Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 2 Issue 9, September - 2015

Numerical Evaluation Of Nonlinear Advection Diffusion Equation Models Using Finite Difference

Emmanuel Anthony
Department of Physics, Imo State University, Owerri, Nigeria
kukuri20042000@yahoo.com

Sunday Alachi and Njoku Linda Chika
Alvan Ikoko Federal College of Education, Owerri, Nigeria
sogood2015@yahoo.com
njoku.chika@yahoo.com

***Abstract*—This study investigates nonlinear Advection-Diffusion equation by making the diffusion constant D=D(c) a function of concentration. Linear model $D(c) = \alpha c$ and Square model $D(c) = \beta c^2$ were constructed for the non constant diffusion coefficient. The two models were substituted in into the advection-diffusion equation and solved. The equations obtained were solved numerically using explicit finite difference method. The results obtained shows that non linear advection diffusion equation gives a better explanation to the transport of contaminant concentration in fluid. The square model reveals that for domestic purposes water is safer at the middle of a river or stream than the banks.**

Keywords—Advectio; Diffusion; Nonlinear; Distribution; Concentration; Model

I. Introduction

The advection-diffusion equation (ADE) is basically one the simplest transport equation which describes the phenomena of transport in many areas of life. The advection-diffusion equation (ADE) has been useful in area like physics, chemistry, geology and biology in prediction of transport system [1] [2][3][4].
Transport phenomena describes the process that takes a system of particles from a non-equilibrium state to an equilibrium state, from an equilibrium state to a non-equilibrium state, or from one non-equilibrium state to another [5].
The ADE is a mathematical model that describes the transport of solutes in groundwater and surface water, the displacement of oil by fluid injection in oil recovery, the movement of aerosols and trace gases in the atmosphere, and miscible fluid flow processes in many other applications. In practical applications, these equations are commonly discretized via the Finite Difference (FD) or the Finite Element (FE) methods [6] [7].
The advection-diffusion equation is given by

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + D\frac{\partial^2 c}{\partial x^2} - \lambda c \qquad (1)$$

where c(x,t) is the concentration in the fluid of the substance in which we are interested,
u is the fluid velocity in the x-direction
D is the diffusion coefficient
λ is the coefficient of decay.
In some cases, *D* can reasonably be taken as constant, while diffusion in high polymers and real application, it depends markedly on concentration.
[8][9] studied a reaction-diffusion equation with a non constant diffusivity, D(x,t), a continuous function. They used an explicit finite difference method to numerically determine the effect of the reaction and found four special form for D(x,t) in which it was possible to reduce the reaction-diffusion equation to an equivalent constant diffusivity equation.
In this study our assumptions are similar to [8] in that our models solve the nonlinear advection-diffusion equation with D being a function of concentration.

II. Methodology

The linear advection diffusion equation is given by (1). Making the diffusion coefficient function of concentration D=D(c), we have

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + \frac{\partial}{\partial x}\left(D(c)\frac{\partial c}{\partial x}\right) - \lambda c \qquad (2)$$

The Linear Model
Here, we assumed a linear model of the form

$$D(c) = \alpha c \qquad (3)$$

where α is a consistent diffusion parameter
substituting (3) into (2) we have

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + \frac{\partial}{\partial x}\left(\alpha c\frac{\partial c}{\partial x}\right) - \lambda c \qquad (4)$$

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + \alpha\left(\frac{\partial c}{\partial x}\right)^2 + \alpha c\frac{\partial^2 c}{\partial x^2} + -\lambda c \qquad (5)$$

Square Model
Here, we assumed a square model the form

$$D(c) = \beta c^2 \qquad (6)$$

where β is a consistent diffusion parameter
substituting (6) into (2) we have

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + \frac{\partial}{\partial x}\left(\beta c^2\frac{\partial c}{\partial x}\right) - \lambda c \qquad (7)$$

$$\frac{\partial c}{\partial t} = -u\frac{\partial c}{\partial x} + 2\beta c\left(\frac{\partial c}{\partial x}\right)^2 + \beta c^2\frac{\partial^2 c}{\partial x^2} + -\lambda c \qquad (8)$$

Finite difference approximation

Explicit finite difference scheme was used to solve (5) and (8) with forward difference representations for the time derivatives and the central difference formulation for the spatial derivatives.

The Linear Model given in (5) discretization becomes

$$\frac{c_i^{n+1}-c_i^n}{\Delta t} = -u\left(\frac{c_{i+1}^n-c_i^n}{\Delta x}\right)+\alpha\left(\frac{c_{i+1}^n-c_i^n}{\Delta x}\right)^2+\alpha(c_i^n)\left(\frac{c_{i+1}^n-2c_i^n+c_{i-1}^n}{\Delta x^2}\right)+ -\lambda c_i^n \qquad (9)$$

$$= -u\left(\frac{c_{i+1}^n-c_i^n}{\Delta x}\right)+\frac{\alpha}{(\Delta x)^2}(c_{i+1}^n-c_i^n)^2+\frac{\alpha}{(\Delta x)^2}(c_i^n)(c_{i+1}^n-2c_i^n+c_{i-1}^n)-\lambda c_i^n$$

$$c_i^{n+1} = c_{i,j}^n-\frac{u\Delta t}{\Delta x}(c_{i+1}^n-c_i^n)+\frac{\alpha\Delta t}{(\Delta x)^2}(c_{i+1}^n-c_i^n)^2+\frac{\alpha\Delta t}{(\Delta x)^2}(c_i^n)(c_{i+1}^n-2c_i^n+c_{i-1}^n)-\lambda c_{i,j}^n \qquad (10)$$

The Square Model in (8) discretization becomes

$$\frac{c_i^{n+1}-c_i^n}{\Delta t} = -u\left(\frac{c_{i+1}^n-c_i^n}{\Delta x}\right)+2\beta(c_i^n)\left(\frac{c_{i+1}^n-c_i^n}{\Delta x}\right)^2+\beta(c_i^n)^2\left(\frac{c_{i+1}^n-2c_i^n+c_{i-1}^n}{\Delta x^2}\right)+ -\lambda c_i^n \qquad (11)$$

$$= -u\left(\frac{c_{i+1}^n-c_i^n}{\Delta x}\right)+\frac{\beta}{(\Delta x)^2}2(c_i^n)(c_{i+1}^n-c_i^n)^2+\frac{\beta}{(\Delta x)^2}(c_i^n)^2(c_{i+1}^n-2c_i^n+c_{i-1}^n)-\lambda c_i^n$$

$$c_i^{n+1} = c_{i,j}^n-\frac{u\Delta t}{\Delta x}(c_{i+1}^n-c_i^n)+\frac{\beta\Delta t}{(\Delta x)^2}2(c_i^n)(c_{i+1}^n-c_i^n)^2+\frac{\beta\Delta t}{(\Delta x)^2}(c_i^n)^2(c_{i+1}^n-2c_i^n+c_{i-1}^n)-\lambda c_{i,j}^n \qquad (12)$$

Equation (11) and (12) were solved with the following initial conditions, boundary conditions and parameters:

c(1,t)=15, c(Nx,t)=5 and c(x,1) =15, Nx=41, Nt=51,α=0.0065, u=0.001 for (11) and c(1,t)=10, c(Nx,t)=3 and c(x,1) =10, Nx=40, β=0.000009, u=0.22 for (12)

III. RESULTS AND DISCUSSION

Linear Model

Fig. 1- fig. 3 show the distribution of contaminant along the downstream (x axis) at difference simulation time for the linear diffusion model. At the initial simulation time for this model fig. 1 (t=4), the distribution of contaminant concentration is almost constant. This may be due to the fact that at the initial simulation time, the effect of the nonlinear advection diffusion equation was not strong enough. Toward the middle of the simulation time (t=40) fig. 2, the distribution of contaminant along downstream decreases uniformly until at x=0.4 to 0.8 where one sees little oscillation with decreasing amplitude. This may be as a result of the effect of our nonlinear model and probably in the distribution of contaminant along the simulation length (river) some traces of other source of contaminants was met. At the end of the simulation time fig. 3 shows similar behavior to fig 2, in this case it decreasing from the beginning of the simulation length until about x=0.7, when it decreases uniformly.

The time variation for the linear advection diffusion equation is shown in fig. 4-6. At an initial point in the simulation plane (x=10), the time variation of contaminant is nearly linear along the simulation time (fig. 4). Getting to the simulation plane, the variation concentration decreases uniformly with little oscillation amplitude (fig. 5). At the end of the simulation plane the contaminant concentration decreases uniformly.

Square Model

Fig. 7 and fig. 8 show the distribution of contaminant along the downstream direction (x-axis) for the square model. At the initial simulation time (t=5), the distribution of contaminant decreases uniformly (fig. 7) except towards the end of the simulation plane were it obeys the boundary condition. This is not too different from the linear situation. It shows that the nonlinear model has no much effect at the initial simulation time. Towards the middle of the simulation time (fig. 8), the distribution of contaminant in the river started with little increase before decreasing uniformly toward the middle of the simulation plane and that was maintained until close to the boundary of the river where there is an increasing contaminant concentration with increase amplitude. This model clearly reveals that to get safe water for domestic purposes and rural dwellers that depend on river or stream, the middle of the river or stream is the best. This is in line with [8]. The time variation concentration for this model is interesting (fig. 9-11), at the beginning of the simulation length (x=3), the time variation of contaminant shows little increase until at t=1.2, where the concentration increases and decreases towards the end of the simulation time. The time variation of contaminant decreases uniformly at x=10 and x=30 (fig. 10-11) as expected for this model.

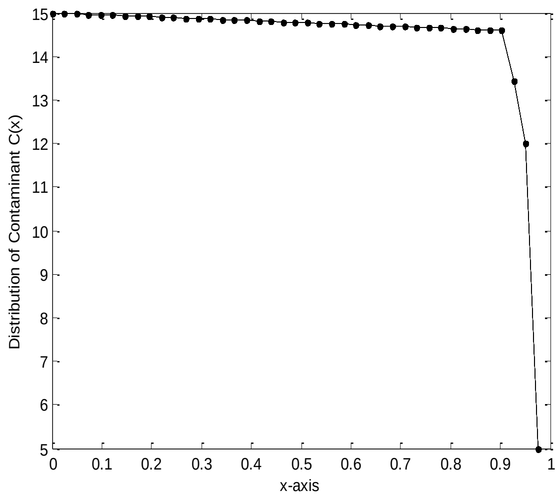

Fig.1 distribution of Contaminant C(x) along downstream direction x at t=4

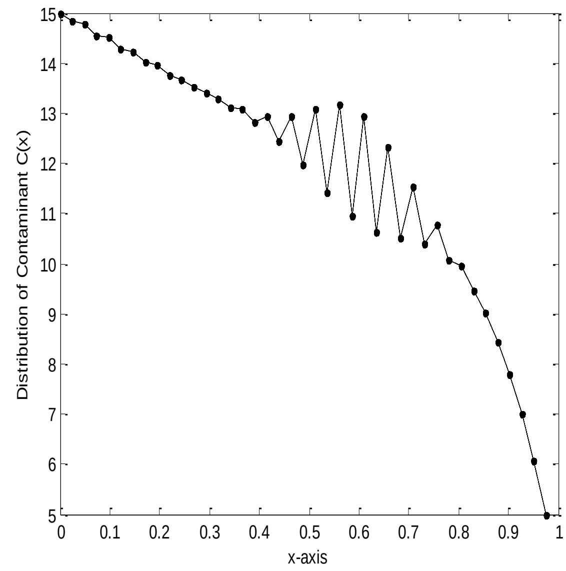

Fig.2: distribution of Contaminant C(x) along downstream direction x at t=40

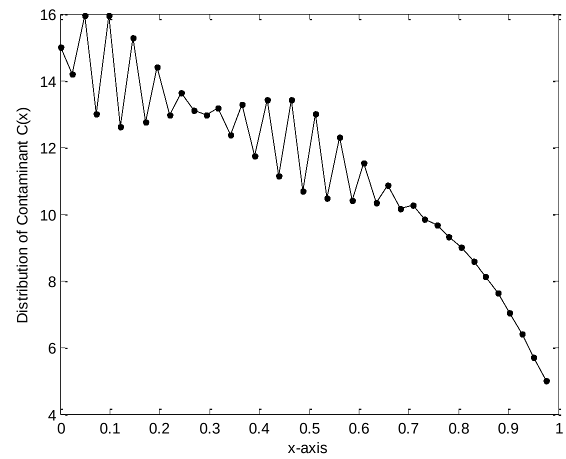

Fig.3: distribution of Contaminant C(x) along downstream direction x at t=50

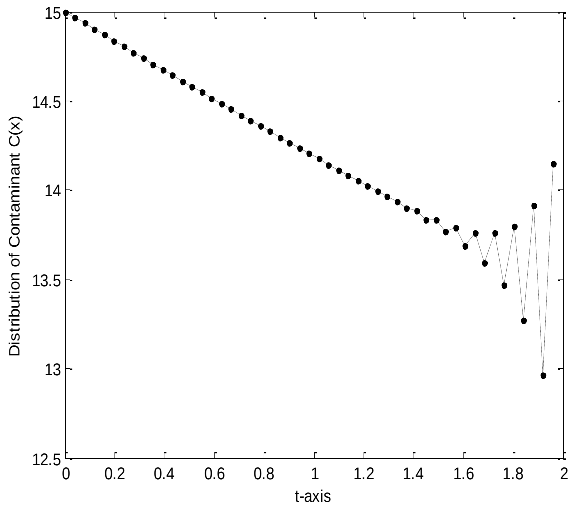

Fig.4: Time variation of Contaminant C(x) at x=10

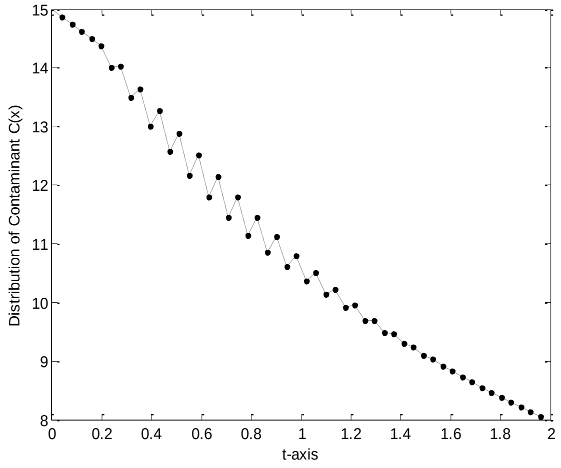

Fig.5: Time variation of Contaminant C(x) at x=36

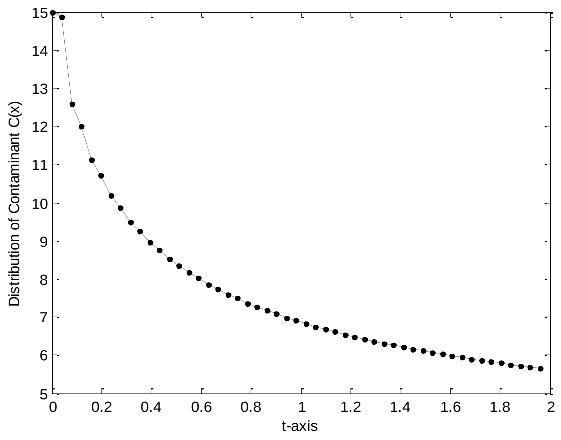

Fig.6: Time variation of Contaminant C(x) at x=40

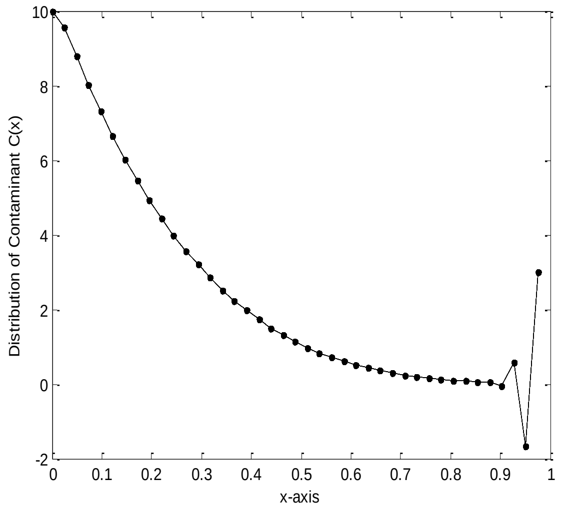

Fig.7: distribution of Contaminant C(x) along downstream direction x at t=5

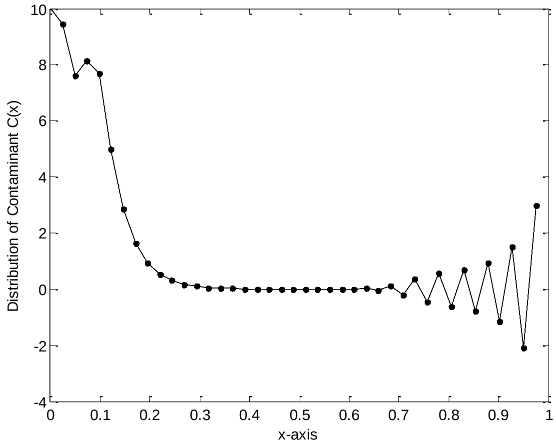

Fig.8: distribution of Contaminant C(x) along downstream direction x at t=25

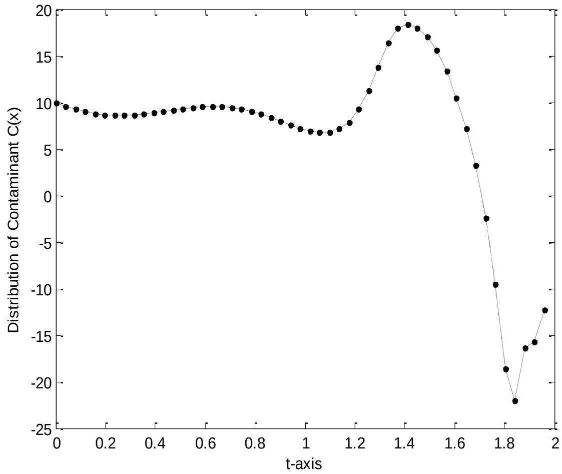

Fig.9: Time variation of Contaminant C(x, y) at x=3

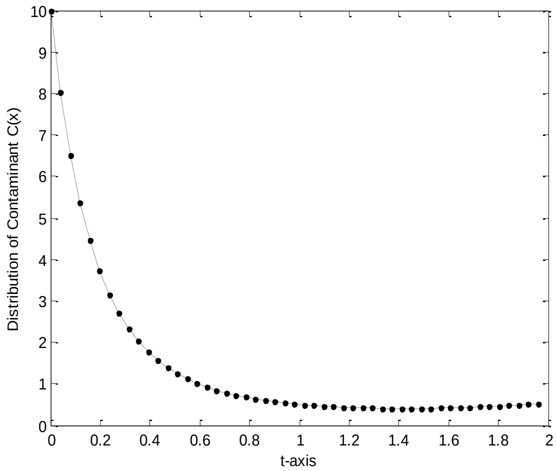

Fig.10: Time variation of Contaminant C(x) at x=10

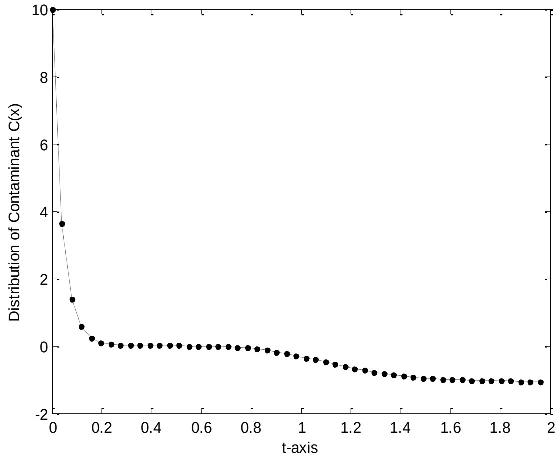

Fig.11: Time variation of Contaminant C(x) at x=30

CONCLUSION

An investigation into our models for nonlinear Advection Diffusion equation reveals that nonlinear model explain the transport of contaminant better than the linear theory. The linear model and the square model considered in our case show that the middle of the river or stream contains safer water for domestic consumption.

ACKNOWLEDGMENT

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g." Avoid the stilted expression "one of us (R. B. G.) thanks .".Instead, try "R. B. G. thanks.". Put sponsor acknowledgments in the unnumbered footnote on the first page.

REFERENCES

[1] A. Thyagaraja, N. Loureiro, P.J. Knight (2002). Spectral and evolutionary analysis of advection diffusion equations and the shear flow paradigm, UKAEA.
[2] D. Debasis, C. Mueller, K. Chen, J.A. Glazier, Solving the advection-diffusion equations in biological context using the cellular Potts model, Phys. Rev. (2005). E 72, 041909.
[3] J.G. Verwer, J.G. Blom, W. Hundsdorfer, An implicit-explicit approach for atmospheric transport-chemistry problems, App. Num. Math., (1996) 20, 191-209.
[4] J. Banavar, A. Maritan, and A. Rinaldo,2000. Nature (London).
[5] Plawsky, Joel.L., 2001, Transport Phenomena Fundamentals, CRC Press.
[6]David Scullen, 1992. Finite Difference Methods for Advection in Variable- velocity fields. The University of Adelaide Department of Applied Mathematics
[7] Abbott, M.B. and Basco, D.R., 1989, Computational fluid dynamics: an introduction for engineers. Longman Scientific & Technical, Harlow
[8]Watt S. D and Webber R. O. 1996. A Reaction Diffusion Equation with a non Constant Diffusivity" J. Austral.Math.Soc.Ser. B37, 458-473
[9]I. M. Echi, A. N. Amah and E. Anthony (2012). Journal of the Nigeria Association of Mathematic Physics.(21),21-134.