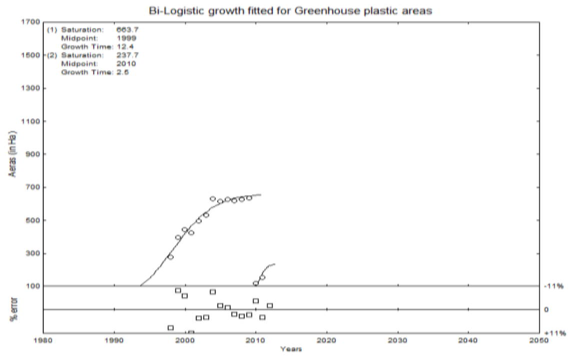

Fig. 8. Two components growth pulses of plastic greenhouse area logistic

The figure 8 gives the informations of two growth pulses that compound bi-logistical process: the first has the midpoint at the year 1999, the saturation level is 663.7 and growth time 12.4 years, the second smaller logistic pulse starts in 2009 has the inflection point at 2010, the saturation level 237.7 and growth time 2.5 years.

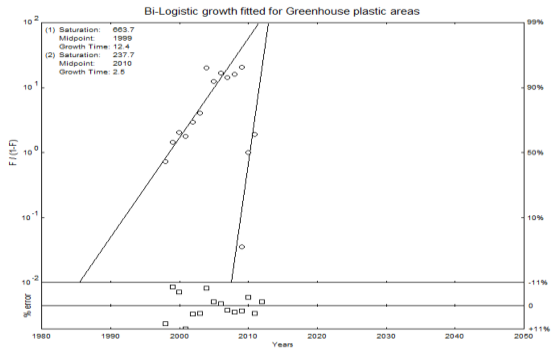

Fig.9. Fisher-Pry transomation of bi-logistical growth of plastic greenhouse areas

A converging Bi-logistical is shown where the first pulse of logistic growth is joint by a second faster pulse, dubbed the logistic model, and the two pulses saturate about the same time. Improving the quality of plastic used in construction of greenhouses will bring both the carrying capacity and the growth rate of the process to increase, causing the second pulse to rise from the first with both a faster characteristic Δt and a higher carrying capacity.

IV. Conclusions

The logistic function can be used as a good forecasting model. It can significantly determine at certain degree of accuracy how effective are the measures implemented to a certain concern such as greenhouses for vegetable production. Based on the findings, the logistic functions give close approximations of the actual values. But, the appropriate choice of time interval should be considered as it affects greatly the predicted values from the logistic equation.

Greenhouses for vegetable production is destined to play more and more a principal part in the Albanian climate environment as a way for sustainable production of vegetables besides the best regulation of product quality and reliability, compatible with the market request, standards and regulations. In addition to supply the local market, the production of greenhouse vegetables is significant for its export potential and represents an important part in the balance of foreign trade. Glass and plastic greenhouses are a new ability of growing vegetables, but the statistics show that poly greenhouses give a better investment profits than the glass ones.

The object of the paper was to improve the results of forecasting for innovations used in Agriculture applying logistic S-shaped curve. It is concluded that application of the rule of natural growth with a logistic S-curve, can improve fundamentally the precision of long-term forecasting.

In this paper we analyzed wide variety of time-series data sets about greenhouse area: heating with glass or plastic, solar with glass or plastic, total heating or solar during the period of time 1998-2012. Firstly, it was observed that the diffusion of greenhouse with glass heating or solar is in it's saturation stage so doesn't follow the growth process. It seems that the diffusion of plastic greenhouse heating or solar tracked S- shaped logistic growth. We focused the study and made forecasting about the total plastic greenhouse areas. The data set were fitted with a simple logistic growth and to check improvement in fit by bi-logistic. So the fitting simple logistic growth gave us the information: the saturation value is 1558 ha which can be achieved after 2032, the midpoint is at 2010 and the growth time is about 44.5 years. Moreover the residuals in percent deviation of data observed from the fitted curve are ± 28. Fitting the growth process by a bi-logistic curve we take some other value: the saturation level is about 910 ha, which can be achieved after 2015. Visual observation of the fitted curves as well as analysis of the residuals show that the bi-logistic model fits better that a simple logistical curve so this indicates the superiority of the bi-logistic. The logistic growth has proven useful in forecasting a wide variety of processes in the growth of systems. However, complex systems rarely follow a single S-shaped curve. The Bi-logistic function is more effective in modeling systems that contain two logistic growth pulses. The Bi-logistic is attractive because it is a prudent model to which we can still make clear physical interpretations.

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