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# Stabilization Of Networked Control Systems Through Packet Dropout Via MJLSs

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***Abstract*—This paper investigates modeling, analysis and design of Networked Control Systems (NCSs).They which are of great interest due to advantages presented here, but are technically complex because of the challenges of this class of systems such as delay and packet-dropout. To study, NCSs with packet dropouts, Markov chains are used to describe historical behaviors of packet dropouts. Therefore, general models are derived under single packet transmission protocols. Furthermore, controllers are designed to stabilize the resulting closed-loop systems. Using this method, an inverted pendulum, which was first controlled in a network with delays, is controlled in a network with packet dropouts. Simulation results show the effectiveness of this method.**

***Keywords—Network Control System, Markovian Jump Linear System, Packet dropout, Stability***

## I. Introduction

In modern industrial and commercial systems, NCS has gained increasing attention due to its cost effectiveness and flexible applications. In other words, the introduction of NCS architectures can improve flexibility and efficiency of common control systems through reducing wiring and thus reducing the installation, reconfiguration and maintenance time and costs.

Networked control systems are spatially distributed systems in which communication between sensors, actuators and controllers is supported by a network (Fig.1). Such a network can be a wired communication network (which is applicable in today's industry) or digital communication network (including unwired networks).

Control systems in large and small scales include sensors, actuators and controllers. Traditionally, linking different parts of control systems is done through wired networks. However, this configuration has its own disadvantages such as high costs and expanding physical setups and functionality.

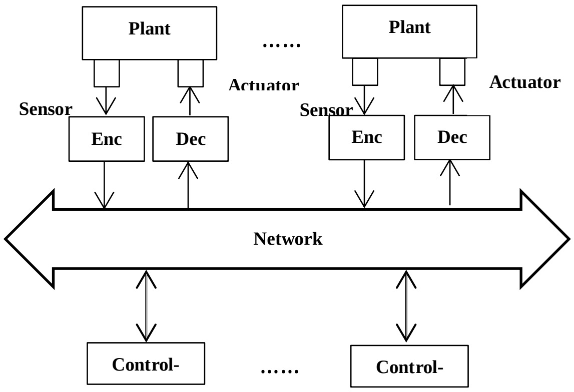


*Fig. 1 a schematic of Network Control System (NCS)*

Hence, traditional control systems are no longer suitable to meet new requirements. Consequently, NCSs have been finding application in a broad range of areas such as mobile sensor networks [2], remote surgery [3], haptic collaboration over the internet [4], automated highway systems, and unmanned aerial vehicles [2-4]. Because of these advantages, many industrial companies have shown interest in applying networks for remote industrial control purposes and factory automation. For example, Control Area Network (CAN) was originally developed in 1983 by the German company Bosch for use in car industries and many other industrial control applications. Other examples of industrial networks are Profibus, Foundation Fieldbus and Device-Net. Most of these protocols are typically reliable and robust for real-time control purposes [5-6].

During these days, the technologies on computer networks, (especially Ethernet) have also progressed rapidly. By decreasing cost, increasing speed and widespread usage, these networks have became major competitors for industrial networks in control applications. Furthermore, the popularity of internet has brought these networks into various organizations. Thus, control applications can utilize these networks to connect to the internet in order to perform remote control from distances farther than in the past [7-8]. Regardless of types of networks used, the overall NCS performance is always affected by network delays. Delays are widely known to degrade the performance of a control system. Network delays may not significantly affect an open loop control system, however, an open loop control system structure may not be appropriate for many control applications such as teleoperation or remote surgery [9-10].

As stated above, data network technology has been widely used in industrial control systems. For decays, connecting different parts of control systems (sensor/actuator/controller) through communication network results in flexible architectures, and generally reduces installation and maintenance costs. So this kind of systems have been applicable more than before, and consequently have been finding application in a broad range of areas.

There are two general NCS configurations:

Direct structure and Hierarchical structure.

The NCS in the direct structure is composed of a controller and a remote system containing a physical plant, sensors and actuators. The controller and the plant are physically located at different locations and are directly linked by a data network (Fig. 2).

The controller signal is encapsulated in a packet and sent to the plant via the network. The plant then returns the system output to the controller by putting the information into a packet as well. Some examples of NCS in this structure include distance learning lab and a DC motor speed control system.

A hierarchical structure consists of a main controller and a remote closed-loop system as depicted in Fig. 3. Periodically, the main controller computes and sends the reference signal in a packet via a network, to the remote system. The remote system then processes the reference signal to perform closed-loop control, and returns the information to the main controller for network control. This structure is widely used in several applications including mobile robots [2] and teleoperation [3]. The use of either the direct structure or the hierarchical structure is based on application requirements and designer's preferences.

Although insertion of network in an NCS has a lot of advantages, it has some drawbacks which make the system analysis and design difficult, and bring many new challenges. The first challenge is network induced-

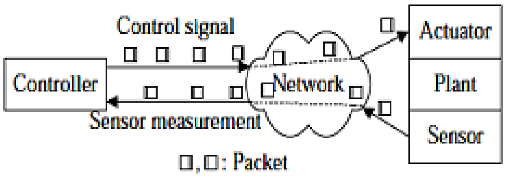


*Fig. 2 Direct structure [11]*

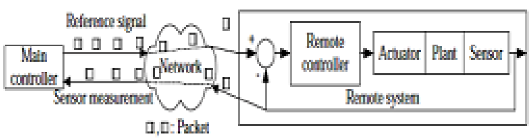


*Fig. 3 Hierarchical Structure [11]*

delays which are inevitable not only due to limited bandwidth, but also due to overhead in the communication nodes and in the network. The second challenge is packet dropouts. Typically, they result from transmission errors in physical network links.

In an NCS, in the case of packet dropout, the unreliable transmission path, single or multiple packet transmission, sampling rate, noise and different issues impact the analysis. Because of this, researchers have encountered packet dropout differently. On the other hand, the modeling of packet dropout affects the analysis.

Packet dropout is a random process. The number of dropped packets never exceeds the specified limit, and the best method of modeling this process is Markovian chain method, since this method describes the behavior of packet dropouts as well.

NCSs with packet dropouts was modeled with ADSs [12]. In [13] NCS with limited packet dropouts was modeled through switching systems, and a recursive method in modeling NCS with packet dropout is described in [14]. But, the effect of C/A packet dropouts was neglected due to complicated NCS modeling and analysis. In [15] the Markovian chain was introduced to model packet dropouts, and a jumping linear estimator was described to improve system's behavior.

Although, because of randomness of packet dropout process, most recent papers have paid much attention to modeling NCS by Markovian chain, they mostly require a prior knowledge of the transition probability of Markovian packet loss. As a result, some papers have recently presented methods, to estimate the uncertainties of transition matrix.

In [16-17] delay, stability analysis and robust controller design in NCSs have been reviewed. In [4] packet dropout has been modeled as a delay and thus, the NCS has changed to MJLS. Then stability analysis and controller design has been performed for this kind of systems.

In this paper, the stability and controller design of NSCs with packet dropout are considered. Both sensor-to-controller and controller to actuator packet dropout are included and described by two independent Markovian chains. Therefore the resulting closed-loop NCS can be transformed to a kind of MJLS with time delay, and thus, the results of this system can be used for analysis. Then the model of an inverted pendulum with delay, which controlled through network [18], has been controlled with packet dropout. Extensive simulation results presented here, have demonstrated the effectiveness of the proposed method.

Before problem statement, we noted out single and multiple transmission usage. Different networks are suitable for different types of transmission. Ethernet originally designed for transmitting information such as data files, can hold a maximum of 1500 bytes of data in a single packet [4]. Hence, it is more efficient to lump the sensor data into one packet and transmit it through single packet transmission. On the other hand, DeviceNet has a maximum 8-byte data field in each packet, thus sensor data often must be shuttled in different packets on DeviceNet. In these cases, analysis of NCSs with packet dropouts is done in two modes according to the type of networks. In this paper only single packet transmission is considered.

## II. Modeling and controller design of NCSs with single packet transmissions and known transition matrix

In this section, the NCSs with packet dropouts are analyzed. At first, packet dropouts are modeled, and then by this definition, the resulting closed loop system can be transformed to a standard "jump linear system" with time delays, which enables us to apply the results of jump linear systems.

Problem statement [4]

A schematic of an NCS with packet dropouts is shown in Fig. 1, where sensors, actuators and controllers are clock-driven.

The plant we consider here is linear time invariant (LTI) and thus we have:

$$x_{k+1} = \Phi x_k + \Gamma u_k \quad (1)$$

In this system, $x_k$ is the state and $u_k$ is the input. $\Phi$ and $\Gamma$ are known real constant matrices with appropriate dimensions. Suppose buffers are long enough to hold all the packets arrived, which will be picked up according to the last-in-first-out rule. For example, when a sensor data $x_k$ is lost, the controller will read out the most recent data $x_{k-1}$ from the buffer and utilize it as $\bar{x}_k$ to calculate the new control input, otherwise the new sensor data $x_k$ will be saved to the buffer and used by the controller as $\bar{x}_k$. Hence for the buffers we have:

$$u_k = \begin{cases} \bar{u}_k & \text{if transmitted successfully} \\ u_{k-1} & \text{otherwise} \end{cases}$$

$$\bar{x}_k = \begin{cases} x_k & \text{if transmitted successfully} \\ \bar{x}_{k-1} & \text{otherwise} \end{cases} \quad (2)$$

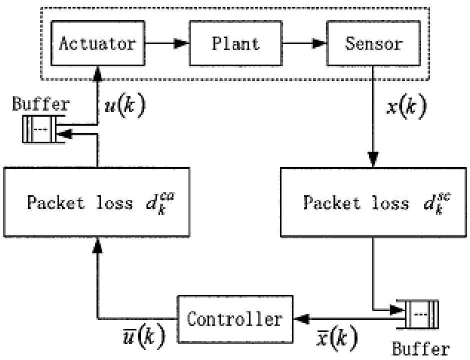


*Fig. 4 An NCS with data packet dropout via state feedback[4]*

Moreover, due to the bandwidth and packet size constraints of the network, the packet transmission can be classified into two types: single and multiple packet transmissions. By this classification, we will have new NCS models.

In the following section, analysis is done for single packet transmission.

## III. Modeling NCSs with single packet transmission [4]

In this section, the packet dropout is considered in sensor/controller and controller/actuator sides. Assume that $d_k^{sc}$ is the quantity of packets dropped at time k on the S/C side, which is calculated from the current time k to the last successful transmission (happened at time $(k - d_k^{sc})$), where $d_k^{ca}$ is the packet quantity dropped on the C/A side between the current time $k$ and its last successful transmission at time $k - d_k^{ca}$, and both of them are bounded. Thus we have:

$$0 \le d_k^{sc} \le d_1, 0 \le d_k^{ca} \le d_2$$

Where $d_1$ and $d_2$ are non-negative integers. We model $d_k^{sc}$ and $d_k^{ca}$as two homogeneous independent Markov chains, which take values in $S_1$= {0,1,…, $d_1$} and $S_2$= {0,1,…, $d_2$} with the generators $\Lambda_1=\rho_{ij}$ and $\Lambda_2=\lambda_{mn}$, respectively. The transition probabilities of $d_k^{sc}$(jumping from mode i t o j) and $d_k^{ca}$ (jumping from mode m to n) are defined by:

$$\rho_{ij} = Pr(d_{k+1}^{sc} = j | d_k^{sc} = i),$$

$$\lambda_{mn} = Pr(d_{k+1}^{ca} = n | d_k^{ca} = m), \quad (3)$$

Where $\rho_{ij}$ >0, $i, j \in S_1$, $\lambda_{mn}$,>0, $m, n \in S_2$.

It is obvious that the transition probabilities satisfy:

$$\sum_{j=0}^{d_1} \rho_{ij} = 1, \sum_{n=0}^{d_2} \lambda_{mn} = 1, \quad (4)$$

Which can be derived by packet dropout definitions. Assume the state feedback control law is:

$$\bar{u}(k) = F(d_k^{sc})\bar{x}(k) \quad (5)$$

Where F($d_k^{sc}$) is a set of controllers and will be designed based on $d_k^{sc}$ . Substituting Eq. (2) and Eq. (5) in Eq. (1), we will have the following closed-loop system:

$$x_{k+1} = \begin{cases} \Phi x_k + \Gamma F(d_k^{sc})\bar{x}_k & \text{if } d_k^{ca} = 0 \\ \Phi x_k + \Gamma u_{k-1} & \text{otherwise} \end{cases} \quad (6)$$

Note that $\bar{x}_k$=$x_{k-d_k^{sc}}$, which can be easily derived by iterations based on Eq. (2) To simplify the expression of the closed-loop system, we introduce a function α(.) to combine the above closed-loop system as:

$$x_{k+1} = \Phi x_k + \alpha(d_k^{ca})\Gamma u_{k-1} + [1 - \alpha(d_k^{ca})]\Gamma F(d_k^{sc}) x_{k-d_k^{sc}}$$

$$u_k = \alpha(d_k^{ca}) u_{k-1} + [1 - \alpha(d_k^{ca})] F(d_k^{sc}) x_{k-d_k^{sc}} \quad (7)$$

Where:

$$\alpha(d_k^{ca}) = \begin{cases} 1, & d_k^{ca} > 0 \\ 0, & d_k^{ca} = 0 \end{cases} \quad (8)$$

Remark1. The value of α(.) depends on whether the designed control signal is successfully transmitted or not (namely, $d_k^{sc}$=0 or $d_k^{sc}$>0), instead of how many designed control signals are dropped (the value of $d_k^{ca}$). This classification can simplify the modeling of the closed-loop system since the control input $u_k$ will not be updated no matter what the value of $d_k^{ca}$>0 will be. That is, the control signal $u_k$ will be the same when $d_k^{ca}$=1,2,3,…, $d_2$. Another advantage of this classification is to avoid introducing the unknown $d_k^{ca}$ in the augmented state vectors and controller design. Thus, we replace $u_k$ with Eq. (2) instead of $u_k$=$\bar{u}_{k-d_k^{ca}}$, the deriving method being the same as the iteration method for $\bar{x}_k$. Combining plant and controller state vectors to obtain a global vector $z_k$=$[x_k^T, u_{k-1}^T]^T$ by Eqs. (7)-(8), we can obtain the closed-loop system for the NCS with single-packet transmissions in Fig. 1 as:

$$z_k = \begin{bmatrix} \Phi & \Gamma\alpha(d_k^{ca}) \\ 0 & \alpha(d_k^{ca}) \end{bmatrix} \begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix} + \begin{bmatrix} (1-\alpha(d_k^{ca}))\Gamma F(d_k^{sc}) & 0 \\ (1-\alpha(d_k^{ca}))F(d_k^{sc}) & 0 \end{bmatrix} = A(d_k^{ca})z_k + B(d_k^{ca}, d_k^{sc}) z_{k-d_k^{sc}} \quad (9)$$

Remark2: The resulting closed-loop system in Eq. (9) is a jump linear system with two modes ($d_k^{sc}$ and $d_k^{ca}$) and one mode-dependent time-varying delay $d_k^{sc}$,where their transitions are described by two Markov chains ,which give the history behavior of S/C and C/A packet dropouts, respectively. This also enables us to apply the results of jumping linear systems with time-delays to the analysis and synthesis of such NCSs.

## IV. Stability analysis and controller design of NCSs with single packet transmission

According to the result of previous section, a sufficient condition on the stochastic stability of the system in 8 with single-packet transmissions is derived. Then we have the following theorem.

Theorem [4]: The system in 8 is stochastically stable if there exist $X_{i,m}$ >0, Q>0and $\Psi_{i,m}$ such that the following LMI:

$$\begin{bmatrix} -X_{i,m} & 0 & X_{i,m}A_m^T\Theta & X_{i,m} \\ * & -Q & \Psi_{i,m}\Theta & 0 \\ * & * & -\Omega & 0 \\ * & * & * & -\frac{1}{\mu}Q \end{bmatrix} < 0 \quad (10)$$

holds for all $i, j \in S_1$ and $m, n \in S_2$ where * denotes blocks that are readily inferred by symmetry and $d_k^{sc} = i \in S_1, d_k^{ca} = m \in S_2$.

Then:

$$\Theta = [\sqrt{\lambda_{m1}\rho_{i1}}I, \ldots, \sqrt{\lambda_{mn}\rho_{ij}}I, \ldots, \sqrt{\lambda_{md_2}\rho_{id_1}}I]$$

$$Q = R^{-1}, \Omega = diag[X_{1,1}, \ldots, X_{j,m}, \ldots, X_{d_1,d_2}]$$

$$\Psi_{i,m} = R^{-1}B_{i,m}^T$$

Here $d_1.d_2$ is the number of matrices. The control law we have is:

$$u_k = \begin{cases} u_{k-1} + \Gamma F(d_k^{sc})\bar{x}_k & d_k^{ca} > 0 \\ K\Psi_{i,0}Q^{-1}[I \quad 0]x_{k-d_k^{sc}} & d_k^{ca} = 0 \end{cases} \quad (11)$$

## V. Simulation of an inverted pendulum

In this section, the confirmation of the presented method is established. For this purpose an inverted pendulum, already controlled through network , when packet dropouts occur in S/C and C/A sides, is stabilized using previous results presented here about MJLSs. Simulation results guarantee the stability of the system. As seen in Fig. 5, $[x_d, \theta, \dot{x}_d, \dot{\theta}]$ are the states and $[x_d, \theta]$ are outputs. The surface is frictionless and parameters of the system are [18]:

$$m_1 = 1kg, m_2 = 0.5kg, L = 1m$$

The discrete dynamic model of system with sampling time of $0.5s$ is given by:

$$A_d = \begin{bmatrix} 1.0000 & 0.1000 & -0.0166 & -0.0005 \\ 0 & 1.0000 & -0.3374 & -0.0166 \\ 0 & 0 & 1.0966 & 0.1033 \\ 0 & 0 & 2.0247 & 1.0996 \end{bmatrix}$$

$$B_d = \begin{bmatrix} 0.0045 \\ 0.0896 \\ -0.0068 \\ -0.1377 \end{bmatrix}$$

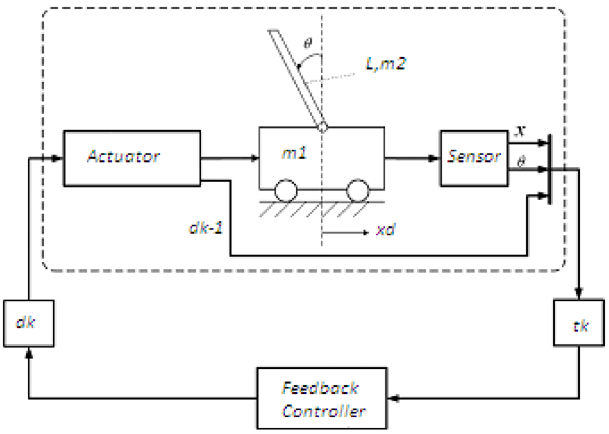


*Fig. 5 an inverted pendulum system*

Eigenvalues of the inverted pendulum system are 1,1,1.5569,0.6423. It's clear that, this discrete system is unstable and should be stabilized through packet dropouts. According to the presented method, packet dropouts in S/C and C/A sides change to delays $(d_k^{sc}, d_k^{ca})$, Where:

$d_k^{ca} = i = \{0,1,2\}, d_k^{sc} = m = \{0,1,\}$

Transition probability matrices are [18]:

$$\Lambda = \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.5 & 0.4 & 0.1 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}, \Pi = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.5 \end{bmatrix}$$

Simulation results

The outputs and states of the simulated system are depicted in Fig. 6.a to Fig. 6.d:

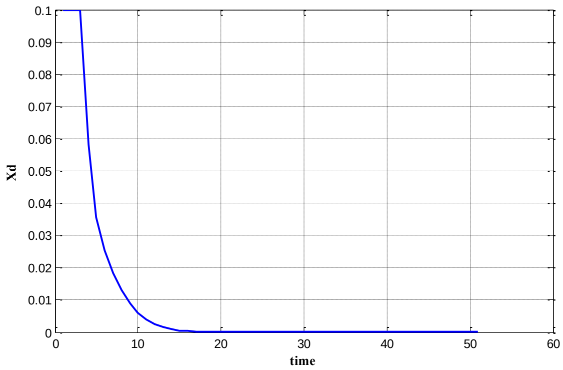


*Fig. 6.a $x_d$ of the system*

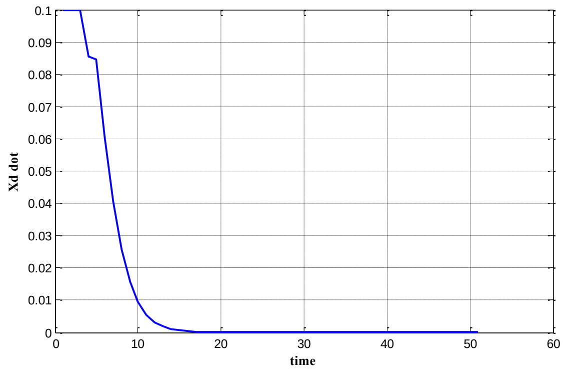


*Fig. 6.b $\dot{x}_d$ of the system*

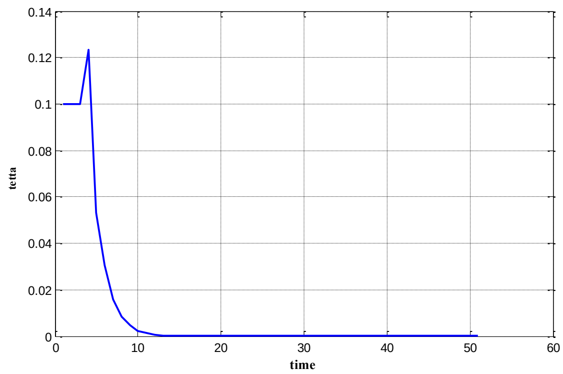


*Fig. 6.c $\theta$ of the system*

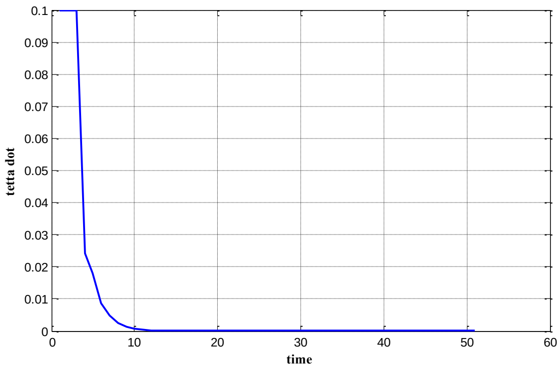


*Fig. 6.d $\dot{\theta}$ of the system*

These figures show the stability of an inverted pendulum, controlled through network by packet dropouts in S/C and C/A sides. Fig. 6.a and Fig. 6.c show the convergence of system states to initial states. Fig. 6.b and Fig. 6.d show continuity of derivative of states and convergence of them. As a result the system has been stabilized through the presented method.

## VI. Conclusion

In this paper, a system which is controlled through network is modeled by an MJLS. Modeling the packet dropout process by delays makes it possible to use the results of MJLSs with time delay. From this, an inverted pendulum with packet dropouts in S/C and C/A sides is stabilized.

## VII. References

[1] James R. Moyne, Member IEEE, and Dawn M. Tilbury.,2007. The Emergence of Industrial Control Networks for Manufacturing Control,Diagnostics, and Safety Data. Vol. 95, No. 1, Proceedings of the IEEE

[2] Y. Tipsuwan and M. Y. Chow.2002,Gain adaptation of networked mobile robot to compensate QoS deterioration. In the 28th Annual Conference of the IEEE Industrial Electronics Society (IECON 02), volume 4, pages 3146-3151,.

[3] T. J. Tarn and N. Xi.1998, Planning and control of Internet-based teleoperation. In Proceedings of SPIE: Telemanipulator and Telepresence Technologies, pages 189-193,.

[4] J. Wu and T. Chen, 'Design of networked control systems with packet dropouts'. IEEE Transactions on Automatic Control, vol. 52, No. 7, pp. 1314-1319, 2007.

[5] Wen-An Zhang, Li Yu,2008," Modelling and control of networked control systems with both network-induceddelay and packet-dropout", Department of Automation, Zhejiang University of Technolog,Article registered in sience direct.

[6] T.C. Yang ,2006,"Networked Control Systems:A Brief Survey"IEEE Proc.-Control Theory Appl.,Vol.153,No 4.

[7] Closed Loop Control Over Wireless Networks, IEEE Control Systems Magazine,Volume: 24 ,page(s): 58 - 71 /June 2004

[8] Jing Wu,2008,"MODELING, ANALYSIS AND DESIGN OF NETWORKED CONTROL SYSTEMS" Department of Electrical and Computer Engineering, University of Alberta , A thesis,degree of Doctor of Philosophy.

[9] Joa˜o P. Hespanha, Payam Naghshtabrizi and Yonggang Xu ,2007," A Survey of Recent Results in Networked Control Systems", Proceedings of the IEEE , Vol. 95, No. 1,

[10] Autonomic control and personalization of a wireless access network/ Dipartimento di Ingegneria Elettronica, University of Rome ''Tor Vergata'', Via del Politecnico, 1-00133 Rome, ItalyD.I.E.I., University of Perugia, Italy/January 2007

[11] Yodyium Tipsuwan, Mo-Yuen Chow,2003,"Control methodologies in networked control systems/"Advanced Diagnosis and Control Lab, Department of Electrical and Computer Engineering, North Carolina State University, Daniels Hall, Control Engineering Practice 11 1099–1111

[12] W.Zhang, M.S. Branicky, and S.M. Philips,'Stability of networked control systems," IET Control Theory Syst. Mag,vol. 21,no 5, pp.1336-1343,2007.

[13] M.G.Rivera,A.Barreiro,'Analysis of networkesd control systems with drops and variable delays,'Automatica,vol 43,pp.2054-2059,2007.

[14] M.Yu,L.Wang et al,'stabilization of networked control systems with data packet dropout and network delays via switching system approach,'in Proceedings of the 43rd IEEE Conference on Decision and Control,2004,pp.3539-3544

[15] S.C. Smith,P.seiler,'Estimation with lossy measurments:jump estimators for jump systems,' IEEE trans. Autom.Control,vol.48,no. 12, pp.2163-2171,2003.

[16] J. Wu, T. Chen, and L. Wang,'Delay-dependent robust stability and H^ control for jump linear systems with delays'. Systems and Control Letters, vol. 55, pp. 939-948, 2006.

[17] J. Wu, T. Chen, and S. Xu,' Stochastic stabilization and H^ control for discrete jumping systems with time delays. Asian Journal of Control, vol. 7, pp. 223-230, 2005

[18] Y.Shi, B.Yu,'Output Feedback Stabilization of Networked Control Systems With Random Delays Modeled by Markov Chains'.IEEE trans on Automatic Control, vol.54,no. 7,2009.