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Vague Rings And Vague Fields

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***Abstract*—In this paper we introduce the concept of vague ring and vague field, vague ideals are introduced and various elementary properties of these concepts are investigated.**

***Keywords*—set, vague additive group, vague ring, vague fields.**

1. Introduction

The concept of fuzzy set was introduced by Zadesh. Since then this idea has been applied to other algebraic structures such as groups, rings etc. With the development of fuzzy set, it is widely used in many fields meanwhile; the deficiency of fuzzy sets is also attract attention. Such as fuzzy set is single function, it cannot express the evidence of supporting and opposing. Based on this reason, the concept of vague set [3] introduced by Gau in 1993. Vague sets as an extension of fuzzy sets, the idea of vague sets is that the membership of every element can be divided into two aspects including supporting and opposing. The notion of fuzzy groups defined by Rosen field [7] is the first application fuzzy set theory in algebra. Ranjit Biswas [1] initiated the study of vague algebra by studying vague groups. We [8] introduced the concept of vague additive groups and its properties.

The object of this paper is to make a study of vague rings, vague fields and its properties.

2. Preliminaries

In this section we collect important results which were already proved for our use in the next section.

Definition2.1: [3] A vague set A in the universal of discourse X is characterized by two membership functions given by:

(1) A truth membership function $t_A : X \to [0,1]$ and

(2) A false membership function $f_A : X \to [0,1]$,

Where $t_A(x)$ is a lower bound of the grade of membership of x derived from the "evidence for x", and $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x" and $t_A(x) + f_A(x) \leq 1$.Thus the grade of membership of x in the vague set A is bounded by subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. The vague set A is written as $A = \{\langle x, [t_A(x), f_A(x)] \rangle / x \in X\}$.

Where the interval $[t_A(x), 1 - f_A(x)]$ is called the value of x in the vague set A and denoted by $V_A(x)$.

Definition2.2:[3] A vague set A of a universe X with $t_A(x) = 0$ and $f_A(x) = 1$ for all $x \in X$, is called the zero vague set of X.

Definition2.3: [3] A vague set A of a universe X with $t_A(x) = 1$and $f_A(x) = 0$ for all $x \in X$, is called the unit vague set of X.

Notation [3]: Let I[0, 1] denote the family of all closed subintervals of [0, 1]. If $I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$ are two elements of I [0, 1], we call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. We define the term imax to mean the maximum of two interval as $imax[I_1, I_2] = [max\{a_1, a_2\}, max\{b_1, b_2\}]$. Similarly, we can define the term imin of any two intervals.

Defenition2.4 [8]: Let (X, +) be a group. A vague set A of X is called a vague additive (briefly VAG) group of X if the following conditions are satisfies:

$\forall x, y \in X$, $V_A(x + y) \leq imax\{V_A(x), V_A(y)\}$, i.e.,

(1). $t_A(x + y) \leq imax\{t_A(x), t_A(y)\}$ and $1 - f_A(x + y) \leq imax\{1- f_A(x), 1-f_A(y)\}$.

(2). $t_A(-x) \leq t_A(x)$ and $1- f_A(-x) \leq 1 - f_A(x)$.

3. Vague rings

Definition 3.1: Let X be a ring and R be a vague set of X. Then R is a vague ring of X if the following conditions are satisfied:

(1). $V_R(x + y) \leq imax\{V_R(x), V_R(y)\}$,
(2). $V_R(-x) \leq V_R(x)$,

(3). $V_R(xy) \geq$ imin { $V_R(x)$, $V_R(y)$} for all x, y ϵX

Example 3.2: Let X = Z = {the set of integers} be the ring under usual addition and multiplication. A vague set R of X defined as

R = $\{\langle 0,[0.5,0.2]\rangle$, $\langle positive\ integer,[0.6,0.1]\rangle, \langle negative\ integer,[0.6,0.1]\rangle\}$.

Clearly R is a vague ring of X.

Example 3.3: Let Z = {0, 1, 2} be the ring under usual addition modulo and multiplication modulo.

A vague set R of defined as

R = $\{\langle 0,[0.6,0.2]\rangle,\langle 1,[0.7,0.1]\rangle,\langle 2,[0.7,0.1]\rangle\}$.

Clearly R is a vague ring of Z.

Example 3.4: Let X = $\frac{Z_2}{\langle 1+x+x^2\rangle}$ = {0, 1, α, 1 + α } is a ring with respect to addition modulo and multiplication modulo 2 [6]. The vague set R of X defined by R= { $\langle 0,[0.5,0.3]\rangle$, $\langle 1,[0.6,0.2]\rangle,\langle \alpha,[0.7,0.1]\rangle,\langle 1+\alpha,[0.7,0.1]\rangle\}$ is a vague ring of X.

Proposition 3.5: If R is a vague ring of a ring X then for all x ϵ X, we have $V_R(-x) = V_R(x)$.

Theorem 3.6: Let R be a vague set of a ring X. Then R is a vague ring of X if and only if

(1). $V_A(x - y) \leq$ imax{$V_A(x)$, $V_A(y)$},

(2). $V_A(xy) \geq$ imin{$V_A(x)$, $V_A(y)$} for all x, y ϵX

Proof: Let R be a vague ring of X. Then we have

(1). $V_R(x - y) \leq$ imax{ $V_R(x)$, $V_R(-y)$}

= imax{ $V_R(x)$, $V_R(y)$}

(2). It follows from the definition of vague ring.

Conversely, suppose that R be a vague set of X and satisfies the above two inequalities. And also $V_R(-x) = V_R(x)$ from the proposition 3.5.

Consider $V_R(x + y) = V_R(x - (-y)) \leq$ imax{ $V_R(x)$, $V_R(-y)$} $\leq$ imax{ $V_R(x)$, $V_R(y)$}.

So R is a vague ring.

Definition 3.7: Let R be a vague ring of the ring (X, +, .). Then a vague set S of X such that S $\subseteq$ R is said to be a vague sub ring of R if the following conditions are satisfied:

(1). $V_A(x - y) \leq$ imax {$V_A(x)$, $V_A(y)$},

(2). $V_A(xy) \geq$ imin {$V_A(x)$, $V_A(y)$} for all x, y ϵX.

Definition3.8: Let f be a mapping from a set X into a set Y. Let B be a vague set in Y. Then the inverse image of B, $f^{-1}(B)$ is the vague set in X by $Vf^{-1}(B)(x) = V_B(f(x))$ for all x ϵ X.

Definition 3.9: Let f be a mapping from a set X into set Y. Let A be a vague set in X. Then the image of A, f (A) is the vague set in Y by

$V_{f(A)}(y)$ = isup { $V_A(z)$ / z ϵ $f^{-1}(y)$, if $f^{-1}(y) \neq \phi$}

= [0, 0] otherwise.

Theorem3.10: Let X and Y be two rings and f be a homomorphism from X into Y. Let R be a vague ring of Y, then the inverse image $f^{-1}(R)$ of R is a vague ring

Proof: For all x, y ϵ X,

$Vf^{-1}(R)(x - y) = V_R(f(x - y))$

$= V_R(f(x) - f(y))$

$\leq$ i max {$Vf^{-1}(R)(x)$, $Vf^{-1}(R)(y)$}

And $Vf^{-1}(R)(x y) = V_R(f(x y))$

$= V_R(f(x) f(y))$

$\geq$ i min {$Vf^{-1}(R)(x)$, $Vf^{-1}(R)(y)$}

Hence the set $f^{-1}(R)$ is a vague ring of X.

Theorem3.11: Let X and Y be two groups and f be a homomorphism from X into Y. Let R be a vague ring of X that has sup property, then the image f(R) of R is a vague ring of Y.

Proof: Let u, v ϵ Y if either $f^{-1}(u)$ or $f^{-1}(v)$ is empty then the properties mentioned in theorem 3.6.

Suppose neither $f^{-1}(u)$ nor $f^{-1}(v)$ is empty.

Let $V_{f(R)}(u)$ = isup { $V_R(x)$ / x ϵ $f^{-1}(u)$ } and

$V_{f(R)}(v)$ = isup { $V_R(y)$ / y ϵ $f^{-1}(v)$ }

Then $V_{f(R)}(u - v)$ = isup { $V_R(r)$ / r ϵ $f^{-1}(u - v)$ }

$\leq$ i max{$V_R(x)$, $V_R(y)$}

= imax {$V_{f(R)}(u)$, $V_{f(R)}(v)$}

Thus the image f(R) of R is a vague ring of Y.

Definition 3.12: Let X be a ring. A be a subset of a vague ring R. Then

(1). $V_A(x - y) \leq$ imax {$V_A(x)$, $V_A(y)$} for all x, y ϵX,

(2). $V_A(rx) \leq V_A(x)$ for all r,x ϵX.

Then A is called a vague ideal of a vague ring R.

Example 3. 14: Clearly the set S = $\{\langle 0,[0.5,0.3]\rangle$, $\langle 1,[0.6,0.2]\rangle,\langle \alpha,[0.7,0.1]\rangle,\langle 1+\alpha,[0.7,0.1]\rangle\}$ is a vague ideal of the vague ring R in example (3.4).

Theorem3.13: The intersection of two vague ideals of a vague ring R of a ring X is a vague ideal of R.

Proof: Suppose A and B are two vague ideals of R and x, y ϵX.

We have $V_{A \cap B}(x - y)$ = min {$V_A(x-y)$, $V_B(x-y)$}

$\leq$ min { max{$V_A(x)$, $V_A(y)$}, max { $V_B(x)$, $V_B(y)$} }

= max {min {$V_A(x)$,$V_B(x)$}, min {$V_A(y)$, $V_B(y)$} }

= max {$V_{A \cap B}(x)$), $V_{A \cap B}(y)$}

Similarly, we can prove that $V_{A \cap B}(rx) \leq V_{A \cap B}(x)$

Therefore A ∩ B is a vague ideal of R.

Definition3.14: Let X be a field and F be a vague set of X. Then F is a vague field of X if the following conditions are satisfied:

(1). $V_F(x + y) \leq$ imax{ $V_F(x)$, $V_F(y)$},

(2). $V_F(-x) \leq V_F(x)$,

(3). $V_F(xy) \geq$ imin{ $V_F(x)$, $V_F(y)$}

for all x, y ϵX

(4). $V_F(x^{-1}) \geq V_F(x)$ for all 0 $\neq$ x ϵX.

Example 3.15: X = { 0, 1, α ,1 + α}is a field with respect to $+_2$ and X_2 as follows:

$.+_2$	0	1	α	$1+\alpha$
0	0	1	α	$1+\alpha$
1	1	0	$1+\alpha$	α
α	α	$1+\alpha$	0	1
$1+\alpha$	$1+\alpha$	α	1	0

X_2	0	1	α	$1+\alpha$
0	0	0	0	0
1	0	1	α	$1+\alpha$
α	0	1	$1+\alpha$	α
$1+\alpha$	0	$1+\alpha$	1	α

Clearly X is a group with respect to addition modulo 2 [6]. The vague set A of X defined by

A= $\{\langle 0,[0.5,0.3]\rangle, \langle 1,[0.6,0.2]\rangle, \langle \alpha,[0.7,0.1]\rangle, \langle 1+\alpha,[0.7,0.1]\rangle\}$ is a vague field of X.

Proposition3.16: If F is a vague field of a field X Then

(1) $V_F(-x) = V_F(x)$ for all $x \in X$,

(2) $V_F(x^{-1}) = V_F(x)$ for all $0 \neq x \in X$.

Proof: We have by definition $V_F(-x) \leq V_F(x)$ for all $x \in X$ …………(1)

Also $V_F(-(-x)) \leq V_F(-x)$, that is $V_F(x) \leq V_F(-x)$………………..(2)

From (1) and (2), $V_F(-x) = V_F(x)$.

Similarly we can prove the second condition also.

Theorem 3.17: Let X be a field and F be a vague field of X. Then F is a vague field of X iff

(1) $V_F(x-y) \leq \max\{V_F(x), V_F(y)\}$ for all $x, y \in X$,

(2) $V_F(xy^{-1}) \geq \min\{V_F(x), V_F(y)\}$ for all $x, 0 \neq y \in X$.

Proof: $V_F(x-y) = V_F(x+(-y)) \leq \max\{V_F(x), V_F(-y)\} \leq \max\{V_F(x), V_F(y)\}$

for all $x, y \in X$.

Similarly we can prove the second one also.

Theorem3.18: Let X and Y be fields and f be a homomorphism from X into Y. Let F be a vague field of Y then the inverse image $f^{-1}(F)$ of F is a vague field of X.

Proof: For all $x, y \in X$, $V_{f^{-1}(F)}(x-y) = V_F[f(x-y)] = V_F[f(x)-f(y)]$

$\leq \max\{V_F(x), V_F(y)\}$

$= \max\{V_{f^{-1}(F)}(x), V_{f^{-1}(F)}(y)\}$.

Also for all $x \in X$, $0 \neq y \in X$, $V_{f^{-1}(F)}(xy^{-1}) = V_F[f(xy^{-1})] = V_F[f(x)f(y^{-1})]$

$\geq \min\{V_F(x), V_F(y)\}$

$= \max\{V_{f^{-1}(F)}(x), V_{f^{-1}(F)}(y)\}$.

So the image $f^{-1}(F)$ of f is a vague field of X.

Theorem 3.19: The intersection of a family of vague fields is a vague field.

Proof: Let $F = \cap F_i$ is a family of fields.

$V_F(x-y) = \inf_{i\in I}\{V_F(x-y)\}$

$\leq \inf_{i\in I}\{\max(V_F(x), V_F(y))\}$

$= \max\{\inf_{i\in I} V_F(x), \inf_{i\in I} V_F(y)\}$.

Similarly we prove that $VF(xy^{-1}) \geq \min\{V_F(x), V_F(y)\}$.

Hence the theorem.

Conclusion: In this paper the concept of vague ring and vague field, vague ideals are introduced and various elementary properties of these concepts are investigated.

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