

# Experimental Determination and Simulation Of Voltage Spectral Densities (VSD) And Corner Frequency Of Thermal And Pink Noises

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**Abstract**—his study estimated and simulates the Voltage Spectral Densities (VSD) as a function of Frequency of an excess noise contains Thermal and Flicker noises. The highest spectrum was found at 60Hz and 600Hz for Thermal and Flicker noises respectively. The work also determines the corner frequency  $F_c$  in 60 $\Omega$  Resistor. The result shows that the corner frequency which is independent of Temperature is 270GHz while the time constant  $\tau$ , was  $5.90 \times 10^{-13}$ s. The frequency is well beyond the frequency that any integrated circuit can operate and therefore the spectrum can be considered White for all practical purposes. Hence, up to quantum frequency limit, our work satisfied with Nyquist theorem.

**Keywords**—Corner frequency, Pink noise, Spectral densities, Thermal Noise, Quantum frequency

## 1. INTRODUCTION

Noise may be defined as any signal which does not convey useful information. It is introduced in measurement and control systems externally by mechanical coupling and coupling from electromagnetic fields. Noise can also be internally introduced within a system by noise sources such as component level noise such as Johnson noise and Shot noise. Noise is any unsteady component of a signal which causes the instantaneous value to differ from the true value [1]. In electrical signals, noise often appears as a highly erratic component superimposed on the desired signal. It is often random in nature and thus it is described in terms of

its average behaviors. This is described in terms of its power spectral density,  $(\varphi_x(f))$ , which shows how the average signal power is distributed over a range of frequencies or in terms of its average power. The average signal power is the power dissipated when the signal voltage is connected across a 1 $\Omega$  resistor,[1]

Johnson noise which is sometimes referred to as Thermal noise is generated by thermal agitation of electrons in a conductor, that is, the thermal fluctuations of the electromagnetic field inside a resistor produce a corresponding random fluctuation in the voltage [2]. Simply put, as a conductor is heated, it will become noisy. Heat disrupts the electrons' response to an applied potential. It adds a random component to their motion. Thermal noise only stops at absolute zero. The actual cause of Johnson noise is blackbody radiation within the conductor.

Flicker noise is a type of noise with a 1/f or pink power density spectrum. It occurs in almost all electronic devices and can show up with a variety of other effects. Flicker noise is often characterized by the corner frequency,  $F_c$ . The corner frequency is the boundary in a system frequency response at which energy flowing through the system begins to be reduced (Attenuated or Reflected). Flicker noise is found in carbon composition resistors. It is often referring to as excess noise because it appears in addition to the Thermal noise [3].

It is meaningless to talk about the noise present at exactly one frequency, since there is zero power in any particular frequency. Instead, power is spread out over a frequency range and the figure of merit is the power spectral density that is the power per unit

bandwidth in a signal. This is found by integrating the power spectral density over the range.

## 2. THEORETICAL CONSIDERATIONS OF THERMAL AND PINK NOISES.

In a narrow band of frequencies,  $\Delta F$ , the root mean square noise voltage of a Thermal noise across any resistance  $R$  in frequency range  $B$  is given by [3]

$$(V_j)^2 = 4KTRB\Delta F \quad (1)$$

Where  $k$  is the Boltzmann constant,  $R$  is the Resistance in ohms and  $T$  is the temperature in degree Kelvin for the resistor,  $B$  is the frequency range. If a sinusoidal input signal  $V_{in}$  of frequency,  $f$  becomes  $V_{out}$  after amplification and filtering, then the effective gain  $g(f)$  is defined by

$$g(f) = \left( \frac{V_{out}}{V_{in}} \right) \quad (2)$$

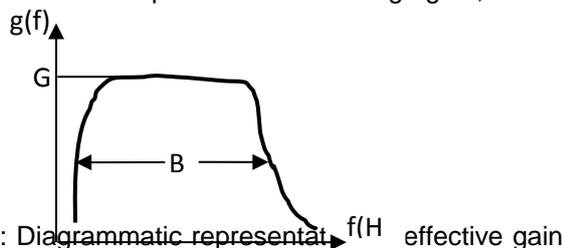
Thus, after amplification the contribution  $dV_j^2$  to the Thermal noise in a differential frequency interval becomes

$$dV_{out}^2 = g(f)^2 dV_j^2 \quad (3)$$

After integrating over the frequencies of the pass band we obtained

$$V^2_j = 4RKT \int_0^\infty g(f)^2 df = 4RKT(G^2B) \quad (4)$$

Where  $(G^2B)$  is essential an effective bandwidth  $B$  multiplied by the square of some average gain,  $G$ .



**Fig. 1:** Diagrammatic representation of effective gain frequency.

The root means square noise voltage and current of a Flicker noise is given by [3]

$$E_n = K_e \sqrt{\left( \ln \frac{f_{max}}{f_{min}} \right)} \quad (5)$$

$$I_n = K_i \sqrt{\left( \ln \frac{f_{max}}{f_{min}} \right)} \quad (6)$$

Where:  $K_e$  and  $K_i$  are proportionality constants representing  $E_n$  and  $I_n$  at 1Hz;

$f_{max}$  and  $f_{min}$  are the maximum and minimum frequencies in Hz.

## 3. EXPERIMENTAL SET UP AND MEASUREMENT

The measurement setup was configured as shown in Figure 2. The measurement chain consists of a low-noise differential amplifier, a band-pass filter, and a voltmeter. The noise measured is very small, typically on the order of microvolt. The problem of electrical interference was minimized in this work by making all the cables be as short as possible.

The filter we use is a Krohn-Hite 3BS8TB-1k/50kg band-pass filter. The filter has fixed frequency band pass range of 100Hz to 3KHz. it has 8 poles, the equivalent of 8 simple filters in series, so the drop off outside of the cutoff frequencies should be quite sharp. The output of the amplifier was connected to the positive input of the Krohn-Hite filter. The output of filter was connected to the digital oscilloscope.

The digital Oscilloscope measures the root mean square voltage of both periodic and random signals. The differential amplifier was set to a nominal gain of 1000, the micro volts noise signals are amplified sufficiently to be measured in the millivolt range of the oscilloscope. Overall amplification of the amplifier/filter combination was determined by feeding a sinusoidal test signal with a Root mean square voltage  $V_i$  in the millivolt and measured the RMS voltage  $V_o$  of the filter output using the digitizing oscilloscope. The gain of the system at the frequency of the test signal is  $(f) = \frac{V_o}{V_i}$ .

### 3.1 PROCEDURE

The oscilloscope was set to display both the  $V_i$  and  $V_o$  sinusoids and the oscilloscope voltage measuring options was used to measure RMS voltage.

The bandwidth used for this work is 2300Hz as in figures (4-6). The inputs are set to AC coupling in order to eliminate DC offset in the signals measuring.

The RMS noise voltage was measured by press 'Display'. The digital oscilloscope amplitude and sweep-speed control was adjusted for the cycles of the sinusoid to appear on the screen. The results are shown in figure (4-6), this is used to determine the upper and lower frequencies and the difference gives the bandwidth used for this work.

The resistors (20 $\Omega$ , 40 $\Omega$  and 60 $\Omega$ ) were mounted as shown in figure 2.

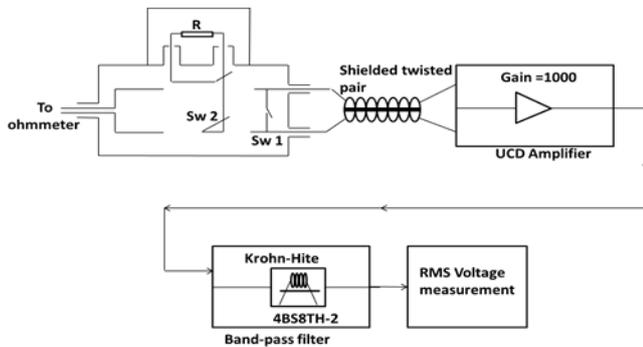
Starting with 20 $\Omega$ , this resistor was covered with metal beaker to shield the input of the system from electrical interference. The measurement was repeated with the shorting switch across the conductor alternately opened and closed. The

measure of the mean square Johnson noise is  $V^2 = V_R^2 - V_S^2$  (5)

Where  $V_R$  and  $V_S$  are the RMS voltages measured with the shorting switch open and closed, respectively at room temperature.

The procedure was repeated using resistors 40 and 60Ω in turns.

Flicker noise was measured experimentally in 60Ω resistor.



**Fig. 2:** Diagrammatic representation of the block diagram used for measuring the noises



**Plate 1:** Snapshot of the apparatus used for measuring the Noises.

### 3.2 SIMULATION OF SPECTRAL DENSITY OF THERMAL AND FLICKER NOISES

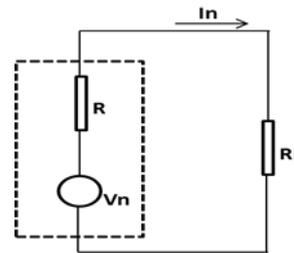
The spectral density of radiated power of thermal noise is given by [4]

$$\frac{dP}{dV} = \frac{hV}{\left[ e^{\frac{hV}{KT}} - 1 \right]} \quad (7)$$

At low frequencies  $hV \ll KT$

$$\text{Hence, } \frac{dP}{dV} \approx \frac{hV}{\left[ 1 + \frac{hV}{KT} \right]} = KT \quad (8)$$

Different resistors of resistance  $R$  was used to determine the power transfer of thermal noise. The resistance of the resistors  $R$  whose thermal noise gives rise to a noise voltage  $V_n$  is simulated using the circuit diagram shown in figure 3.



**Fig.3:** Circuit diagram used to simulate Noise Voltages.

The power dissipated in the load resistor  $R_L$  is given by

$$\frac{V_n^2 L}{R_L} = I_n^2 R_L = \frac{V_n^2 R_L}{(R+R_L)^2} \quad (9)$$

The maximum power transfer occurs when  $R_L = R$  and power transferred to  $R_L = KT B$

Hence, the power spectral density is

$$S = 4KTR \quad (10)$$

While the voltage spectral density is

$$N = (4KTR)^{1/2} \quad (11)$$

The noise voltage spectral density (Wiener and Khintchine) theorem shows that

$$S(f) = \frac{4K_B T}{R} \left( \frac{1}{1 + \omega^2 \tau^2} \right) \quad (12)$$

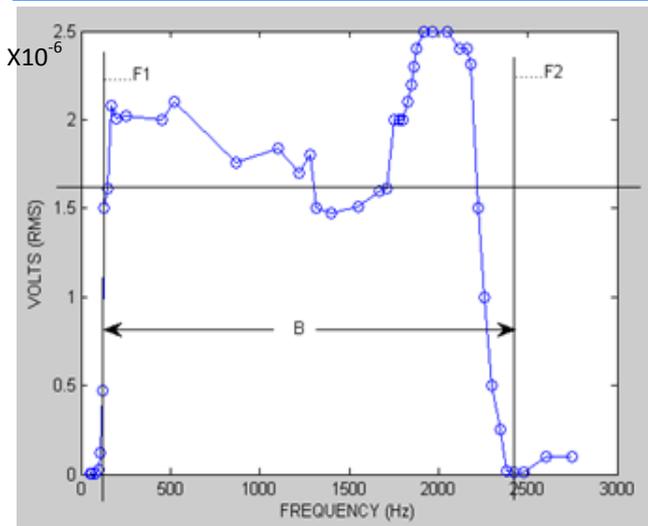
Where  $R = \frac{L}{\sigma A}$ . The frequency dependence in equation (12) is known as Lorentzian characteristics with time constant,  $\tau$ , this set the corner frequency [5] at  $1/2\pi\tau$ .

### Results and discussion

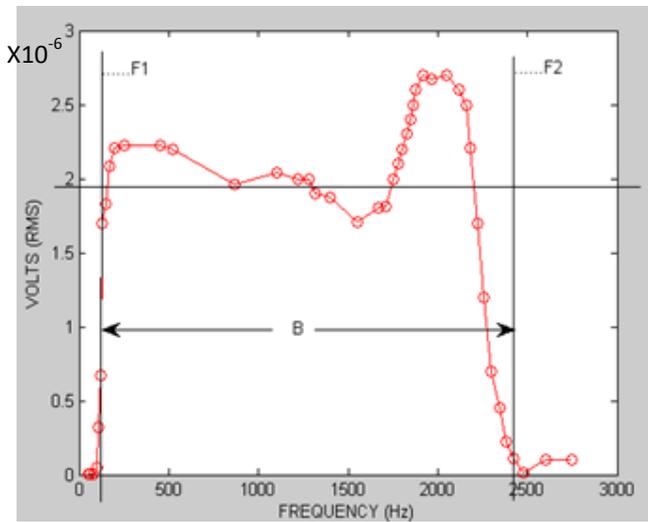
Figure 4 to 6 shows the noise voltage against frequency in three different resistors. The bandwidth in our measurement is the same 2320Hz but the peak voltages are 2.50μV, 2.75μV and 3.00μV respectively in 20Ω, 40Ω and 60Ω used.

Fig. 7 and 8 shows the experimental measurement of Thermal and Pink noises in 60Ω resistor. In figure 8, the noise voltage is directly proportional to Temperature but it is inversely proportional to frequency as shown in figure 8.

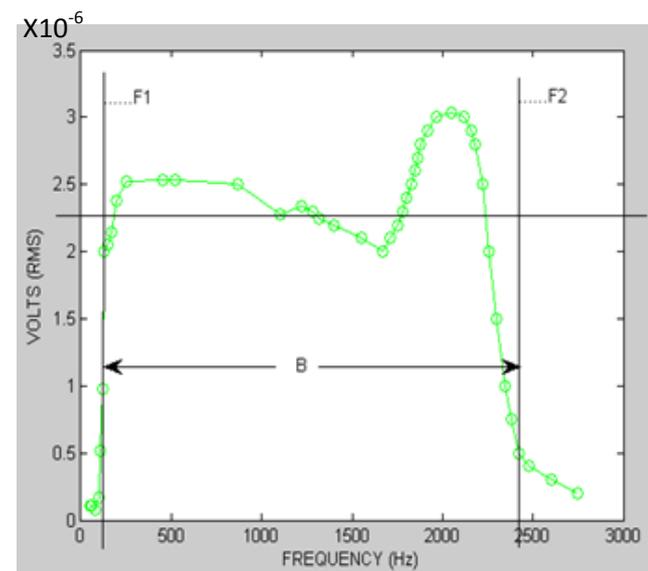
Our results shows that the noise voltage spectral density is independent of frequency and the noise power is independent of the resistance – it only depends on the temperature. At low frequencies, the spectral densities are independent of frequency and for a total bandwidth  $B$ , the noise power that can be transferred to an external device  $P_n = KTB$ .



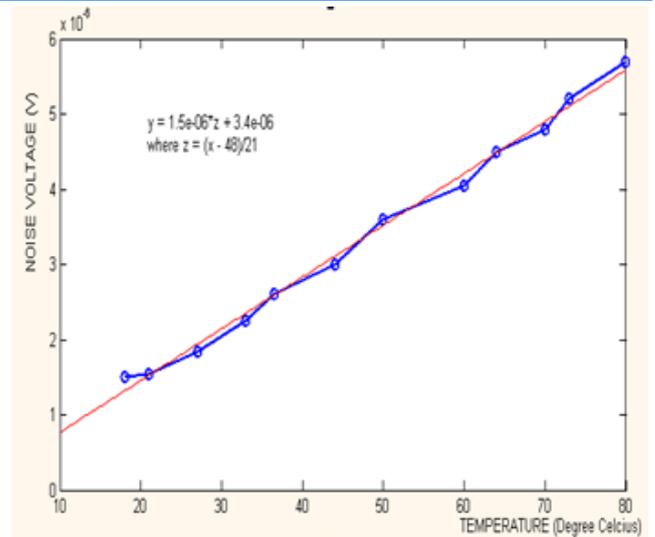
**Fig.4:** Noise voltage measured against frequency in 20Ω Resistor.



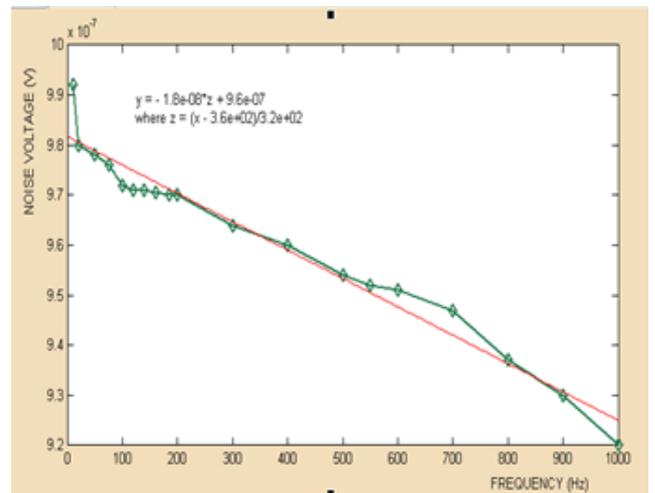
**Fig.5:** Noise voltage measured against frequency in 40Ω Resistor.



**Fig.6:** Noise voltage measured against frequency in 60Ω Resistor.



**Fig.7:** Thermal Noise voltage measured at different temperatures.



**Fig.8:** Flicker Noise voltage measured at different frequencies.

A fitting straight line of Polynomial of "order one" was used to propose a model for both noise voltages measured.

**For Thermal Noise:**

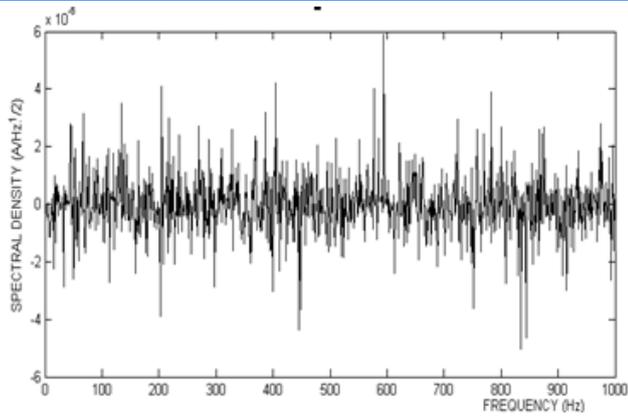
$$V = 1.5e^{-0.6} z + 3.4e^{0.6} \quad (13)$$

Where,  $z = (T - 48)/320$

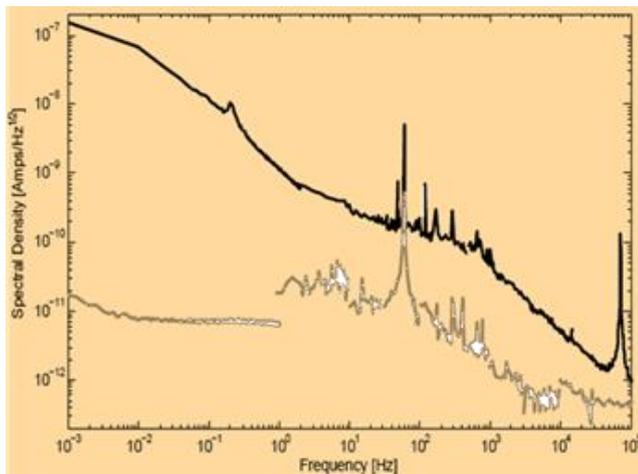
**For Pink Noise:**

$$V = -1.8e^{08} z + 9.6e^{-7} \quad (14)$$

Where,  $z = (F - 360)/320$



**Fig.9:** Flicker noise spectral density against frequency.

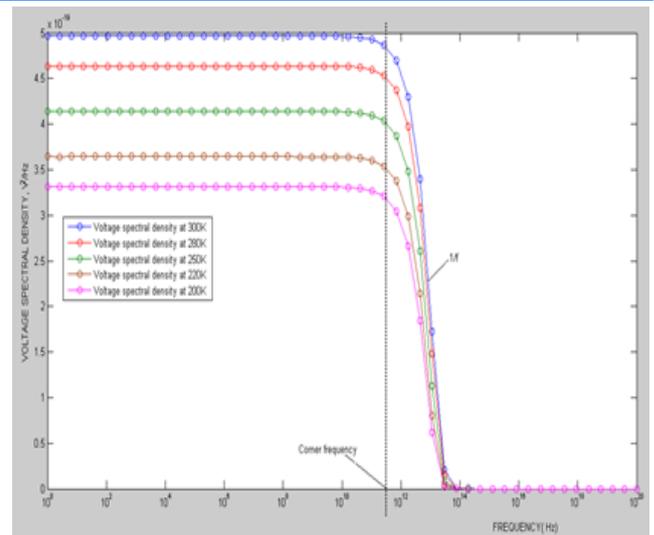


**Fig. 10:** Thermal Noise Spectral density against frequency.

There are several prominent peaks in the Noise spectrums for both Flicker and Thermal noises as shown in figure 9 and 10. In Figure 9 there is a peak 60Hz and near a few tenths of a Megahertz for Thermal but the highest peak is at 60Hz.

In Flicker noise, the highest peak is around 600Hz.

The corner frequency (cut-off frequency) which depends upon the details of electron transport in the 60Ω resistor is around 270GHz as in fig. 11 below.



**Fig. 11:** Simulation of voltage spectral densities of Thermal and Pink noises.

**CONCLUSION:**

We studied experimentally and computationally Johnson and Flicker noises in a Resistor. For Thermal noise, we found that the spectral densities of the fluctuating voltage across the 60Ω resistor depend linearly on temperature as predicted by Nyquist. The Flicker noise voltage measured was found to be independent of temperature but inversely proportional to the frequency. We compute the spectral densities for both noises. The work shows that there has to be a cut-off to the Thermal noise spectrum in any physical device. The corner frequency (cut-off frequency) which depends upon the details of electron transport in the 60Ω resistor is around 270GHz which is well beyond the frequency that any integrated circuit can operate and therefore the spectrum can be considered White for all practical purposes. However, the transport details also reflect in the high-frequency impedance of the resistor, so the Nyquist theorem in the form  $S_p(f) = K_B T$ , is satisfied up to the quantum frequency limit.

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