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Squeezed State Of Light, Multiple Reflection And Spatial Hole-Burning

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***Abstract*—In the present work we have reported a possible analogy that exists in three different domains in physics. These are the phenomena of multiple reflection, spatial hole burning and squeezed states of light. There are three pairs of parameter which have been included for our analogy. In the first case it is the intensity versus phase angles at different reflectivity. In the second case it is the normalized population difference versus axial coordinates (Phase angle) at various values of dimensionless intensities and in the third case it is the variance of a squeezed state vs. squeezing angles.**

I. Introduction

In many ways some phenomena in physics appearing in different contexts are quite analogous. In some cases the phenomena appearing in physics show up also in non-physics contexts. In the present work we report a possible analogy in three different phenomena appearing in classical optics, laser and quantum optics. These are respectively the phenomena of multiple reflection [1], spatial hole burning [2] and squeezed state of light [3, 4]. An analogy between spatial hole burning and the intensity contour of the fringes in multiple reflection inside a Fabry-Perot cavity is already established in an earlier work [5] reported by one of the authors of the present work. In this work it is shown that the socalled "dimensionless intensity" I, a parameter in the semi classical theory of laser is identical to the parameter reflectance (*r*). In the present work we carry the analogy further to include the squeezed state of light. In all the three cases phase sensitive events are involved.

II. Squeezed State of Light

The zero-point fluctuations or vacuum fluctuation is a universal phenomenon which arises as a result of quantization of the electromagnetic field in the quantum theory of radiation. Many phenomena like spontaneous emission. Casimir effect, Lamb-shift, laser line width etc. can be explained adequately only with the help of the quantum theory. The quantum fluctuations in coherent state are equal to the zero-point fluctuation and are randomly distributed in phase. These zero-point fluctuations represent the standard quantum noise limit (QNL) to the reduction of noise in a signal. Even an ideal laser operating in a pure coherent state would still possess quantum noise due to zero-point fluctuation. We know that the electric field for a nearly monochromatic plane wave may be decomposed into two quadrature components with time dependent *coswt* and *sinwt* respectively. In a coherent state closent counterpart of a classical field, the fluctuations in the two quadratures are equal and minimize the uncertainty product given by Heisenberg's uncertainty product. We cannot suppress the fluctuations altogether, that would violate the uncertainty principle. But it is possible to rearrange them between the quadratures so that it is possible to generate light which has less noise in one selected quadrature than the quantum noise limit (QNL) dictates. Light with this property is called squeezed light. We can build experiments or applications which measure only one quadrature at a time. Consequently it will be possible to reduce fluctuations in this one quadrature only.

We consider here a formal quantum model of squeezed states [6] which could exist and can explained quantum theoretically from the vacuum state, the lowest number state, Squeezed states are represented by

$$|\alpha,\xi\rangle \text{ and } \xi = r_s \exp(i2\theta_s) \quad (1)$$

Where, α^2 is the intensity of the states, θ is the orientation of the squeezing angle and r_s is the degree of squeezing. The squeezing states are generated from the lowest number state.

$$|\alpha,\xi\rangle = \bar{D}(\alpha)\bar{S}(\xi)|0\rangle \quad (2)$$

The squeezing operator $\bar{S}(\xi)$ is defined as

$$\bar{S}(\xi) = \exp\left(\frac{1}{2}\xi^*\bar{a}^2 - \frac{1}{2}\xi\bar{a}^{\dagger 2}\right) \quad (3)$$

And the properties of the squeeze operator are

$$\bar{S}^{\dagger}(\xi)=\bar{S}^{-1}(\xi)=\bar{S}(-\xi)$$
$$\bar{S}^{\dagger}(\xi)\bar{a}\bar{S}(\xi) = \bar{a}\cosh(r_s)-\bar{a}^{\dagger}\exp(-2i\theta_s)\sinh(r_s)$$
$$\bar{S}^{\dagger}(\xi)\bar{a}^{\dagger}\bar{S}(\xi) = \bar{a}^{\dagger}\cosh(r_s)-\bar{a}\exp(-2i\theta_s)\sinh(r_s) \quad (4)$$

It may be noted that the reverse order of $\bar{D}(\alpha)$ and $\bar{S}(\xi)$ in (2) is possible. This results in the so called two photon correlated state. The concept of two photon correlated state was originally introduced by Yuen [7] in the work concerning two photon coherent state of the radiation field, even before the name squeezed state was coined. The formal definition allows us to derive the noise properties of a squeezed state exactly. The squeezed states have the following expectation values for the creation and annihilation operators

$$\langle \alpha,\xi|\bar{a}|\alpha,\xi\rangle = \propto$$
$$\langle \alpha,\xi|\bar{a}^2|\alpha,\xi\rangle = \alpha^2-\cosh(r_s)\sinh(r_s)\exp(2i\theta_s)$$
$$\langle \alpha,\xi|\bar{a}^{\dagger}\bar{a}|\alpha,\xi\rangle = |K|^2+\sinh^2(r_s) \quad (5)$$

From the last equation it is seen that the intensity of a squeezed light is greater that of the coherent state with the same value $|\alpha|$. This is, however, a minute effect for any laser beam for detectable intensity. Using these properties one can work out the variance of the generalized quadrature

$$X(\theta)=\exp(-i\theta)\bar{a}+\exp(i\theta)\bar{a}^{\dagger}$$

the quadrature which would be measured at the rotation angle θ. This variance is given by

$$Var(\bar{X}(\theta))=\cosh(2r_s)-\sinh(2r_s)\cos(2(\theta-\theta_s)) \quad (6)$$

The expression as given by (6) shows that the variance is a periodic function of the rotation angle as one would expect from the concept of an ellipse being rotated. It has a minimum when $\theta=-\theta_s$ and maximum in the orthogonal direction, that is, $\theta=-\theta_s+\frac{\pi}{2}$. This variance is the central point of our discussion in connection with other domains in physics. The variance is plotted in Fig.1 for one fixed squeezing angle, $\theta_s=-\pi/3$, and for angles of squeezing parameters $r_s=0.25, 0.5, 0.75$ and 1.00. Fig.1 is a linear plot of the variance.

It is seen that as the squeeze parameter is increased the minimum variance decreases and the maximum increases. This is similar to the concept of a stretching ellipse. It is worthwhile to note here that the uncertainty area for the squeezed state is generally drawn as an ellipse. Detecting squeezed light means detecting noise fluctuations. These fluctuations are very small and we need to have a minimum detectable energy to overcome the limitations imposed by electronic noise in the detecting system.

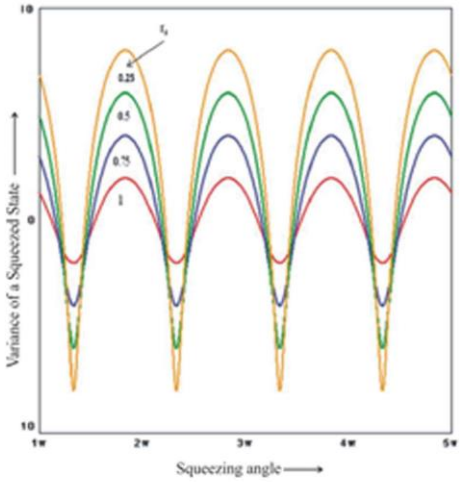

Fig.1 : A plot of the variance of a squeezed state as a function of the squeezing angle and different squeezing

III. Spatial Hole Burning

In this section we consider the phenomenon of the so-called spatial hole burning which is inherently present in a laser cavity and explained beautifully in the semiclassical theory of laser [2]. Spatial hole burning inhibits gain in laser originating from a Febry Perrot cavity.

The normalized population difference in terms of density matrix ρ_{aa} and ρ_{bb} is given by

$$\frac{\rho_{aa}-\rho_{bb}}{N(z,t)}=\frac{1}{1+R/R_s} \quad (7)$$

Where the constant R_s is known as saturation parameter and is given by

$$R_s=\gamma_a\gamma_b/2\gamma_{ab}, \quad \gamma_{ab}=\frac{1}{2}(\gamma_a+\gamma_b)$$

γ_a and γ_b are the decay rates from the upper and lower levels respectively. *R* is called the rate constant, given by

$$R=\frac{1}{2}\left(\frac{\wp E_n}{\hbar}\right)^2|U_n|^2\gamma^{-1}\mathcal{L}(\omega-\upsilon_n)$$

Where $\mathcal{L}(\omega-\upsilon_n)=\frac{\gamma^2}{\gamma^2+(\omega-\upsilon_n)^2}$

For a rate constant R with $|U_n(z)|^2=\sin^2 K_n Z$ dependence we have

$$\frac{R}{R_s} = I_n\left(\frac{2\gamma_{ab}}{\gamma}\right)\frac{\gamma^2}{\gamma^2+(\omega-\upsilon_n)^2}$$

Where $I_n = \frac{1}{2}\frac{\wp^2 E_n^2}{\hbar^2\gamma_a\gamma_b}$ is known as the dimensionless intensity, $K_n = 2\pi/\lambda$. For central tuning $\omega-\upsilon_n = 0$ and using $\gamma = 2\gamma_{ab}$ Eqn. (7) becomes

$$P_{aa} - P_{bb} = \frac{N(z,t)}{1+I_n \sin^2 K_n Z} \qquad (8)$$

Eqn. (8) has been used to draw the graph depicting the normalized population difference versus axial coordinate *z*. This is shown in Fig.2.

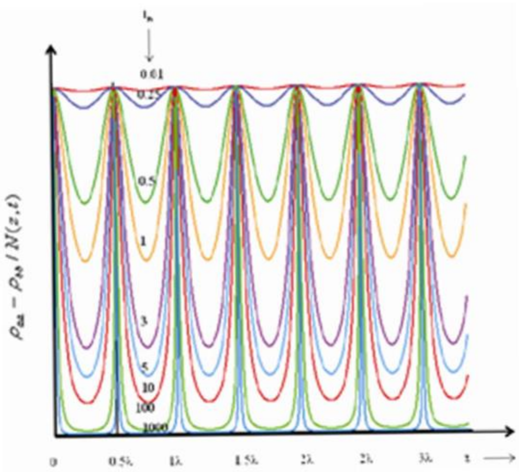

Fig.2: Normalized population difference vs axial co-ordinate**.**

In this figure dips are produced at regular intervals in the normalized population difference at various values of dimensionless intensity I_n . These dips created by the laser field are known as spatial holes and they are clearly depicted. We observe here that these dips are more prominent at higher values of dimensionless intensities. Complete depletion of population difference takes place at a value of dimensionless intensity which is equal to 1000. With this brief introduction of the phenomenon of spatial hole burning we consider the topic of the multiple reflections inside a Febry Perrot cavity which has been described in the section to follow.

IV. Intensity Contour of Fringes Due to Multiple Reflections

When a broad source of light is allowed to be incident on a pair of parallel plates, as in the case of Febry Perrot mirrors, the transmitted ray interfere at a point outside the plates after a series of multiple reflection. The intensity of the transmitted ray is given by

$$I_T = \frac{I_o}{1+\left[4r^2/(1-r^2)^2\right]\sin^2\frac{\delta}{2}} \qquad (9)$$

Fig.3 depicts the intensity contour of Fringes due to multiple reflections worked out with the help of Eqn. (9). In (9) $\delta = 2\pi m$, $m = 1,2,3.....$ At maxima $\sin^2\frac{\delta}{2} = 0$, and $I_T = I_O$. When the reflectance r^2 is very large and approaching unity, the quantity $4r^2/(1-r^2)^2$ will also be large and as a result, even a small departure of δ from its value for maximum will result in a rapid drop of intensity. This is also true in the case of Fig.2 where normalized population difference is plotted against axial coordinates. We observe in Fig.2 that when the parameter *I* is large even a small departure of the axial coordinate in phase from its value for maximum will result in a rapid drop of population difference. This case of decrease in the magnitude of the normalized population difference has its parallel in the intensity contour of fringes due to multiple reflections where it is shown that the sharpness of fringes depends on reflectance. We term these holes in the intensity contour of fringes due to multiple reflections as 'dark holes'. Comparing Eqns. (8) and (9) we note that

$$\rho_{aa} - \rho_{bb} \equiv I_T,\quad I_n \equiv \frac{4r^2}{(1-r^2)^2},\quad N(z,t) \equiv I_O$$

$$\frac{2\pi}{\lambda} z \equiv \delta = 2\pi m$$

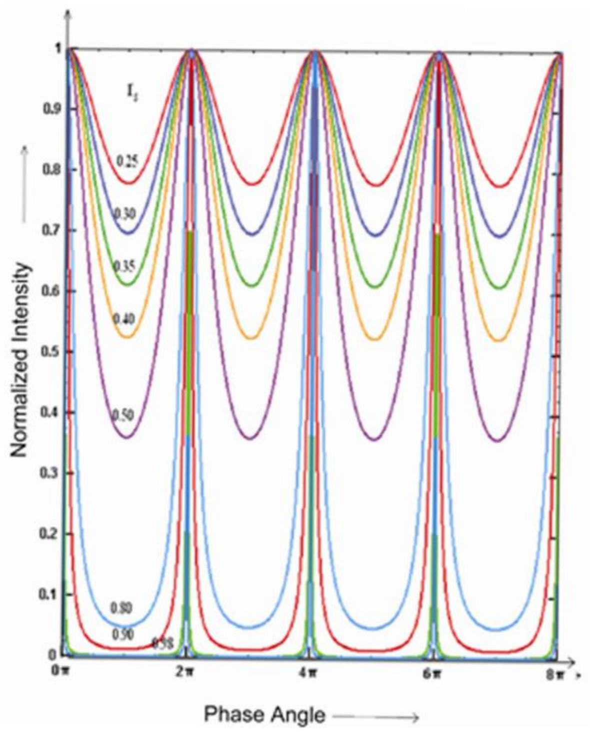

Fig.3: Intensity contour of Fringes due to multiple reflections

IV. A Comparative Analysis

From what has been discussed above it is now appropriate to provide an analogy that exists in three phenomena from classical optics to quantum optics. But it is also true that squeezed states represent a class of quantum state which has actually no classical analogy. There is already a worthwhile analogy between the intensity contour of multiple reflections and spatial hole burning. These are both phase sensitive events in the sense that in spatial hole burnings the normalized populated difference $(\rho_{aa}-\rho_{bb})/N(z,t)$ depends in phase. Similarly, the intensity contour of the fringes in multiple reflections is also phase sensitive process. Carrying the analogy further we observe that the squeezed filed is also a phenomenon involving phase sensitive quantum fluctuations which at certain phase angles are less than those of a perfectly coherent field, or of no field at all. Table-1 includes the co-efficient appearing in these domains.

Table-1: Comparison of the parameters and phenomena appearing in three domains

Multiple Reflection	**Spatial Hole Burning**	**Squeezed State**
Classical	Semi-classical	Quantum
Phase sensitive event	Phase sensitive event	Phase sensitive event
F.P. Cavity	F.P. Cavity	F.P. Cavity
Normalized intensity I_T	Normalized population difference $(\rho_{aa}-\rho_{bb})/N(z,t)$	Variance $Var\{\bar{X}(\theta)\}$
Reflectance (r^2)	Dimensionless Intensity	Squeezing parameter r_s
$4r^2/(1-r^2)^2$	$I_n=\frac{1}{2}\frac{\wp^2 E_n^2}{\hbar^2\gamma_a\gamma_b}$	

The spatial hole depletes the population and consequently inhibits gain in a laser. Similarly the 'dark holes' in the intensity contour reduces the intensity of transmitted beam. In the third case the dips in the variance vs. phase angle curve lowers the intensity of variance which results in the reduction of quantum fluctuations.

V. Conclusion

In the present work we have attempted to provide an analogy in three domains of physics. There is a worthwhile analogy between the phenomena of spatial hole burning in the semiclassical theory of laser and the so-called dark holes in the intensity contour of fringes due to multiple reflection. Spatial holes reduce the population difference and inhibit gain in a laser cavity. Dark holes reduce the intensity of the intensity contour of the fringes due to multiple reflections. Dip in the plot of variance vs. squeezing angles reduces the variance of a squeezed state and quantum fluctuations.

References

1. F.A. Jenkins and H.E. White, Fundamentals of Optics, McGraw Hill International Edition, (1991).
2. M. Sargent III, M.O. Scully and W.E. Lamb Jr., Laser Physics, (Addison–Wesley Publishing Company, Reading, Masschusetts, 1974).
3. D.F. Walls, Nature 306, 141 (1983).
4. Special Issues on Squeezed States of Light, J. Opt. Soc. Am. B4, (1987); J. Mod. Opt., 34 (1987).
5. R.M. Borah and G.D. Baruah, Pramana, Journal of Physics, 54, 269 (2000).
6. Hans A. Bachor and T.C. Ralph, A guide to Experiments in Quantum Optics, KGaA, Weinheim (2004), p.242.
7. H.P. Yuen, Phys. Rev. A 13, 2226 (1976).

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