Fluctuations Of Cylindrical Shells When Exposed Internal Unsteady Load

[1] **Safarov Ismail Ibrahimovich**- Doctor of physical and mathematical sciences, professor.
[2] **Umarov Aladdin Ochilovich**- senior Lecturer
[3]**Leyla Rustamova Asadovna** - teacher
lazizbek.axmedov.2011@mail.ru
[1,2]Bukhara Technological- Institute of Engineering
[3]1 Academic lyceum under Bukhara State University.
Republic of Uzbekistan, 15 K. Murtazoyev Street.

***Abstract*—This paper considers the vibrations of cylindrical shells when exposed internal explosive loads. Explosive load applied to the axis of symmetry of the cylindrical body. The problem is reduced to the study of the bending of the transverse oscillations relative to the element sheath. An analytical expression for the radial displacement and the corresponding numerical results.**

***Keywords*—Shell, explosive load, transverse vibrations, displacement, normal force.**

Introduction

The study sewn construction in the form of rods and shells is considered in [1,2]. Calculation sewn construction is often based on field and laboratory experiments [3].

In this paper we consider the cylindrical protective structures [4]. Under dynamic stress in the wall of the cylindrical protective structure having transverse vibrations of bending. Explosive load applied to the axis of symmetry of the cylindrical body. The problem is reduced to the study of the bending of the transverse oscillations relative to the element sheath.

Statement of the problem.

Fig. 1 shows an element with the current in it internally. The figure shows the bending moments along the generator shell M_x, annular M_φ, normal ring of force N_φ and transverse forces Q_x, and the directions of the axes of coordinates and the corresponding displacement. The equilibrium equation shell element is the sum of the projections of all efforts on the axis Z. It can be written in the form.

$$mad\varphi dx\frac{\partial^2 w}{\partial x^2}+\frac{\partial Q_x}{\partial x}dxad\varphi+2N\varphi dx\sin\frac{d\varphi}{2}=p(x,t)ad\varphi dx$$

Given, that $\sin\frac{d\varphi}{2}\cong\frac{d\varphi}{2}$,

have

$$\frac{\partial Q_x}{\partial x}=\frac{\partial^2 M_x}{\partial x^2}$$

We denote by σ, h, ρ и p circumferential compressive stress, respectively, wall thickness, density, and external pressure; then the equation of the transverse vibrations can be written as:

$$m\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2 M_x}{\partial x^2}+\frac{1}{a}N_\varphi=p(x,t)\ , \qquad (1)$$

where $N_{x\varphi} = N_{\varphi x}$; the torques $M_{x\varphi} = M_{\varphi x}$ in both sections are zero.

By the same reason, annular an effort M_φ and N_φ must be the same along the entire perimeter shell; w - transverse movements; M_x - bending moment along the generator shell; N_φ - normal ring force, m - mass per unit length; p(x,t) - external load, which is attached inside the cylindrical body; ν - the Poisson coefficient.

$$N_x=\frac{Eh}{1-\mu^2}(\varepsilon_x+\nu\varepsilon_\varphi)=0, \qquad (2)$$

$$N_\varphi=\frac{Eh}{1-\mu^2}(\varepsilon_\varphi+\nu\varepsilon_x) \qquad (3)$$

From (2) we obtain $\varepsilon_x = -\mu\varepsilon_\varphi$; Substituting this into (3) and taking into account that ε_φ = - w/ r_0, have

$$N_\varphi = -\frac{Eh}{r_0} w \qquad (4)$$

Because of symmetry conditions can be seen that the curvature of the circumference of the enclosure should be constant. Then it follows from the theory of plates, that:

$$M_x = D\frac{\partial^2 w}{\partial x^2}. \quad M_\varphi = \nu \cdot M_x \qquad (5)$$

Substituting the expressions (4) and (5) into equation (1), we obtain the equation of forced vibrations of a closed cylindrical shell [4, 5].

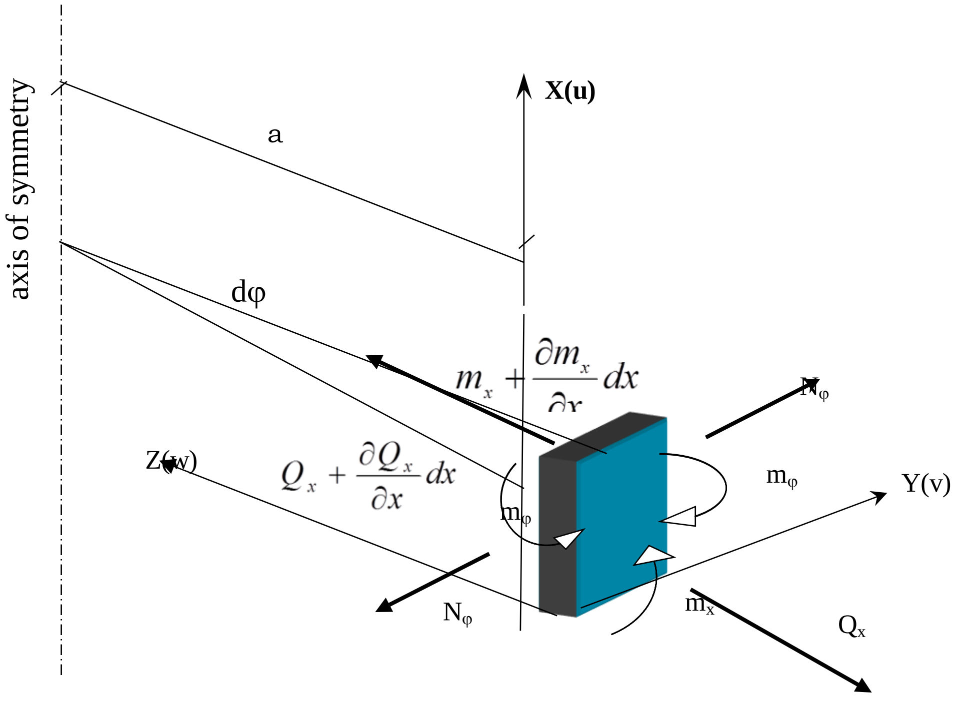
$$w^{IV} + 4\beta^4 w + \lambda^2 w'' = p(x_1 t)/D, \qquad (6)$$

where $$\beta^4 = \frac{Eh}{4R^2 D} = \frac{3(1-\nu^2)}{E^2 h^2}; \quad D = \frac{Eh^3}{12(1-\nu^2)}; \quad \lambda^2 = \frac{\nu h}{gD}$$

$$w^{1v} = \partial^4 w/\partial x^4; \quad w'' = \partial^2 w/\partial t^2.$$

D - Cylindrical rigidity; v - the Poisson coefficient; R - radius of the middle surface; h - thickness of the shell wall; g=980 см/сек^2 - acceleration of gravity; w - radial movement the shell wall.

Equation (6) differs from equation shell deformation under static loading [1] member λ^2 w'', introduced to account for the inertial forces mass shell wall at a dynamic pressure is applied. This equation, as well as the equations studied [2], does not account for the effect of the rotational inertia of the cross sections and shear forces. Consider a finite shell length L (Figure 2). Influence consolidate all shell will take into account the appropriate boundary conditions.



I. FIG.1. SETTLEMENT SCHEME ELEMENTARY AREA OF A CLOSED CYLINDRICAL SHELL WITH TRANSVERSE VIBRATIONS

As the origin of time t will take the start of the deformation. Thus, the problem of deformation of the cylindrical shell of the explosion is reduced to finding the solution of equation (6) satisfying the zero initial conditions and the corresponding boundary conditions. The solution of this problem is reduced to the determination w (x, t), since the forces and stresses after determining w (x, t) are located.

With all the variety of loads, resulting in the explosion, the nature of deformation of the shell provided with stiffening rings, can be found by solving for a single load instantly attached at a distance x = L_o (Fig. 2.a).

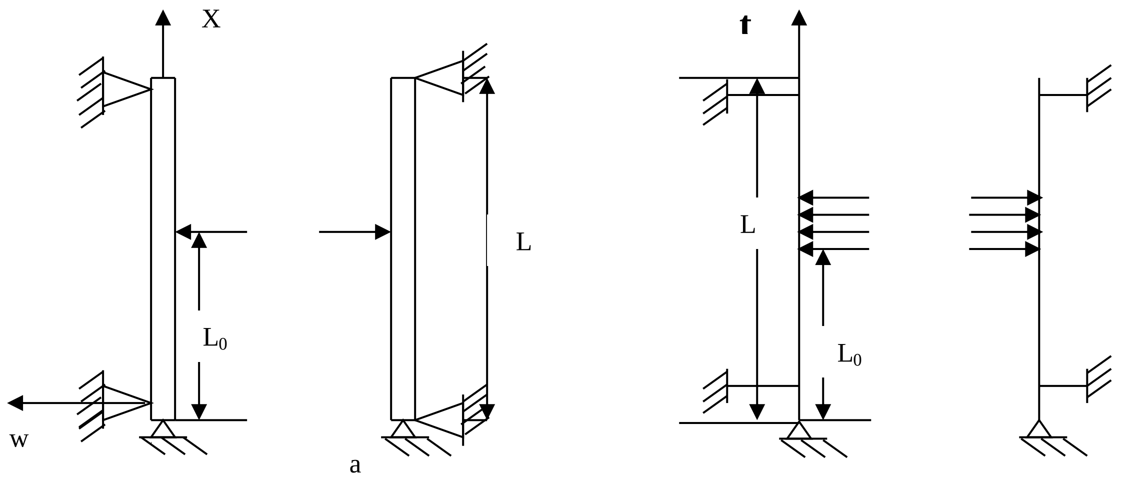


Fig. 2. Settlement scheme of a cylindrical shell

Let us find the solution, ie, define the movement of the shell wall at the instantaneous application of a load of the form

$$P(x,t)=\begin{cases}1, & npu \quad x=L_0\\ 0, & npu \quad x\neq L_0.\end{cases} \qquad (7)$$

The solution of equation (7) at the right side of (7) in the form of a number of

$$w=\sum_{k=1}^{\infty}\beta_k(t)W_k(\beta_k x), \qquad (8)$$

where W (β_κ, x) - fundamental functions satisfying the equation

$$W_k^{IV}(\beta_{k,},x)-\beta_k^4 W_k(\beta_k x)=0 \qquad (9)$$

and appropriate boundary conditions; $\beta_\kappa(t)$ - unknown coefficients to be determined. As is known, the function U_κ for the boundary conditions $W_k = W_\kappa^1 = 0$ at x = 0 and x = L have the following form;

$$W_k=\sin\beta_k x-sh\beta_k x-\frac{\sin\beta_k L-sh\beta_k L}{cos\beta_k L \quad c\beta_k L}(\cos\beta_k x-Ch\beta_k x),$$

where β_κ - the roots of the transcendental equation.

$$Ch\beta_k L \ \cos\beta_k L=1 \qquad (10)$$

Several numerical values of the solutions of equations (10):

$\beta_1 L$ = 4,73; $\beta_2 L$ = 7.8542; $\beta_3 L$ = 10,9956, at κ > 3 , $\beta_\kappa L=\frac{2\kappa+1}{2}\pi.$

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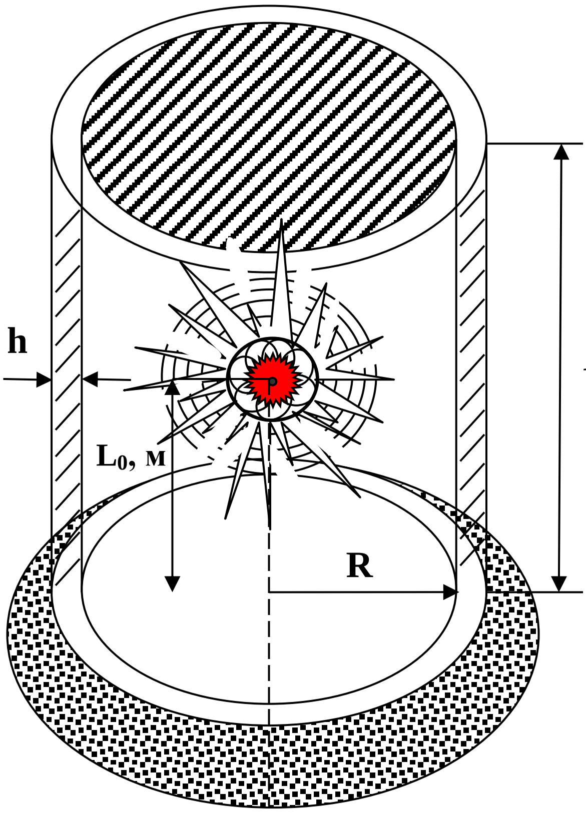


Figure 3. Settlement scheme. Cylindrical protective structure

For the boundary conditions $W_к = W_к^{II} = 0$ at $x = 0$ and $x = L$,

$$W_k = \sin\frac{k\pi}{L}x, \qquad \beta_k = \frac{k\pi}{L}, \quad k = 1, 2, 3, \ldots$$

We expand the unit load (1) in a series of fundamental functions W_k. To do this, we will consider it as the ultimate load (Fig. 2)

$$P(x)=\sum_{\kappa=1}^{\infty} A_\kappa W_\kappa(\beta_\kappa x), \qquad A_\kappa = \frac{W_\kappa(\beta_\kappa e_0)}{\int_0^e W_\kappa^2 dx}. \qquad (11)$$

To find the unknown coefficients $A_к$ series (8), in the form we are looking for solution of the problem, put the number as well as the expression (11) in equation (1). As a result of this substitution, taking into account the equality (11) we obtain the following differential equation for $B_к$

$$\ddot{B}_\kappa + \frac{\beta_\kappa^4 + 4\beta^4}{\lambda^2}B_\kappa - \frac{A_\kappa}{\lambda^2 D} = 0 \qquad (12)$$

A solution of these equations for zero initial conditions the following

$$B_\kappa = \frac{C_n(1-cjsq_k t)}{(\beta_\kappa^4 + 4\beta^4)D},$$

where

$$q_к = \sqrt{\frac{\beta_\kappa^4 + 4\beta^4}{\lambda^2}}.$$

Thus, the solution of equation (1) we have the following form:

$$W = \sum_{k=1}^{\infty} \frac{W_\kappa(\beta_\kappa)(1-\sin q_k)}{D\int_0^L W_k^2 dx\left(\beta_k^4 + 4\beta^4\right)} W_k\left(\beta_k x\right). \quad (13)$$

Wake assume that $t = T_B$ load removed. Solution for $t > T_B$ we find the principle of superposition, suggesting that at the time t_1 applied to the system unit load directed in the opposite direction. In this case the decision is determined by the formula

$$W = 2\sum_{k=1}^{\infty} \frac{W_k(\beta_k L_0)\sin\frac{q_k t_1}{2}\sin q_k\left(t-\frac{T}{2}\right)}{D\left(\beta_k^4+4\beta^4\right)\int_0^e W_k^2 dx} U_k(\beta_k x) \qquad (14)$$

The last formula can be obtained movement caused by the instantaneous unit impulse, i.e. when $t_1 \to 0$ PT = 1

$$W_m(e_0, x, t) = \sum_{k=1}^{\infty} \frac{W_k(\beta_k L_0) q_k \sin q_k \psi_k(\beta_k x)}{D\left(\beta_k^4+4\beta^4\right)\int_0^e W_k^2 dx}.$$

Now consider the load P (t) as a set of pulses. The action of the force F (t) at time T for a short time interval dt can be considered as a pulse R (T) dT. Moving the shell wall at time t (t> T), caused by this pulse is equal to

$$dW = \sum_{k=1}^{\infty} \frac{W_k(\beta_k e_0) q_k \sin q_k (t-T) p(T) dT W_k(\beta_k x)}{D\left(\beta_k^4+4\beta^4\right)\int_0^e U_k^2 dx}.$$

Moving all of the pulses applied during the time interval $0 \div T$ will

$$W = \sum_{k=1}^{\infty} \frac{W_k(\beta_k a) W_k q_k \int_0^e p(T)\sin q_k (t-T) dt}{D\left(\beta_k^4+4\beta^4\right)\int_0^e W_k^2 dx}. \qquad (15)$$

Thus, we obtain a formula for finding the moving wall of the sheath when the load is applied at the point X = l_0, varies according to an arbitrary law P (t). Formula (13), (14) and (15) can be used for constructing solutions in all cases, with the explosion of the shell of loading.

Let us consider some particular cases:

1. Pressure P (x) = P = const. Pressure is applied instantaneously, and act on the membrane for a time t_1. At the time t_1 pressure instantaneously is removed. We find the solution for the given case load applied to the sheath having boundary conditions W = W′ = 0 at x = 0 and x = l.

For time interval $0 \div t_1$, Using (13) we obtain

$$W = \sum_{k=1}^{\infty} \frac{\int_0^e P(L_0) W_k(\beta_k L_0) e_0 (1-\cos q_k t) W_k(\beta_k x)}{D\left(\beta_k^4+4\beta^4\right)\int_0^e W_k^2 dx}.$$

$$\int_0^e P(L_0) W_k(\beta_k L_0) de_0 = \begin{cases} \frac{4P}{\beta_k} & npu \quad \kappa = 1,3,5,.... \\ 0 & npu \quad \kappa = 2,\ 4,...... \end{cases}$$

$$\int_0^e W_k^2 dx = \frac{sh\beta_k t - \sin\beta_k L}{sh\beta_k L + \sin\beta_k L} L$$

and hence,

$$W = \frac{4P}{D}\sum_{k=1,3,5,..} \frac{(1-\sin Q_k t)}{\beta_k L \beta_1} \eta_k W_k(\beta_k x)$$

at t < T. Introduced here the notation

$\beta_1 = \beta_\kappa^4$; $\eta_\kappa = \dfrac{sh\beta_k L + \sin\beta_k L}{sh\beta_k L - \sin\beta_k L}$.

Accordingly at t > t_1, Using (14) we obtain

$$W = P_1 \sum_{K=1,3,5..}^{\infty} \frac{Q(t)\eta_K W_K(x)}{\beta_K L \beta_1},$$

Where $Q(t) = \sin(q_k T/2)\sin q_k\left(t - \dfrac{T}{2}\right)$

2. The pressure P (x) = 0,084 x + 0,72x^2 + 0,7x^3 is applied to the shell and is valid within the time time interval T. For 0 ÷ T using (14) we obtain

$$W = \sum_{K=1}^{\infty} \frac{Q_2(t) W_K(\beta_K x)}{D\beta_1 \int_0^e W_K^2 dx},$$

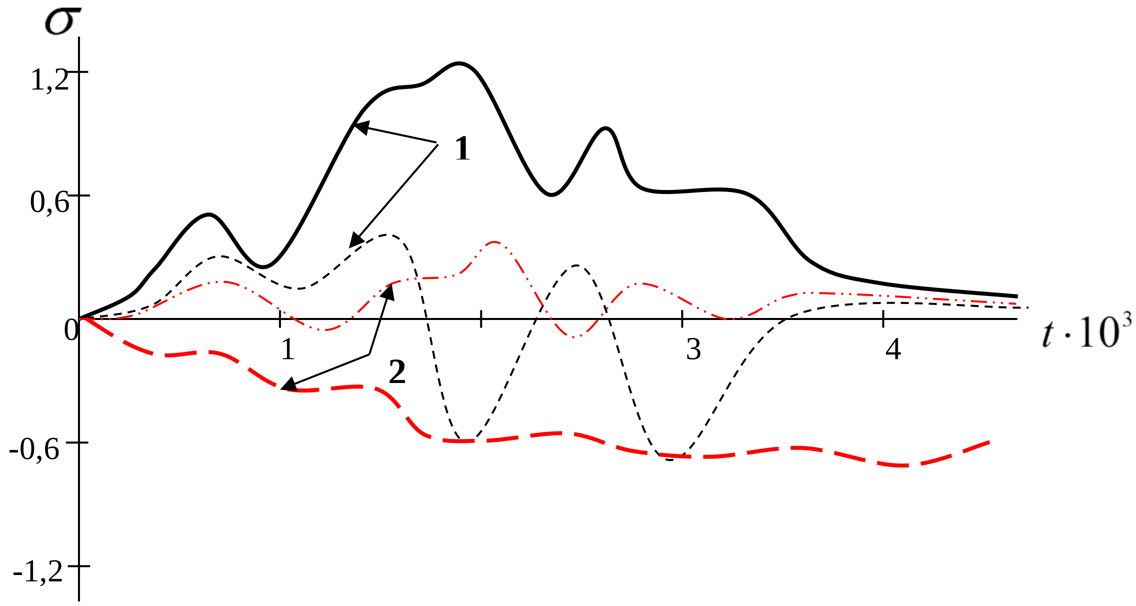
Where $Q_2(t) = \left(0{,}084\int_0^L xW_K(\beta_K L_0) + 0{,}72\int_0^L x^2 W_K(\beta_K L_0) + 0{,}67\int_0^L x^3 W_K(\beta_K L_0)\right) dL_0 (1 - \cos q_K t)$.

3. The pressure P(x) = $\sum_{n=0}^{N_1} P_n \sin\dfrac{n\pi}{L}x$ applied instantaneously and is valid for the time 0 ÷ T. At time T_1 pressure instantaneously withdrawn. We find the solution for the given case load, taking boundary conditions W = W" = 0 at x = 0 and x = L. For the data the boundary conditions at N = 1:

$$W = \frac{2P_0 \sin\sqrt{\left(\frac{\pi}{L}\right)^4 + 4\beta^4}\, \frac{T}{2\lambda} \sin\frac{\sqrt{\left(\frac{\pi}{L}\right)^2 + 4\beta^4}\left(t - \frac{T}{2}\right)}{\lambda} \sin\frac{\pi}{L}x}{D\left[\left(\frac{\pi}{L}\right)^4 + 4\beta^4\right]}.$$

The numerical results and their analysis are given below.

When calculating take the following initial data: R_0/h_1 =20; ν = 0,25, E = 2,1.10^5 кг/см^2. Some solutions of particular problems identified natural frequencies, which are listed in Table 1. The results of calculations are compared with the results presented in [4]. The discrepancy between the results of up to 20% Calculation of cylindrical shell on the effect of dynamic load.



1 – N_φ – annular of force τ = 0,05 and 0,1.2 – M_φ - moments τ = 0,05 and 0,1

Fig. 4. Cross-chain and bending stresses in the shell

Table 1. The eigenvalues.

№	The paper [4]	our the results	The difference
1.0	0,97394	0,97103	0,29
	1,47003	1,46996	0,007
	1,83890	1,83792	0,0068
2.0	2,89321	2,89436	0,00111
	3,14526	3,14627	0,00101
	3,76525	3,76423	0,00102

Consider a cylindrical shell (Fig. 3) clamped around the edges made of reinforced concrete and having the following dimensions and physical constants: D = 4 m, the height h = 4 m; elastic modulus E = $2{,}1.10^5$ кг/см^2, ν = 0,25; volumetric weight of the material of the dome γ = $2{,}1.10^3$ кг/см^3. On the inner cylinder acts uniformly distributed load, time-varying linearly (Fig. 2). Figure 4 shows the chain and bending stresses in the shell when exposed to pulsed loads as $P(x,\varphi,t)=\sigma e^{-t/\tau_э}$ (σ_0 - amplitude load).

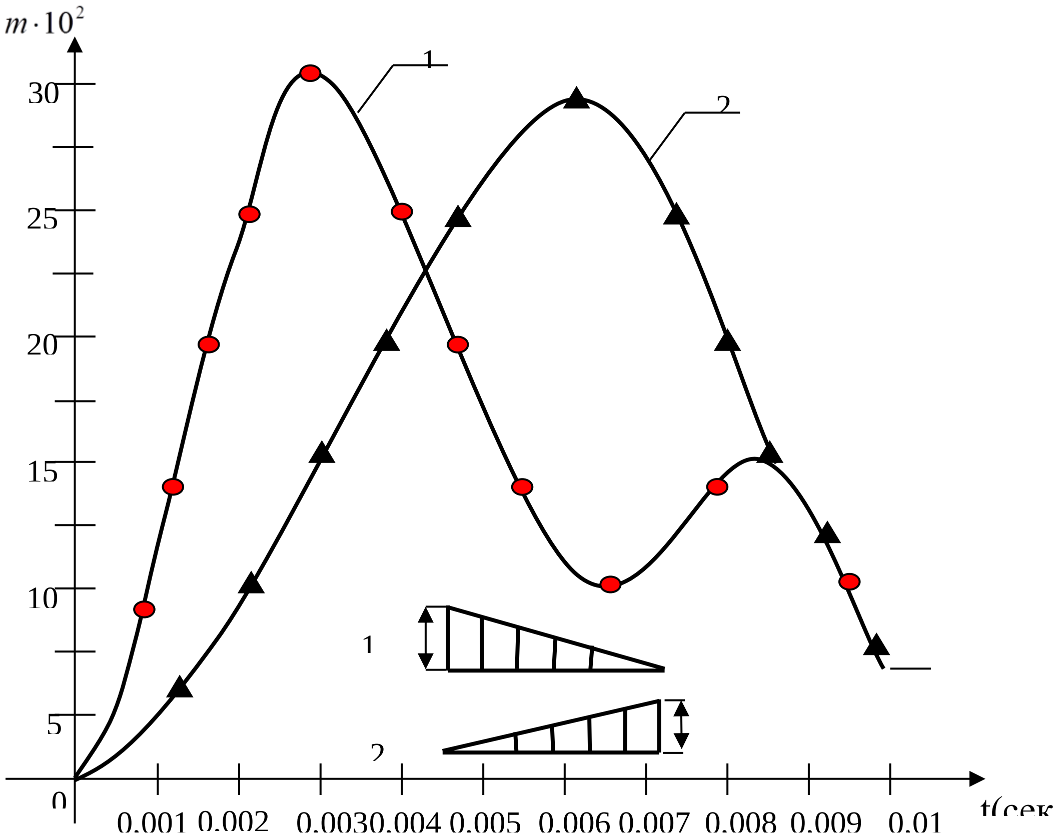


Figure 5 Change the greatest moment in time

From Figure 5 it is clear that the time reaches its maximum value at the initial time, and then gradually decreases.

Dynamic stress-strain state of an infinitely long cylindrical shell when exposed to explosive load.

Moving wall infinitely long cylindrical shell with instant annexed thereto unit load found by solving for the ultimate shell passing to the limit l → ∞. Form this case, the solution (14) takes the form

$$W = \frac{1}{\pi}\int_0^{\infty} \frac{\cos yx_1}{\left(y^4 + 4\beta^4\right)D}\left(1 - \cos\frac{1}{x}\sqrt{y^4 + 4\beta^4 t}\right)$$

This expression coincides with the expression obtained A.I. Zeitlin [5] with the cosine - Fourier transformation.

Table 2. Estimated efforts in the wall

X./H	X	W, мм	X''	M_X, кн. м/м	M_φ, кн.м/м	N_φ, кн/м	X'''	Q_x, кн/м
0	0	0	-11, 011	-1030,68	171,78	0	6,192	0
0,1	1,773	0,063	-7,294	-682,76	113,79	318,8	6,15	575,67
0,2	6,136	0,216	-3,661	-342,69	57,11	1093,1	5,885	550,87
0,3	11,856	0,419	-0,296	-27,71	4,62	2120,4	5,267	493,02
0,4	17,706	0,625	2,582	241,69	40,28	30162,9	4,24	396,89
0,5	22,635	0,779	4,727	442,47	73,75	4043,5	2,835	265,37
0,6	25,88	0,914	5,934	555,45	92,58	4625,5	1,138	106,52
0,7	27,029	0,964	6,072	568,37	94,73	4827,9	-0,705	-65,99
0,8	26,024	0,919	5,083	475,79	79,3	4650,8	-2,555	-239,16
0,9	23,227	0,82	3,028	283,44	47,24	4149,8	-4,27	-399,69
1	19,335	0,683	0	0	0	3456,5	-5,75	-538,23

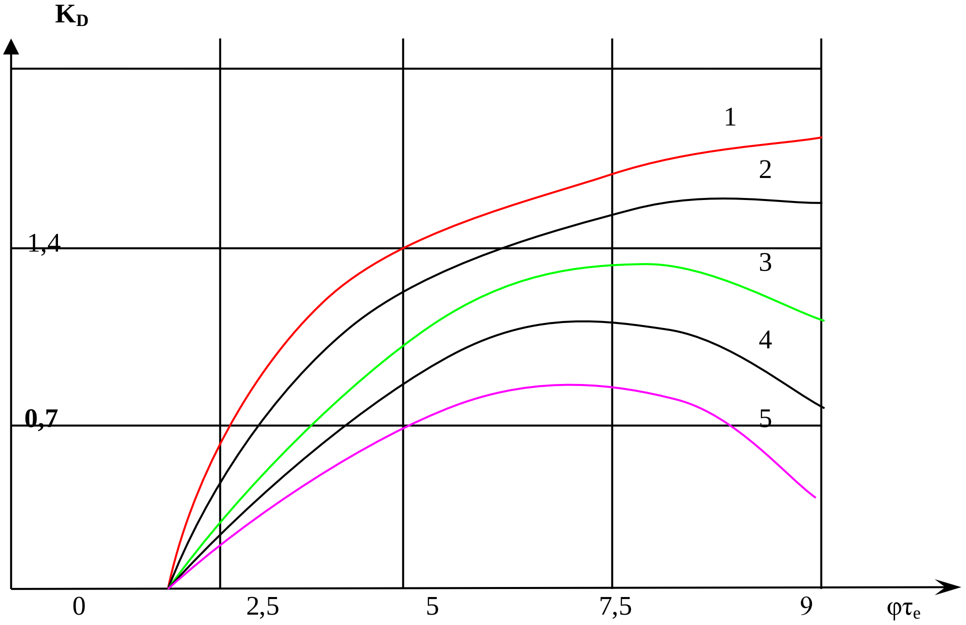
If the applied load instantaneously after time T will be charged, the decision in this case, was found by a superposition, will have the following form

$$W = \frac{1}{\pi}\int_0^\infty \frac{\cos yx_1 \sin \frac{1}{2\lambda}\sqrt{y^4 + 4\beta^2 T\, y_1(t)}}{D_1}$$

At t > T where y_1 (t) = sin $\frac{1}{\lambda}\sqrt{y^4 + 4\beta^2}\left(t - \frac{T}{2}\right)$

$D_1 = D\,(y^4 + 4\beta^4)$

Figure 5 and 6 shows the stress in the cylindrical body by the action of impulse and step load. Subsequent waves effects are less and less energy, so for practical purposes is sufficient to apply no more than two - three waves. Calculated efforts shell wall are given in Table 5.



1-α = 0,01; 2 – α = 0.05; 3 – α = 0,2; 4 – α = 0,4; 5 – α= 0,6

Fig. 6. Schedule to determine the values of the dynamic coefficients

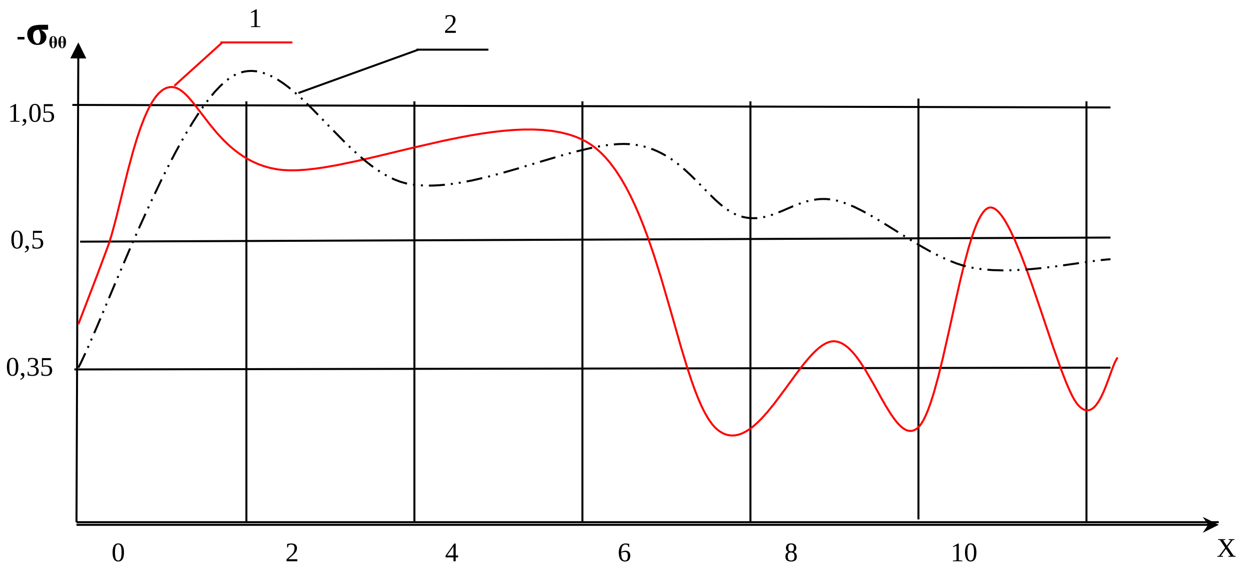
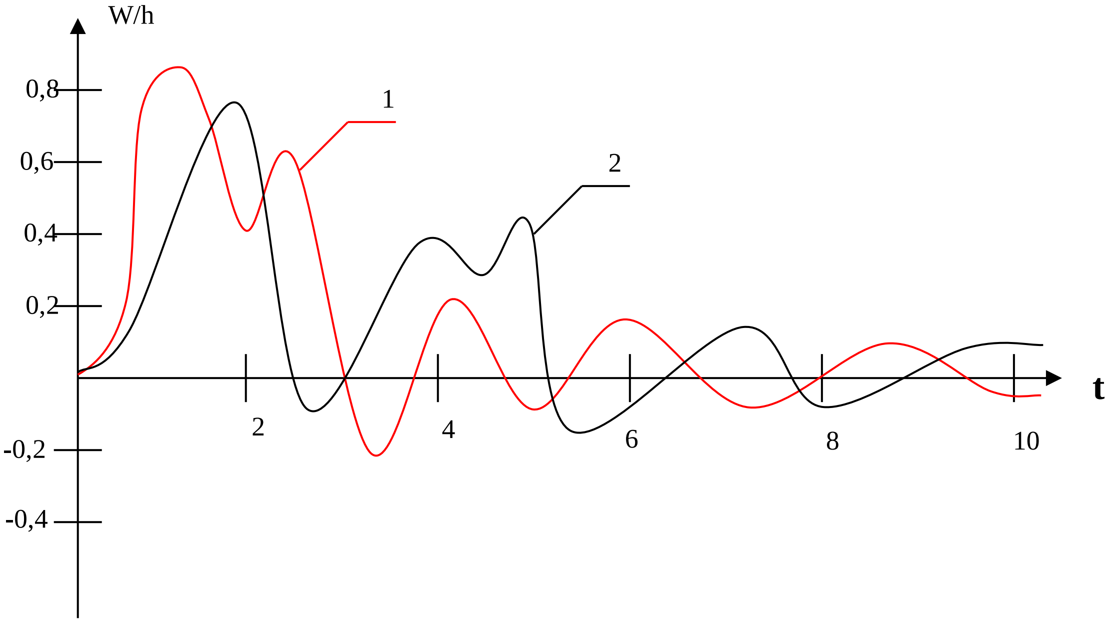


Fig. 7. Stresses in cylindrical body with a pulse speed and load

In view of the required accuracy of the results of calculations are presented in Figure 3.8, with υ = 0,25; E = 2,1.10^5 кг/см^2 $\frac{h}{R} = 0{,}1$; R = 1; 2; 3. N = 10^{-4}; 1, 2, 3, ..5, and step h = 0,1; 0,01; 0,001. At N = 5 and N = 6 value w differs from the previous fifth decimal place. change w depending on t shown in Figure 8 .



1 – $\lambda_1\Theta_1 = 2{,}5$; 2 – $\lambda_1\Theta = 10$.

Fig. 8 Change displacement versus time.

It can be seen that with increasing time (t> 0,03 sec) movement reaches its maximum value, and then approaches zero. The results are presented in Figure 8.

Conclusions

1. An algorithm and a program is to address the problem of the impact of shock waves on the cylindrical shell. Numerical results and to analyze their error. This technique is not very important in terms of structural strength. In the axially symmetric case, the effects of reflection are extended mainly.

2. The results show that the effect of the reflected waves is significant at relatively small scales charge. The largest county deformation concentrated in the central zone of the cylinder, near the line of the meeting, and the highest - longitudinal related to edge effects - in the vicinity of the ends. Predominant among the are district deformation.

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