Thermoelastic Stresses in a Rod Subjected to Periodic Boundary Condition: An Analytical Treatment

Ahmet N. Eraslan
Department of Engineering Sciences
Middle East Technical University
Ankara 06531, Turkey
aeraslan@metu.edu.tr

Tunc Apatay
Department of Mechanical Engineering
Gazi University
Ankara 06570, Turkey
tapatay@gazi.edu.tr

***Abstract*—Analytical solution to thermoelastic response of a long rod (solid cylinder) subjected to periodic boundary condition is derived. The solution to the heat conduction equation is obtained by the application of Duhamel's theorem. Manipulating on the basic equations of elasticity, the governing equation describing thermoelastic behavior of the solid cylinder is formulated in terms of the radial displacement. This equation turns out to be of type nonhomogeneous second order Cauchy-Euler. The distributions of thermally induced stress, strain and displacement are determined by the analytical solution of the governing equation. Two different periodic boundary conditions are handled. In the first, the boundary temperature is assumed to vary sinusoidal, and in the second vary periodic but decaying eventually to zero.**

Keywords—Transient heat conduction; Periodic heating; Duhamel's theorem; Thermoelasticity; Generalized Plane Strain

I. INTRODUCTION

The heat conduction in cylinders and spheres is one of the common problems in engineering. Especially heat conduction with periodic boundary conditions has many applications in engineering such as in the heat treatment of metals, air conditioning, wall temperature oscillation of internal combustion engines, components of nuclear reactors, spacecrafts, automobiles, brake systems, chemistry and energy [1-6]. These applications may involve thermal disturbances that are large enough to force failures in materials. In order to predict these failures, determining the thermoelastic stresses causing them is important [7]. So, researchers have developed many analytical models and numerical methods to evaluate the transient thermal fields and resulting stresses in various parts and geometries.

The analytical solution for a temperature field and resulting thermal stresses in a hollow cylinder under arbitrary boundary conditions was first derived by Trostel [8]. The temperature field in a solid cylinder and caused by sinusoidal oscillation of the ambient temperature is presented in VDI [9]. Some general solutions of the transient temperature fields in solid and hollow cylinders under specific boundary conditions are given by Timoshenko and Goodier [2] and Hahn and Ozisik [1]. Mahmudi and Atefi [7] obtained an analytical solution for thermal stresses in a hollow cylinder, subjected to periodic time-varying thermal loading on the inner circular and insulated outer circular surfaces. An analytical solution for the transient thermal stresses in a hollow cylinder under constant boundary conditions by the use of the finite Hankel transform is given by Shahani and Nabavi [10]. Lee and Huang [11] developed an analytical solution method, without integral transform, to find the exact solutions for the transient heat conduction in functionally graded circular hollow cylinders with time-dependent boundary conditions. Fazeli [12] derived the mathematical model of two-dimensional heat conduction at the inner and outer surfaces of a hollow cylinder which are subjected to a time-dependent periodic boundary condition. In this work Duhamel's theorem is used to solve the problem for the periodic boundary condition which is decomposed into Fourier series. In the studies of Segall [13-15] the solution for an unsteady temperature field for a thick-walled hollow cylinder under generalized thermal loading on the inner circular surface is offered.

In this paper, an analytical model is derived to solve the thermoelastic behavior of a solid cylinder subjected to a periodic boundary temperature, applying the uncoupled theory of thermoelasticity. The heat conduction problem is solved with Duhamel's theorem. Two different time dependent surface temperature functions are handled. The transient stress and displacement distributions are obtained and the von Mises equivalent stresses are determined.

II. MODEL DEVELOPMENT

A. *Transient Temperature Distribution in the Rod*

An infinitely long solid cylinder of radius b is considered. The solid cylinder is initially at zero temperature and for times $t > 0$ its surface is subjected to a prescribed time-dependent boundary condition. The transient temperature distribution in the cylinder is described by the heat conduction equation [1]. The mathematical formulation of the problem in terms of dimensionless variables is

$$\frac{\partial \bar{T}}{\partial \tau} = \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{r}^2}\,; \quad 0 \le \bar{r} < 1, \;\; \tau > 0, \qquad (1)$$

$\bar{T}(0,\tau) = \text{finite},$

$$\bar{T}(1,\tau) = F(\tau), \tag{2}$$
$$\bar{T}(\bar{r},0) = 0,$$

where $\bar{r} = r/b$ is the dimensionless radial coordinate, $\bar{T} = T/T_0$ is the dimensionless temperature, T_0 is a reference temperature, $\tau = \alpha_T t/b^2$ is the dimensionless time with α_T the thermal diffusivity and $F(\tau)$ is the time-dependent surface temperature. The solution of this problem is obtained by the use of Duhamel's theorem as [1]

$$\bar{T}(\bar{r},\tau) = \int_0^\tau F(\beta)\frac{\partial}{\partial\tau}\Phi(\bar{r},\tau-\beta)d\beta, \tag{3}$$

in which $\Phi(\bar{r},\tau)$ represents the solution of the auxiliary problem defined by

$$\frac{\partial\Phi}{\partial\tau} = \frac{1}{\bar{r}}\frac{\partial\Phi}{\partial\bar{r}} + \frac{\partial^2\Phi}{\partial\bar{r}^2}\ ; \quad 0<\bar{r}<1,\ \ \tau>0, \tag{4}$$

$$\Phi(0,\tau) = finite,$$
$$\Phi(1,\tau) = 1, \tag{5}$$
$$\Phi(\bar{r},0) = 0,$$

The nonhomogeneous boundary condition $\Phi(1,\tau) = 1$ is handled by proposing a solution of the form

$$\Phi(\bar{r},\tau) = Y(\bar{r},\tau) + Z(\bar{r}) \tag{6}$$

Substituting into the auxiliary problem described by equation (4) results in two different problems

$$\frac{1}{\bar{r}}\frac{\partial Z}{\partial\bar{r}} + \frac{\partial^2 Z}{\partial\bar{r}^2} = 0\ ; \quad Z(0) = \text{finite}\,,\ Z(1) = 1, \tag{7}$$

and

$$\frac{\partial Y}{\partial\tau} = \frac{1}{\bar{r}}\frac{\partial Y}{\partial\bar{r}} + \frac{\partial^2 Y}{\partial\bar{r}^2}\ ; \quad 0<\bar{r}<1,\ \ \tau \tag{8}$$

subject to

$$Y(0,\tau) = \text{finite},$$
$$Y(1,\tau) = 0, \tag{9}$$
$$Y(\bar{r},0) = -Z(\bar{r}),$$

The solutions are

$$Z(\bar{r}) = 1, \tag{10}$$

and

$$Y(\bar{r},\tau) = -2\sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\frac{J_0(\lambda_n\bar{r})}{\lambda_n J_1(\lambda_n)} \tag{11}$$

in which J_0 and J_1 represent Bessel functions of the first kind of order zero and one, respectively, and λ_n for $n = 1,2,$ are the positive roots of

$$J_0(\lambda_n) = 0\,. \tag{12}$$

Hence, the solution for $\Phi(\bar{r},\tau)$ takes the form

$$\Phi(\bar{r},\tau) = 1 - 2\sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\frac{J_0(\lambda_n\bar{r})}{\lambda_n J_1(\lambda_n)}\,. \tag{13}$$

Substituting this solution into (3), the solution to the transient temperature distribution is obtained as

$$\bar{T}(\bar{r},\tau) = 2\sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\frac{\lambda_n J_0(\lambda_n\bar{r})}{J_1(\lambda_n)}\int_0^\tau e^{\lambda_n^2\beta}F(\beta)d\beta. \tag{14}$$

Furthermore, integrating this solution by parts, it can be put into an alternate and more convenient form as

$$\bar{T}(\bar{r},\tau) = F(\tau) - 2\sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\frac{J_0(\lambda_n\bar{r})}{\lambda_n J_1(\lambda_n)}\left[F(0) + \int_0^\tau e^{\lambda_n^2\beta}\frac{dF(\beta)}{d\beta}d\beta\right] \tag{15}$$

or

$$\bar{T}(\bar{r},\tau) = F(\tau) - 2\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\bar{r})}{\lambda_n J1(\lambda_n)}\int_0^\tau e^{-\lambda_n^2(\tau-\beta)}\frac{dF(\beta)}{d\beta}d\beta \tag{16}$$

since $F(\tau)$ is arranged so that $F(0) = 0$.

Two different boundary conditions are used and the corresponding integrals

$$I(\tau,\lambda_n) = \int_0^\tau e^{-\lambda_n^2(\tau-\beta)}\frac{dF(\beta)}{d\beta}d\beta \tag{17}$$

are evaluated.

B.C. 1. The condition is

$$F(\tau) = A\sin\tau, \tag{18}$$

with A being a load parameter. Then we find

$$I = I_1 = \frac{A\left[\lambda_n^2\cos\tau + \sin\tau - \lambda_n^2 e^{-\lambda_n^2\tau}\right]}{1+\lambda_n^4}. \tag{19}$$

B.C. 2. The condition is

$$F(\tau) = A\tau e^{-\tau/2}\cos\tau. \tag{20}$$

Then

$$I = I_2 = \frac{4A\lambda_n^2(3+4\lambda_n^2-4\lambda_n^4)e^{-\lambda_n^2\tau}}{(5-4\lambda_n^2+4\lambda_n^4)^2} - \frac{4Ae^{-\tau/2}\lambda_n^2[4+5\tau+4\lambda_n^4\tau-4\lambda_n^2(2+\tau)]\sin\tau}{(5-4\lambda_n^2+4\lambda_n^4)^2} - \frac{Ae^{-\tau/2}[8\lambda_n^6(\tau-2)-25\tau+6\lambda_n^2(2+5\tau)-4\lambda_n^4(7\tau-4)]\cos\tau}{(5-4\lambda_n^2+4\lambda_n^4)^2}. \tag{21}$$

Note that although the condition is periodic, it eventually decays to zero because of the existence of the term $e^{-\tau/2}$.

B. Thermoelastic Behavior of the Cylinder

Small deformations and a state of generalized plane strain are assumed. In order to determine transient stresses and deformations in the solid cylinder, dimensionless forms of the basic equations are used [2,3]. These are the following.

The equation of equilibrium

$$\frac{d\bar{\sigma}_r}{d\bar{r}} + \frac{\bar{\sigma}_r - \bar{\sigma}_\theta}{\bar{r}} = 0\,, \tag{22}$$

the strain-displacement relations

$$\bar{\epsilon}_\theta = \frac{\bar{u}}{\bar{r}}\ ,\ \bar{\epsilon}_r = \frac{d\bar{u}}{d\bar{r}}\ , \tag{23}$$

and the equations of the generalized Hooke's law

$$\bar{\epsilon}_r = \bar{\sigma}_r - \nu(\bar{\sigma}_\theta + \bar{\sigma}_Z) + \bar{\alpha}\bar{T}, \tag{24}$$

$$\bar{\epsilon}_\theta = \bar{\sigma}_\theta - \nu(\bar{\sigma}_r + \bar{\sigma}_Z) + \bar{\alpha}\bar{T}, \tag{25}$$

$$\bar{\epsilon}_z = \bar{\sigma}_z - \nu(\bar{\sigma}_r + \bar{\sigma}_\theta) + \bar{\alpha}\bar{T}\,. \tag{26}$$

In these equations $\bar{\sigma}_j = \sigma_j/\sigma_Y$ represents a dimensionless stress component, $\bar{\epsilon}_j = \epsilon_j E/\sigma_Y$ a normalized strain component, $\bar{u} = Eu/\sigma_Y b$ the dimensionless radial displacement, $\bar{\alpha} = E\alpha T_0/\sigma_Y$ the dimensionless coefficient of thermal expansion, ν the Poisson's ratio, E the modulus of elasticity and σ_Y the uniaxial yield stress of the material. In a state of generalized plane strain $\epsilon_Z = \epsilon_0$ is constant and (26) can be solved for the axial stress to give

$$\bar{\sigma}_z = \epsilon_0 + \nu(\bar{\sigma}_r + \bar{\sigma}_\theta) - \bar{\alpha}\bar{T} \tag{27}$$

Combination of this equation with strain-displacement relations, (23) and the equations of generalized Hooke's law, (24) and (25) allows one to formulate the stress displacement relations as

$$\bar{\sigma}_r = \frac{1}{(1+\nu)(1-2\nu)}\left[\nu\epsilon_0 + \frac{\nu\bar{u}}{\bar{r}} + (1-\nu)\bar{u}'\right] - \frac{\bar{\alpha}\bar{T}}{1-2\nu}\,, \tag{28}$$

$$\bar{\sigma}_\theta = \frac{1}{(1+\nu)(1-2\nu)}\left[\nu\epsilon_0 + \frac{(1-\nu)\bar{u}}{\bar{r}} + \nu\bar{u}'\right] - \frac{\bar{\alpha}\bar{T}}{1-2\nu}\,, \tag{29}$$

A prime above denotes differentiation with respect to the dimensionless radial coordinate $\bar{r}$. Substituting these stresses in the equation of equilibrium, (22), leads to the thermoelastic equation in terms of radial displacement

$$\bar{r}^2\frac{d^2\bar{u}}{d\bar{r}^2} + \bar{r}\frac{d\bar{u}}{d\bar{r}} - \bar{u} = \frac{1+\nu}{1-\nu}\bar{\alpha}\bar{r}^2\frac{\partial\bar{T}}{\partial\bar{r}}\,. \tag{30}$$

This is a second order nonhomogeneous Cauchy-Euler type differential equation which assumes the exact solution

$$\bar{u}(\bar{r}) = C_1\bar{r} + \frac{C_2}{\bar{r}} + \left(\frac{1+\nu}{1-\nu}\right)\left(\frac{\bar{\alpha}}{\bar{r}}\right)\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta\,. \tag{31}$$

It is important to note that

$$\lim_{\bar{r}\to 0}\frac{1}{\bar{r}}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta = \lim_{\bar{r}\to 0}\bar{r}\bar{T}(\bar{r},\tau) = 0. \tag{32}$$

Since the radial displacement $\bar{u}$ must be finite at the center of the solid cylinder ($\bar{r} = 0$) C_2 must be zero. Hence, the solution becomes

$$\bar{u}(\bar{r}) = C_1\bar{r} + \left(\frac{1+\nu}{1-\nu}\right)\left(\frac{\bar{\alpha}}{\bar{r}}\right)\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta\,. \tag{33}$$

The stresses are then obtained from (27)-(29). These equations contain two unknowns; namely C_1 and $\epsilon_0$to be determined. Since the surface of the solid cylinder is free of stress we have the condition

$$\bar{\sigma}_r(1) = 0. \tag{34}$$

Another condition can be formulated by making use of the fact that the total axial force F_Z must vanish as the ends of the solid cylinder are free. This leads to

$$F_Z = \int \bar{\sigma}_Z dA = 2\pi\int_0^1 \bar{r}\bar{\sigma}_Z d\bar{r} = 0. \tag{35}$$

Application of these conditions reveal that

$$C_1 = \frac{\bar{\alpha}(1-3\nu)}{1-\nu}\int_0^1 \bar{r}\,\bar{T}(\bar{r},\tau)d\bar{r}, \tag{36}$$

$$\epsilon_0 = 2\bar{\alpha}\int_0^1 \bar{r}\,\bar{T}(\bar{r},\tau)d\bar{r}. \tag{37}$$

Finally, the complete thermoelastic solution of the solid cylinder takes the form

$$\bar{u} = \frac{\bar{\alpha}}{1-\nu}\left[\bar{r}(1-3\nu)\int_0^1 \bar{r}\,\bar{T}(\bar{r},\tau)d\bar{r} + \frac{1+\nu}{\bar{r}}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta\right] \tag{38}$$

$$\bar{\sigma}_r = \frac{\bar{\alpha}}{(1-\nu)}\left(\int_0^1 \bar{r}\,\bar{T}(\bar{r},\tau)d\bar{r} - \frac{1}{\bar{r}^2}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta\right) \tag{39}$$

$$\bar{\sigma}_\theta = \frac{\bar{\alpha}}{(1-\nu)}\left(\int_0^1 \bar{r}\,\bar{T}(\bar{r},\tau)d\bar{r} + \frac{1}{\bar{r}^2}\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta - \bar{T}(\bar{r},\tau)\right) \tag{40}$$

$$\bar{\sigma}_Z = \frac{\bar{\alpha}}{(1-\nu)}\left(2\int_0^1 \bar{r}\,\bar{T}(\bar{r},\tau)d\bar{r} - \bar{T}(\bar{r},\tau)\right), \tag{41}$$

The integrals in the solution are evaluated as

$$\int_0^1 \bar{r}\,\bar{T}(\bar{r},\tau)d\bar{r} = \frac{F(\tau)}{2} - 2\sum_{n=1}^{\infty}\frac{1}{\lambda_n^2}I(\tau,\lambda_n)\,, \tag{42}$$

and

$$\int_0^{\bar{r}}\eta\bar{T}(\eta,\tau)d\eta = \frac{F(\tau)}{2}\bar{r}^2 - 2\sum_{n=1}^{\infty}\frac{\bar{r}J_1(\lambda_n\bar{r})}{\lambda_n{}^2 J_1(\lambda_n)}I(\tau,\lambda_n) \tag{43}$$

in which $I(\tau,\lambda_n)$ refers to the integral in (17). For the first temperature boundary condition it is to be replaced by I_1 in (19) and for the second by I_2 in (21).

C. Mathematical Verification of the Model

1. At the center of the cylinder $\bar{T}(0,\tau) = \text{finite}$, i.e. $\partial\bar{T}(0,\tau)/\partial\bar{r} = 0$. Based on the heat conduction solution presented by (16) the temperature gradient at any radial position and time instant turns out to be

$$\frac{\partial\bar{T}(\bar{r},\tau)}{\partial\bar{r}} = 2\sum_{n=1}^{\infty}\frac{J_1(\lambda_n\bar{r})}{J_1(\lambda_n)}\int_0^{\tau} e^{-\lambda_n^2(\tau-\beta)}\frac{dF(\beta)}{d\beta}d\beta, \tag{44}$$

as $J_1(0) = 0$ we determine $\partial\bar{T}(0,\tau)/\partial\bar{r} = 0$.

2. At the center of the cylinder $\bar{u} = 0$. By virtue of (32) we arrive at

$$\bar{u} = \frac{\bar{\alpha}}{1-\nu}\left\{(0)(1-3\nu)\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} + (1+\nu)\lim_{\bar{r}\to 0}\left[\frac{1}{\bar{r}}\int_0^{\bar{r}} \eta\bar{T}(\eta,\tau)d\eta\right]\right\} = 0 \quad (45)$$

3. At the center of the cylinder $\bar{\sigma}_r = \bar{\sigma}_\theta$ by geometry. Since

$$\lim_{\bar{r}\to 0}\frac{1}{\bar{r}^2}\int_0^{\bar{r}} \eta\bar{T}(\eta,\tau)d\eta = \frac{1}{2}\bar{T}(0,\tau), \quad (46)$$

it is easy to show that

$$\bar{\sigma}_r(0) = \bar{\sigma}_\theta(0) = \frac{\bar{\alpha}}{1-\nu}\left(\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} - \frac{1}{2}\bar{T}(0,\tau)\right) \quad (47)$$

4. At the surface of the cylinder $\bar{\sigma}_r(1)$ =0. From (39)

$$\bar{\sigma}_r(1) = \frac{\bar{\alpha}}{1-\nu}\left(\int_0^1 \bar{r}\bar{T}(\bar{r},\tau)d\bar{r} - \int_0^1 \eta\bar{T}(\eta,\tau)d\eta\right) = 0 \quad (48)$$

III. Numerical Results and Discussion

Another verification of the entire model can be made numerically. The computer program based on finite element collocation method developed by Eraslan and Varli [16] was modified to handle time-dependent boundary conditions for this purpose. The distributions of the von Mises stress in the cylinder at various time instants are calculated and compared with the results of the numerical solution. The von Mises stresses $\bar{\sigma}_{vM}$ are obtained from [17]

$$\bar{\sigma}_{vM} = \sqrt{\frac{1}{2}[(\bar{\sigma}_r-\bar{\sigma}_\theta)^2 + (\bar{\sigma}_r-\bar{\sigma}_z)^2 + (\bar{\sigma}_\theta-\bar{\sigma}_z)^2]}. \quad (49)$$

It is well known that the deformation is elastic as long as $\bar{\sigma}_{vM} \leq 1$. The reason why the von Mises stress is selected for comparison is that it is a function of all three principal stress components, a small error in one of them may result in apparent discrepancies in the results. The parameter values $\nu = 0.3$, $A = 2.0$ and $\bar{\alpha}$ =1.75 are used in these calculations. The results of these calculations are presented in Fig. 1a and Fig. 1b on which solid lines belong to the results of this work and dots to the results of numerical solution. Fig. 1a is based on B.C. 1 given by (18) while Fig. 1b on B.C. 2 given by (20). As seen in these figures, perfect agreement is obtained between the results of this work and finite element solution verifying the present analytical model for both boundary conditions. Note also that the largest values of $\bar{\sigma}_{vM}$ occur at the surface of the cylinder, hence failure with respect to plastic deformation occurs at the surface of the cylinder as the load parameter A is further increased.

The parameter values ν =0.3, A =2.0 and $\bar{\alpha}$ =1.75 are not altered in the following calculations. The change in temperature $\bar{T}$ and temperature gradient $d\bar{T}/d\bar{r}$ with time at the radial position $\bar{r} = 0.5$ are calculated and plotted in Fig. 2a and Fig. 2b. The periodic boundary condition described by (18) leads to periodic variation of $\bar{T}$ and $d\bar{T}/d\bar{r}$ with time at any radial position as seen in Fig. 2a. This behavior is repeated with a period 2π. The variation of $\bar{T}$ and $d\bar{T}/d\bar{r}$ with time die out for times τ >15 in case of B.C. 2 as shown in Fig. 2b. Variations of the stress components, the von Mises stress and the radial displacement with time at the same radial location ($\bar{r} = 0.5$) are plotted in Fig. 3a and Fig. 3b. It is apparent in these figures that the deformation is purely elastic as $\bar{\sigma}_{vM} \leq 1$ throughout and compressive as well as tensile stresses may be found in the cylinder according as the temperature gradient is negative or positive. The variations shown in Fig. 3a correspond to thermal response depicted in Fig. 2a, and the ones in Fig. 3b correspond to thermal structure of the cylinder pictured in Fig. 2b.

The cylinder may expand or contract in the axial direction as the ends are free. The change in height of it at any time instant is obtained by multiplying the axial strain with its height. The results of these calculations corresponding to B.C. 1 are depicted in Fig. 4. As seen in this figure the height of the cylinder follows a path similar to the boundary temperature and returns to the original height as soon as one period is completed.

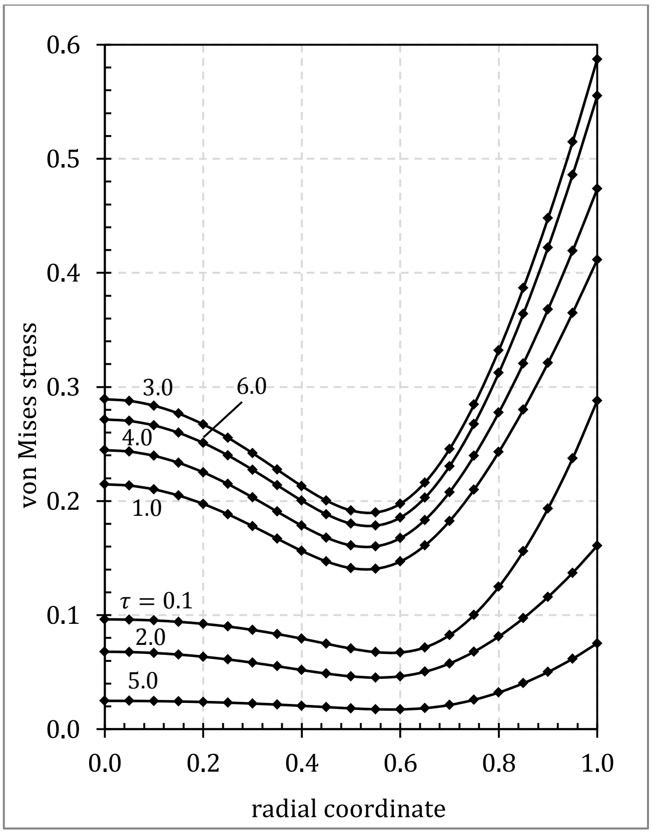

Fig. 1a. Distributions of the von Mises stress in the cylinder at various time instants based on B.C.1 for the parameter set $\nu = 0.3$, $A = 2.0$ and $\bar{\alpha} = 1.75$. The solid lines belong to the results of this work and dots to the numerical solution [16].

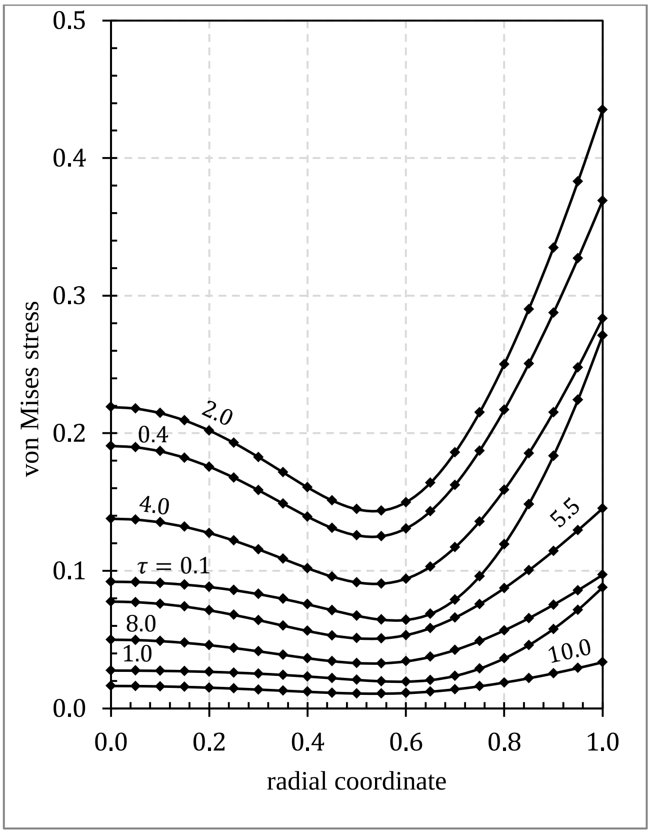

Fig.1b. Distributions of the von Mises stress in the cylinder at various time instants based on B.C.2 for the parameter $\boldsymbol{\nu} = \mathbf{0.3}$, $\boldsymbol{A} = \mathbf{2.0}$ and $\bar{\boldsymbol{\alpha}} = \mathbf{1.75}$. The solid lines belong to the results of this work and dots to the numerical solution [16].

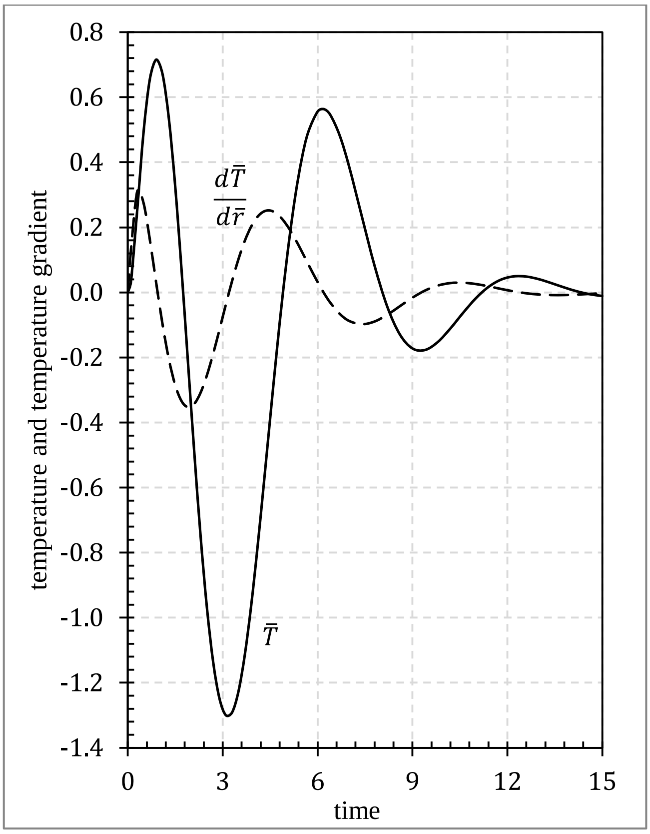

Fig. 2b. Variations of temperature and temperature gradient with time at $\bar{\boldsymbol{r}} = \mathbf{0.5}$ based on B.C.2 for the parameter $\boldsymbol{\nu} = \mathbf{0.3}$, $\boldsymbol{A} = \mathbf{2.0}$ and $\bar{\boldsymbol{\alpha}} = \mathbf{1.75}$.

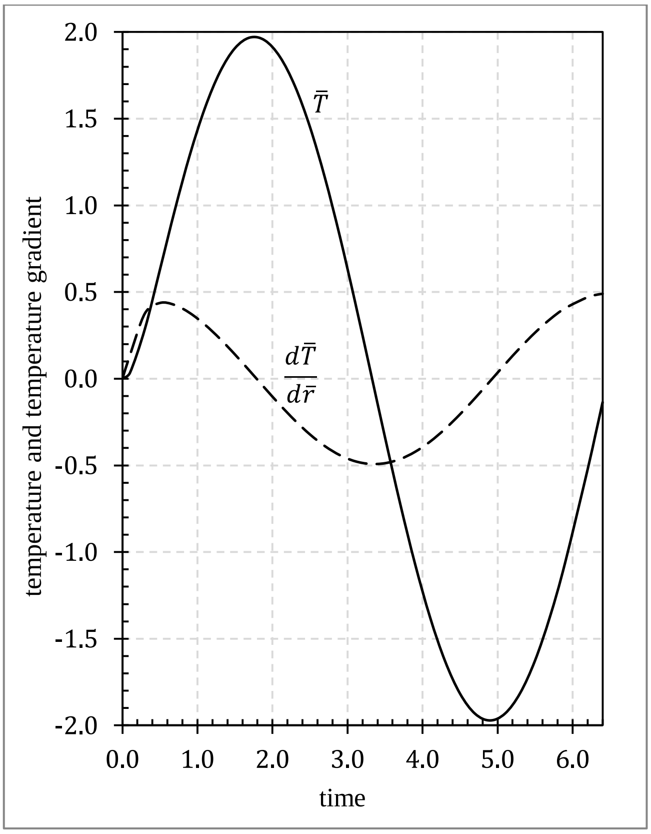

Fig. 2a. Variations of temperature and temperature gradient with time at $\bar{\boldsymbol{r}} = \mathbf{0.5}$ based on B.C.1 for the parameter set $\boldsymbol{\nu} = \mathbf{0.3}$, $\boldsymbol{A} = \mathbf{2.0}$ and $\bar{\boldsymbol{\alpha}} = \mathbf{1.75}$.

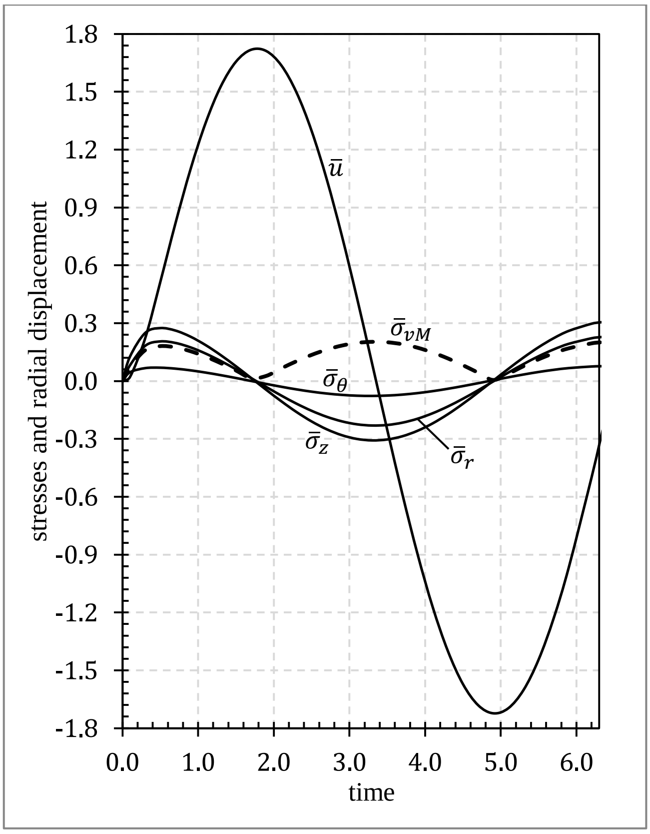

Fig. 3a. Variations of the stress components, the von Mises stress and the displacement with time at $\bar{\boldsymbol{r}} = \mathbf{0.5}$ based on B.C. 1 for the parameter set $\boldsymbol{\nu} = \mathbf{0.3}$, $\boldsymbol{A} = \mathbf{2.0}$ and $\bar{\boldsymbol{\alpha}} = \mathbf{1.75}$.

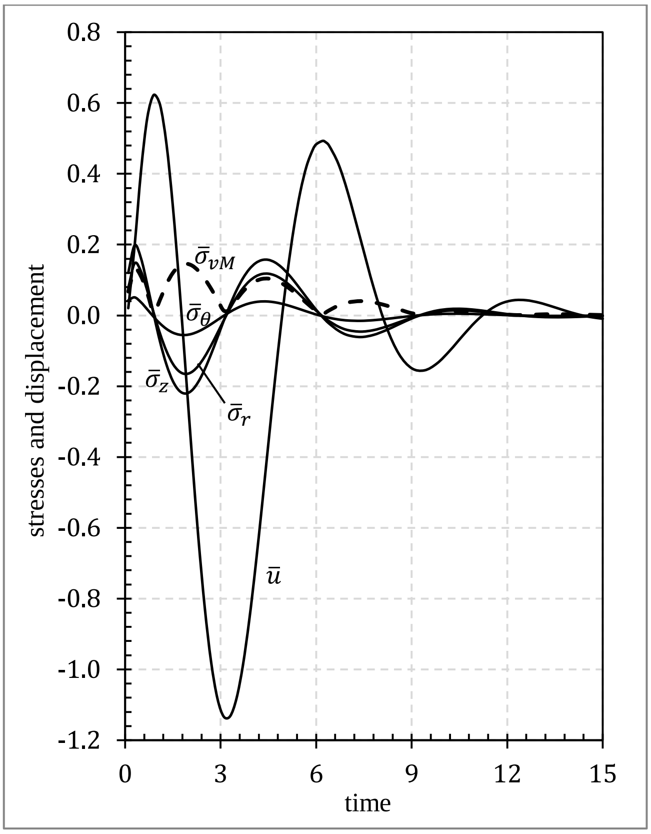

Fig. 3b. Variations of the stress components, the von Mises stress and the displacement with time at $\bar{r} = 0.5$ based on B.C. 2 for the parameter set $\nu = 0.3$, $A = 2.0$ and $\bar{\alpha} = 1.75$.

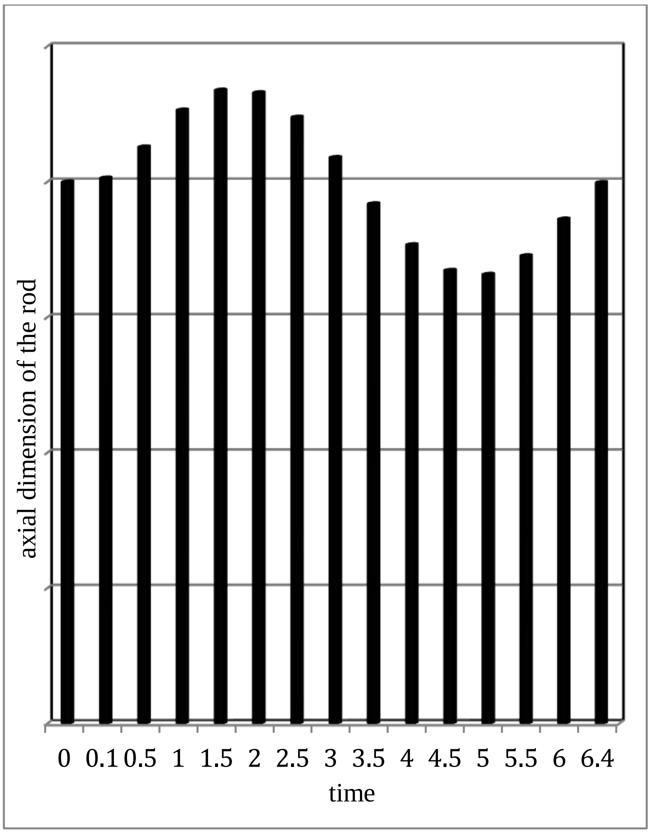

Fig. 4. The change in the height of the cylinder with time based on B.C. 1 for the parameter set $\nu = 0.3$, $A = 2.0$ and $\bar{\alpha} = 1.75$.

REFERENCES

[1] Hahn, W.D. and Ozisik, M.N., Heat Conduction. John Wiley & Sons, New Jersey, 2012.

[2] Timoshenko, S. and Goodier, J.N., Theory of Elasticity. McGraw-Hill, New York, 1970.

[3] Ugural, A.C. and Fenster, S.K., Advanced Mechanics of Materials and Applied Elasticity, Prentice-Hall, London, 2012.

[4] I. Dincer, “Simplified solution for temperature distributions of spherical and cylindrical products during rapid air cooling”,. Energy Conversion and Management-An International Journal 36 (12), 1175-1184, 1995.

[5] X. Lu, P. Tervola, and M. Viljanen, “Transient analytical solution to heat conduction in multi-dimensional composite cylinder slab”, International Journal of Heat and Mass Transfer 49 (5-6), 1107-1114, 2006.

[6] C.D. Rakopoulos, K.A. Antonopoulos, D.C. Rakopoulos, and E.G. Giakoumis, “Investigation of the temperature oscillations in the cylinder walls of a diesel engine with special reference to the limited cooled case”, International Journal of Energy Research 28 (11), 977-1002, 2004.

[7] H. Mahmoudi, and G. Atefi, “Analytical solution for thermal stresses in a hollow cylinder under periodic thermal loading”, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science 226 (7), 1705—1724, 2011.

[8] R. Trostel, “Instationare Warmespannungen in Hohlzylindern mit Kreisringquerschnitt”, Archive of Applied Mechanics 2, 1-26, 1956.

[9] Verein Deutscher Ingenieure, 1974. Werkstoffe und Bauteilfestigkeit. VDI Verlag, Dusseldorf.

[10] A.R. Shahani, and S.M. Nabavi, “Analysis of the thermoelasticity problem in thick-walled cylinders”, Proceedings of the 10th Annual (International) Mechanical Engineering Conference, Tehran. Iran: ISME 4, 2056-2062, 2002.

[11] S.Y. Lee, and C.C. Huang, “Analytic solutions for heat conduction in functionally graded circular hollow cylinders with time-dependent boundary conditions”, Hindawi Publishing Corporation Mathematical Problems in Engineering, 2013.

[12] H. Fazeli, M.A. Abdous, H. Karabi, N. Moallemi, and M. Esmaeili, “Analysis of transient heat conduction in a hollow cylinder using Duhamel theorem”, International Journal of Thermophysics 34 (2), 350-365, 2013.

[13] A.E. Segall, “Useful polynomials for the transient response of solids under mechanical and thermal excitations”, International Journal of Mechanical Sciences 44 (4), 809-823, 2002.

[14] A.E. Segall, “Transient analysis of thick-walled piping under polynomial thermal loading”, Nuclear Engineering and Design 226 (3), 183-191, 2003.

[15] A.E. Segall, "Thermoelastic stresses in an axisymmetric thick-walled tube under an arbitrary internal transient", ASME Journal of Pressure Vessel Technology 126 (3), 327-332, 2004.

[16] A.N. Eraslan, E. Varli, "Elastic Response of a Heat Generating Rod at a Variable Generation Rate", Paper presented at 12th International Conference of Numerical Analysis and Applied Mathematics (ICNAAM), 22 - 28 September, Rhodes, Greece, 2014.

[17] Mendelson, A., Plasticity: Theory and Application. The Macmillan Company, New York, 1968.