Rayleigh Surface Waves In A Transversely Isotropic Microstretch Elastic Solid Half Space

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***Abstract*—The linear governing equations of a transversely isotropic microstretch elastic solid medium are formulated and solved for surface wave solutions. The appropriate solutions satisfying the radiation conditions are applied to the required boundary conditions at the free surface of the half- space of the medium. A frequency equation is obtained for Rayleigh surface wave in the medium. The non-dimensional speed of the propagation of Rayleigh surface wave is computed for a specific model of the material and is shown graphically to observe the effects of non-dimensional constants and non-dimensional frequency.**

Keywords—Rayleigh surface wave, Frequency equation, Speed of propagation, Transverse isotropy, Microstretch elastic solid.

I. INTRODUCTION

The linear theory of elasticity has applications in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent linear elasticity describes the mechanical behaviour of the other common solid materials, for example, concrete, wood and coal. However, the linear theory of elasticity is unable to explain the behaviour of many of the new synthetic materials of the elastomer and polymer type. The theory of micropolar elasticity is adequate to represent the behaviour of such materials. For the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant. Microstretch continuum is a model for Bravias lattice with a basis on the atomic level and a two phase dipolar solid with a core on the macroscopic level. For example, composite materials reinforced with chopped elastic fibres, porous media whose pores are filled with gas or inviscid liquid, asphalt or other elastic inclusions and 'solid-liquid' crystals, etc. represent the microstretch solids. Eringen [1] developed theory of Micropolar elastic solids with stretch. Eringen [2] proposed a theory of thermo-microstretch elastic solid in which he included microstructural expansions and contractions. Eringen [3] formulated a electromagnetic theory of microstretch elasticity and bone modelling.

Surface waves in elastic solids were first studied by Lord Rayleigh [4] for an isotropic elastic solid. The extension of surface wave analysis and other wave propagation problems to anisotropic elastic materials has been the subject of many studies. For example, Musgrave [5] reported the propagation of elastic waves in crystals and other anisotropic media. Anderson, [6] derived periodic equations for waves of Rayleigh, Stoneley and Love types in a transversely isotropic medium. Buchwald [7] also studied the Rayleigh waves in transversely isotropic media using an approach based on potential functions. Chadwick and Smith [8] discussed the theory of surface waves in anisotropic elastic materials. Royer and Dieulesaint [9] obtained Rayleigh wave velocity and displacement in orthorhombic, tetragonal, hexagonal and cubic crystals. Barnett and Lothe [10] studied free surface (Rayleigh) waves in anisotropic elastic half-space using the surface impedance method. Dowaikh and Ogden [11] discussed the surface waves and deformation in a compressible elastic half-space. Destrade [12] derived the explicit secular equation for surface acoustic waves in monoclinic elastic crystals. Ahmad [13] studied the guided waves in a transversely isotropic cylinder immersed in a fluid. Ting [14] derived an explicit secular equation for surface waves in an elastic material of general anisotropy. Ting [15] derived explicit secular equations for surface waves in monoclinic materials with the symmetry plane $x_1 = 0$, $x_2 = 0$ or $x_3 = 0$. Ogden and Vinh [16] obtained the secular equation for the Rayleigh wave speed in an incompressible orthotropic elastic solid in a form that does not admit spurious solutions. Singh et al. [17] studied the effect of rotation on non-dimensional speed of Rayleigh wave in an orthotropic micropolar solid half-space.

Many researchers have attempted various plane wave and surface wave problems in isotropic microstretch elasticity. For example, Singh [18] has shown the existence of five plane waves in an isotropic microstretch elastic solid and studied reflection of these waves from free surface to obtain reflection coefficients and energy ratios. Nowinski [19] studied the non-local surface wave in a linear isotropic micropolar and microstretch. Sharma et al. [20-21] studied the propagation of generalized Rayleigh surface waves in a homogeneous, isotropic, microstretch thermoelastic solid half-space. Kumar et al. [22] studied the Rayleigh waves in isotropic microstretch thermoelastic diffusion solid half space. A transversely isotropic material is one with physical properties which are symmetric about an axis that is normal to a plane of isotropy. Rayleigh surface wave in a transversely isotropic microstretch elastic solid is not considered in literature yet. In the present paper, the basic equations of motion for transversely isotropic microstretch elastic solid are formulated and solved for surface wave sulutions. A frequency equation of Rayleigh surface wave is obtained. The non-dimensional wave speed of Rayleigh is computed and shown graphically for a particular model of the half-space.

II. GOVERNING EQUATIONS

We consider a body that at some instant occupies the region *B* of the euclidean three- dimensional space and is bounded by the piecewise smooth surface ∂B. The motion of the body is referred to the reference configuration B and a fixed system of rectangular Cartesian axes Ox_i $(i = 1,2,3)$. We denote by *n* the outward unit normal of ∂B. We consider the linear theory of microstretch elasticity. The basic equations of linear theory of microstretch elasticity are:

The equations of motion

$$t_{ji,j} = \rho \ddot{u}_i , \tag{1}$$

$$m_{ik.i} + \varepsilon_{ijk} t_{ij} = \rho j \ddot{\varphi}_k , \tag{2}$$

$$\pi_{k,k} - \sigma = j_0 \ddot{\varphi} \tag{3}$$

The constitutive equations

$$t_{ij} = A_{ijrs} e_{rs} + B_{ijrs} \kappa_{rs} + D_{ij} \Phi + F_{ijk} \zeta_k \tag{4}$$

$$m_{ij} = B_{rsij} e_{rs} + C_{ijrs} \kappa_{rs} + E_{ij} \Phi + G_{ijk} \zeta_k \tag{5}$$

$$\sigma = D_{ij} e_{ij} + E_{ij} \kappa_{ij} + \xi \Phi + h_k \zeta_k \tag{6}$$

$$\pi_k = F_{ijk} e_{ij} + G_{ijk} \kappa_{ij} + h_k \Phi + A^*_{kj} \zeta_j \tag{7}$$

and, the geometrical equations

$$e_{ij} = u_{j,i} + \varepsilon_{jik} \varphi_k, \kappa_{ij} = \varphi_{j,i} , \zeta_j = \Phi_{,j} \tag{8}$$

Here, t_{ij} is the stress tensor, ρ is the reference mass density, u_i is the displacement vector, m_{ij} is the couple stress tensor, ε_{ijk} is the alternating symbol, I_{ij} is the microinertia tensor, φ_i is the microrotation vector, π_k is the microstretch stress vector, Φ is the microstretch function, σ is the microstress function, j is the microinertia, j_0 is microstretch inertia, e_{ij} , κ_{ij} and ζ_k are kinematic strain measures and A_{ijrs}, B_{ijrs}, C_{ijrs}, D_{ij}, E_{ij} , F_{ijk}, G_{ijk}, h_i, ξ, A_{ij} , κ_{ij} are constitutive coefficients. Latin subscripts are understood to range over the integers (1, 2, 3), summation over repeated subscriptsis implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. Superposed dot denotes partial differentiation with respect to the time *t*. The microinertia tensor and the constitutive coefficients are assumed to satisfy the symmetry relations

$$A_{ijrs} = A_{rsij}, B_{ijrs} = B_{rsij},\ C_{ijrs} = C_{rsij},\ A_{ij} = A_{ji}, \kappa_{ij} = \kappa_{ji}. \tag{9}$$

III. FORMULATION OF THE PROBLEM AND SOLUTION

We consider a homogeneous transversely isotropic microstretch solid half space. We take the origin of the coordinate system on the free surface and z axis is pointing normally into the half-space,which is thus represented by z ≥ 0. We assume the components of the displacement and microrotation vector of the form $u = (u_1 ,0, u_3)$ and $\varphi = (0, \varphi_2 ,0)$. With the help of equations. (4) to (9), the equations (1) to (3) are written in x-z plane as

$$A_{11}\frac{\partial^2 u_1}{\partial x^2} + (A_{13} + A_{56})\frac{\partial^2 u_3}{\partial x \partial z} + A_{55}\frac{\partial^2 u_1}{\partial z^2} + K_1\frac{\partial \varphi_2}{\partial z} + D_{11}\frac{\partial \Phi}{\partial x} = \rho\frac{\partial^2 u_1}{\partial t^2} \tag{10}$$

$$A_{66}\frac{\partial^2 u_3}{\partial x^2} + (A_{13} + A_{56})\frac{\partial^2 u_1}{\partial x \partial z} + A_{33}\frac{\partial^2 u_3}{\partial z^2} + K_2\frac{\partial \varphi_2}{\partial x} + D_{33}\frac{\partial \Phi}{\partial z} = \rho\frac{\partial^2 u_3}{\partial t^2} \tag{11}$$

$$B_{77}\frac{\partial^2 \varphi_2}{\partial x^2} + B_{66}\frac{\partial^2 \varphi_2}{\partial z^2} - K_1\frac{\partial u_1}{\partial z} - K_2\frac{\partial u_3}{\partial x} - \chi\varphi_2 = \rho j\frac{\partial^2 \varphi_2}{\partial t^2} \tag{12}$$

$$A^*_{11}\frac{\partial^2 \Phi}{\partial x^2} + A^*_{33}\frac{\partial^2 \Phi}{\partial z^2} - \xi\Phi - D_{11}\frac{\partial u_1}{\partial x} - D_{33}\frac{\partial u_3}{\partial z} = j\frac{\partial^2 \Phi}{\partial t^2} \tag{13}$$

where

$A_{11} = A_{1111}$, $A_{55} = A_{3131}$, $A_{13} = A_{1133} = A_{3311}$, $A_{56} = A_{3113} = A_{1331}$, $A_{66} = A_{1313}$, $A_{33} = A_{3333}$, $K_1 = A_{56} - A_{55} = A_{3113} - A_{3131}$, $K_2 = A_{66} - A_{56} = A_{1313} - A_{1331}$, $B_{77} = C_{1212}$, $B_{66} = C_{3232}$, $\chi = K_2 - K_1$.

We seek the surface wave solutions of equations (10) to (13) as

$$\{u_1, u_3, \varphi_2, \Phi\} = \{\overline{u_1}(z), \overline{u_3}(z), \overline{\varphi_2}(z), \overline{\Phi}(z)\} e^{ik(x-ct)} \tag{14}$$

where *k* is the wave number, c is phase speed of the wave, and $\omega = kc$ is the angular frequency.

Making use of equation (14) in equations (10) to (13), we obtain four homogeneous equations

$$(A_{55}D^2 - Lk^2)\overline{u_1}(z) + ikMD\overline{u_3}(z) + K_1 D\overline{\varphi_2}(z) + ikD_{11}\overline{\Phi}(z) = 0 \tag{15}$$

$$ikMD\overline{u_1}(z) + (A_{33}D^2 - Nk^2)\overline{u_3}(z) + +ikK_2\overline{\varphi_2}(z) + D_{33}D\overline{\Phi}(z) = 0 \tag{16}$$

$$-K_1 D\overline{u_1}(z) - ikK_2\overline{u_3}(z) + (B_{66}D^2 - Pk^2)\overline{\varphi_2}(z) = 0 \tag{17}$$

$$ikD_{11}\overline{u_1}(z) + D_{33}D\overline{u_3}(z) + (-A^*_{33}D^2 + Rk^2)\overline{\Phi}(z) = 0 \tag{18}$$

where

$$L = (A_{11} - \rho c^2), M = (A_{13} + A_{56}), N = (A_{66} - \rho c^2),$$

$$P = \left(B_{77} - \rho j c^2 + \frac{\chi}{k^2}\right),\ R = \left(A^*_{11} - j c^2 + \frac{\xi}{k^2}\right).$$

The equations (15) to (18) have non-trivial solutions if

$$a_0 D^8 - a_1 D^6 + a_2 D^4 - a_3 D^2 + a_4 = 0 \tag{19}$$

where $D = \frac{d}{dz}$ and a_0, a_1, a_2, a_3, a_4 are given in *Appendix.*

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Let $m_1, m_2, m_{3,}, m_{4,}$ be the roots of auxiliary equation corresponding to equation (19). Using the radiation conditions $u_1 \rightarrow 0$, $u_3 \rightarrow 0$, $\varphi_2 \rightarrow 0$, $\rightarrow 0$ as $z \rightarrow \infty$, we obtainthe following solutions in the half-space

$$u_1 = (A_1 e^{-m_1 z} + A_2 e^{-m_2 z} + A_3 e^{-m_3 z} + A_4 e^{-m_4 z}) e^{ik(x-ct)}, \quad (20)$$

$$u_3 = (\zeta_1 A_1 e^{-m_1 z} + \zeta_2 A_2 e^{-m_2 z} + \zeta_3 A_3 e^{-m_3 z} + \zeta_4 A_4 e^{-m_4 z}) e^{ik(x-ct)}, \quad (21)$$

$$\varphi_2 = (\eta_1 A_1 e^{-m_1 z} + \eta_2 A_2 e^{-m_2 z} + \eta_3 A_3 e^{-m_3 z} + \eta_4 A_4 e^{-m_4 z}) e^{ik(x-ct)}, \quad (22)$$

$$\Phi = (\xi_1 A_1 e^{-m_1 z} + \xi_2 A_2 e^{-m_2 z} + \xi_3 A_3 e^{-m_3 z} + \xi_4 A_4 e^{-m_4 z}) e^{ik(x-ct)} \quad (23)$$

where $\eta_1, \eta_2, \eta_{3,}, \eta_4$ and $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ *and* ξ_1, ξ_2, ξ_3 *and* ξ_4 are given in *Appendix*, and

$$m_1^2 + m_2^2 + m_3^2 + m_4^2 = \frac{a_1}{a_0},$$

$$m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_4^2 + m_4^2 m_1^2 = \frac{a_2}{a_0},$$

$$m_1^2 m_2^2 m_3^2 + m_2^2 m_3^2 m_4^2 + m_3^2 m_4^2 m_1^2 = \frac{a_3}{a_0},$$

$$m_1^2 m_2^2 m_3^2 m_4^2 = \frac{a_4}{a_0}.$$

IV. BOUNDARY CONDITIONS

The mechanical boundary conditions at z = 0 are

$$t_{33} = 0, \; t_{31} = 0, \; m_{32} = 0, \; \pi_3 = 0, \quad (24)$$

where

$$t_{33} = A_{13} u_{1,1} + A_{33} u_{3,3} + D_{33} \Phi,$$

$$t_{31} = A_{56} u_{3,1} + A_{55} u_{1,3} + (A_{56} - A_{55}) \varphi_2,$$

$$m_{32} = B_{66} \varphi_{2,3},$$

$$\pi_3 = h_3 \Phi + A^*_{31} \Phi_{,1} + A^*_{33} \Phi_{,3}.$$

The solutions given by equations (20) to (23) satisfy the boundary conditions (24) at the free surface z = 0, and we obtain the following frequency equation

$$A_1^* B_2^* C_3^* D_4^* - A_1^* B_2^* C_4^* D_3^* - A_1^* B_3^* C_2^* D_4^* + A_1^* B_3^* C_4^* D_2^* + A_1^* B_4^* C_2^* D_3^* - A_1^* B_4^* C_3^* D_2^* - A_2^* B_1^* C_3^* D_4^* + A_2^* B_1^* C_4^* D_3^* + A_2^* B_3^* C_1^* D_4^* - A_2^* B_3^* C_4^* D_1^* - A_2^* B_4^* C_1^* D_3^* + A_2^* B_4^* C_3^* D_1^* + A_3^* B_1^* C_2^* D_4^* - A_3^* B_1^* C_4^* D_2^* - A_3^* B_2^* C_1^* D_4^* + A_3^* B_2^* C_4^* D_1^* + A_3^* B_4^* C_1^* D_2^* - A_3^* B_4^* C_2^* D_1^* - A_4^* B_1^* C_2^* D_3^* + A_4^* B_1^* C_3^* D_2^* + A_4^* B_2^* C_1^* D_3^* - A_4^* B_2^* C_3^* D_1^* - A_4^* B_3^* C_1^* D_2^* + A_4^* B_3^* C_1^* D_2^* = 0, \quad (25)$$

where

$$A_i^* = ikA_{13} - m_i \zeta_i A_{33} + \xi_i D_{33}, \; (i = 1,2,.,4)$$

$$B_i^* = ik\zeta_i A_{56} - m_i A_{55} + (A_{56} - A_{55}) \eta_i,$$

$$C_i^* = m_i \eta_i B_{66}, \quad D_i^* = h_3 \xi_i + ik\xi_i A^*_{31} - m_i \xi_i A^*_{33}.$$

In absence of microstretch parameters, i.e., if we put $A^*_{11} = A^*_{33} = A^*_{31} = D_{11} = D_{33} = \xi = 0$, the frequency equation (25) reduces for Rayleigh wave in a transversely isotropic micropolar elastic solid half-space.

V. NUMERICAL RESULTS AND DISCUSSION

To compute the non-dimensional speed of Rayleigh wave, the following relevant parameters for a transversely isotropic microstretch material are taken

$A_{11} = 17.8 \times 10^{10}$ Nm^{-2}, $A_{33} = 18.43 \times 10^{10}$ Nm^{-2}, $A_{13} = 7.59 \times 10^{10}$ Nm^{-2}, $A_{56} = 1.89 \times 10^{10}$ Nm^{-2}, $A_{55} = 4.357 \times 10^{10}$ Nm^{-2}, $A_{66} = 4.42 \times 10^{10}$ Nm^{-2}, $A_{65} = 4.32 \times 10^{10}$ Nm^{-2}, $B_{77} = 0.278 \times 10^{9}$ N, $B_{66} = 0.268 \times 10^{9}$ N, $A^*_{11} = 0.03 \times 10^{10}$ Nm^{-2}, $A^*_{33} = 0.04 \times 10^{10}$ Nm^{-2}, $A^*_{33} = 0.05 \times 10^{10}$ Nm^{-2}, $D_{11} = 0.062 \times 10^{10}$ N, $D_{33} = 0.063 \times 10^{10}$ N, $\rho = 1.74 \times 10^{3}$ Kg m^{-3}, $j = 0.196$ m^2.

The non-dimensional speed of Rayleigh wave $\sqrt{\frac{\rho c^2}{A_{33}}}$ plotted against non-dimensional frequency $\{\omega^* = \omega^2/(\frac{\chi}{\rho j})\}$ in Figure 1. For transversely isotropic microstretch case, the non-dimensional speed is 0.5071 at $\omega^* = 2.5$. It increases to 0.6153 at $\omega^* = 10$. For transversely isotropic micropolar case, the non-dimensional speed is 1.902 at $\omega^* = 2.5$. It increases to 2.862 at $\omega^* = 10$. The comparison of solid and dotted curves in figure 1 shows the effect of microstretch on non- dimensional speed of Rayleigh wave at different non-dimensional frequency.

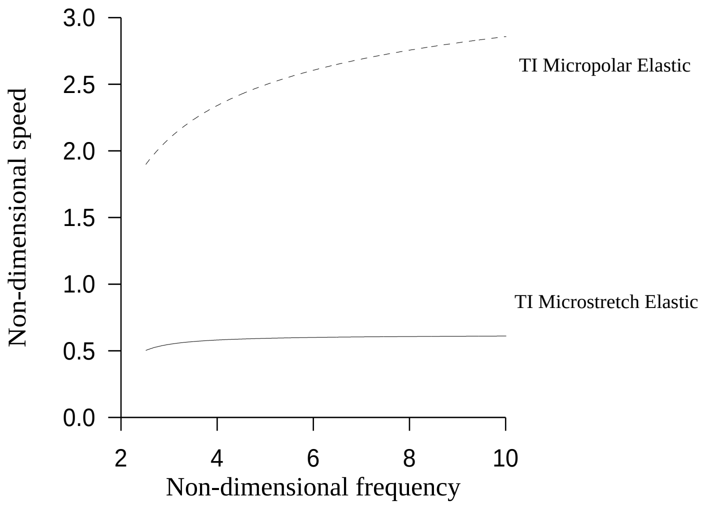

Figure 1. *Variation of non-dimensional speed* $\sqrt{\frac{\rho c^2}{A_{33}}}$ *against non-dimensional frequency* $\{\omega^* = \omega^2/(\frac{\chi}{\rho j})\}$.

The variations of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ against non-dimensional frequency $\{\omega_1^* = \omega^2/(\frac{\xi}{\rho j_0})\}$ are shown graphically in Figure 2, when $\omega^* = 5, 10$ and 15. For $\omega^* = 5$, the non-dimensional speed of Rayleigh wave is 0.6231 at $\omega_1^* = 2.5$ and 0.5847 at $\omega_1^* = 10$. For $\omega^* = 10$, the non-dimensional speed of Rayleigh wave is 0.6341 at $\omega_1^* = 2.5$ and 0.6053 at $\omega_1^* = 10$. For $\omega^* = 15$, the non-dimensional speed of Rayleigh wave is 0.6370 at $\omega_1^* = 2.5$ and 0.6104 at $\omega_1^* = 10$. The comparison of solid and dotted curves in Figure 2 shows the effect of non-dimensional frequency on non-dimensional speed of Rayleigh wave.

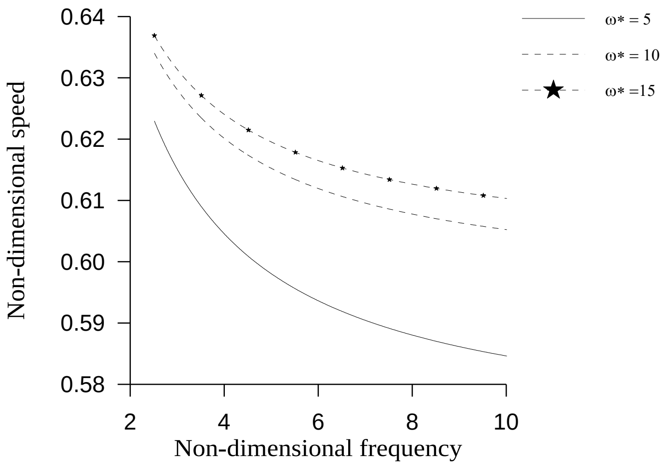

Figure 2. *Variations of non-dimensional speed* $\sqrt{\frac{\rho c^2}{A_{33}}}$ *against non-dimensional frequency* $\{\omega_1^* = \omega^2/(\frac{\xi}{\rho j_0})\}$, *when* $\omega^* = 5, 10$ *and 15.*

The variations of non-dimensional speed $\sqrt{\frac{\rho c^2}{A_{33}}}$ against non-dimensional constant ($0 \leq \frac{A_{66}}{A_{33}} \leq 0.3$) are shown graphically in Figure 3 when $\omega^* = 5$ (solid), 10 (dotted) and 15 (dotted with star) and $\omega_1^* = 5$. The comparison of solid and dotted curve show the effect of non-dimensional frequency at different non-dimensional material constant.

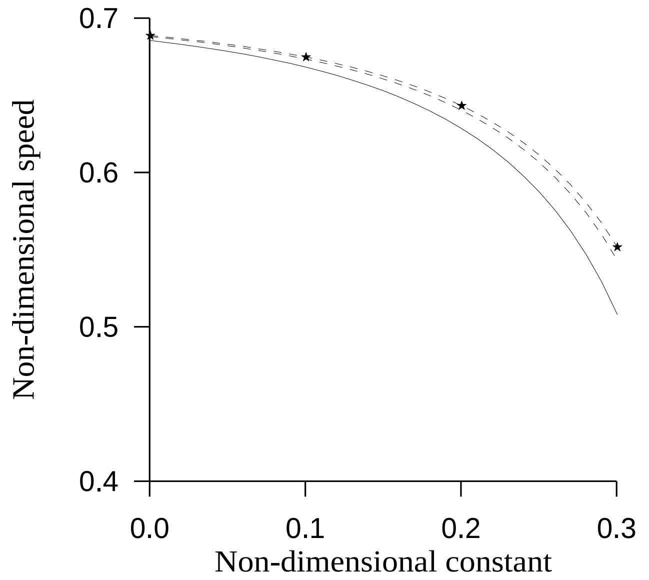

Figure 3. *Variation of non-dimensional speed* $\sqrt{\frac{\rho c^2}{A_{33}}}$ *against non-dimensional constant* $(\frac{A_{66}}{A_{33}})$ *when* $\omega^* = 5$ *(solid), 10 (dotted) and 15 (dotted with star) and* $\omega_1^* = 5$.

VI. CONCLUSIONS

Linear microstretch elasticity is applied to study the Rayleigh wave in a transversely isotropic medium. A secular equation of Rayleigh wave is obtained. The numerical results show the effects of non-dimensional material constant and non-dimensional frequency on wave speed of Rayleigh wave. The exact nature of the layers under the earth surface is not known. Various appropriate models are considered for theoretical investigation about the Earth's interior. The problems of waves and vibrations become more important in the field of seismology, when one studies the problem with additional parameters, for example, microstretch, thermal disturbance, microrotation, porosity, viscosity, and other paramaters.

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APPENDIX

The coefficients a_0, a_1, a_2, a_3, a_4 in equation (19) are given as $a_0 = A_{33}A_{55}A^*_{33}B_{66}$,

$$a_1 = [k^2(A_{33}A_{55}A^*_{33}P + A_{33}A_{55}B_{66}R + A_{55}B_{66}A^*_{33}N + A_{33}B_{66}A^*_{33} - B_{66}A^*_{33}M^2) - A_{55}B_{66}D^2_{33} - A_{33}A^*_{33}{K_1}^2],$$

$$a_2 = [k^4(A_{55}B_{66}RN + A_{33}A_{55}RP + A^*_{33}A_{55}PN + A_{33}B_{66}RL + A_{33}A^*_{33}LP + B_{66}A^*_{33}LN - B_{66}M^2R - A^*_{33}M^2P) + k^2(-A_{55}A^*_{33}{K_2}^2 - A_{55}D^2_{33}P - B_{66}D^2_{33}L - A_{33}{K_1}^2R - A^*_{33}{K_1}^2N - A_{33}D^2_{11}B_{66} + 2B_{66}D_{33}D_{11}M + 2A^*_{33}K_1K_2M) + D^2_{33}{K_1}^2],$$

$$a_3 = [k^6(A_{55}RNP + A_{33}PRL + B_{66}RLN + A^*_{33}LPN - PRM^2) + k^4(-A_{55}R{K_2}^2 - A^*_{33}L{K_2}^2 - D^2_{33}PL + 2D_{11}D_{33}PM - RN{K_1}^2 - A_{33}D^2_{11}P - D^2_{11}B_{66}N + 2K_1K_2RM) + k^2(-2K_1K_2D_{33}D_{11})]$$

$$a_4 = [k^8(PRNL) - k^6(RL{K_2}^2 + D^2_{11}PN) + k^4(D^2_{11}{K_2}^2)].$$

The coefficients η_i, ζ_i, ξ_i ($i = 1,2,..,4$) are given as

$$\eta_i = \frac{\left[iK_2\zeta_i - K_1\frac{m_i}{k}\right]}{k},$$

$$\zeta_i = \frac{\left[iA_{55}K_2\frac{{m_i}^2}{k^2} - iLK_2 - iMK_1\frac{{m_i}^2}{k^2} - \left(K_2D_{11} + K_1D_{33}\frac{{m_i}^2}{k^2}\right)\frac{\xi_i}{k}\right]}{\left[NK_1\frac{m_i}{k} - MK_2\frac{m_i}{k} - A_{33}K_1\frac{{m_i}^3}{k^3}\right]},$$

$$\frac{\xi_i}{k} = \frac{p_i + q_i}{r_i + s_i}$$

where

$$p_i = iD_{33}\left[A_{55}K_2\frac{{m_i}^3}{k^3} - LK_2\frac{m_i}{k} - MK_1\frac{{m_i}^3}{k^3}\right],$$

$$q_i = iD_{11}\left[MK_2\frac{m_i}{k} + A_{33}K_1\frac{{m_i}^3}{k^3} - NK_1\frac{m_i}{k}\right]$$

$$r_i = K_1\left[D^2_{33}\frac{{m_i}^5}{k^5} + RN\frac{m_i}{k} - RA_{33}\frac{{m_i}^3}{k^3} - A^*_{33}N\frac{{m_i}^3}{k^3} + A^*_{33}A_{33}\frac{{m_i}^5}{k^5}\right],$$

$$s_i = K_2\left[D_{11}D_{33}\frac{m_i}{k} - MR\frac{m_i}{k} + A^*_{33}M\frac{{m_i}^3}{k^3}\right]$$