Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 2 Issue 6, June - 2015

Reliability Based Design Of Two-Way Slab In Comparison With Design Based On BS 8110 (1997)

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***Abstract*— This research presents the comparison between Reliability and deterministic design of two-way solid slab in accordance with BS8110 (1997). First Order Reliability Method (FORM) and the theory of Statistics are reviewed and adopted for designing the floor to a pre-determined safety level using a FORTRAN subroutine created and linked with the reliability software (FORM5). Reliability and deterministic design yielded satisfactory results for long span (mid-span and continuous edge) and short span (mid span only) with average safety index of 3.20 in bending and 1.48 in deflection which satisfies the JCSS (2000) code's requirement. Reliability design yielded an economical section in the other of 10% while deterministic design yielded an unsatisfactory result for short span (continuous edge only). This revealed the weakness in ultimate limit state (ULS) design of the two-way slab along the shorter span (continuous edge only). Correction factor was also proposed to counter this weakness.**

Keywords—Two-way solid slab, deterministic, FORM, FORTRAN, BS 8110(1997), structural reliability.

I. Introduction

The main aim of structural design is the achievement of a structure with an acceptable low probability of failure that will be safe, serviceable and economical during its life time. Reinforced concrete structures present significant nonlinear behavior and consequently nonlinear analysis of this kind of structure has been subject of research for many years [1], [2]; [3], [4], [5]. Several authors have proposed structural models, which can take into account most aspects of the nonlinear behavior of concrete structures [6], [7], [8], [9], [10]. However, reinforced concrete structures are subjected to strong uncertainties, both related to the properties of the material and the applied load effect. Consequently, the design of structures that will need to work under real conditions need to take into account these uncertainties to some extent. These uncertainties in the early 60's were considered by applying some safety factor during the design stage [11]. It is true that in the 21st century, great edifices have littered the globe but every structure is built with some factors of ignorance known as factors of safety [12]. These safety factors were established only by means of "engineering judgment", and not by a rigorous scientific approach. The next step was the design of structures according to limit states [11] that is the approach recommended by most structural design standards nowadays. In this case, the properties of each material and the magnitude of each load is decreased/increased according to its respective factor. These factors are evaluated based on probabilistic analysis and presented as fixed values in design standards. For this reason design using limit states is also known as semi-probabilistic design. The factors were actually evaluated using probabilistic analysis, but the designer makes a deterministic analysis using reduced/increased resistances/loads. It turns out that design standards are not able to cover the full range of application that engineers are able to conceive. Even if some kind of design standards are available for most kind of constructions (such as buildings, bridges and dams), sometimes the engineers need to design some structure that does not fit exactly in any standard due to its size, complexity or multidisciplinary nature. In these cases (or in cases that the engineer wants to) probabilistic analysis can be pursued. Full probabilistic analysis, where one aims for a full probabilistic characterization of the behavior of the structure, needs in general much computational effort [13]. Fortunately, in many cases it is enough to study the structure from the optics of "fail" versus "do not fail". In these cases one can substitute a full probabilistic analysis by a reliability analysis, where only the failure probability is evaluated. This takes much less computational effort, and can be successfully applied to several structural problems. One common feature of most reliability analysis methods is that they need to evaluate the response of the system and its gradient according to the probabilistic variables several times. [11], [13], [14], [15]. Consequently, the application of such methods to nonlinear problems must be made with care, since making a single nonlinear analysis can be a time consuming process.

II. First order reliability method

Reliability and deterministic approaches to design differ in principle. Deterministic design is based on total discounting of the occurrence of failure. Partial factors of safety are used to cater for these uncertainties. On the other hand, reliability design is concerned with the probability that the structure will realize the functions assign to it. It is a measure of the ability of the structure to perform, or to be capable of performing, a required function without Failure under Stated Condition for a stated period of time on unit of operation. Reliability is usually specified in terms of probability of failure.

A common measure of reliability of structural members is through safety index (β). This is expressed in terms of resistance (R) and load effect (S) of the structure. R and S are random variables. The purpose of reliability analysis of any system or component is to ensure that R is greater than or equal to S. In practice, R and S are usually functions of different variables. In order to evaluate the effect of the, variables on the performance of the structural system, a limit state equation is required. This limit state equation is called performance function and expressed in equation 1.

$$g(x_i) = g(x_1, x_2, x_{3\ldots\ldots\ldots\ldots}, x_n) \\ = R - S \ldots\ldots\ldots\ldots\ldots \qquad (1)$$

Where, x_i represents basic variables for $i = 1,2,3,4 \ldots\ldots.., n$

$$g(x_i) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

Graphically, when

$g(x_i) = 0$ Represents the failure surface

$g(x_i) > 0$ Represents safe boundary

$g(x_i) < 0$ Represents unsafe boundary

For uncorrelated reduced variates,

$$x' = \frac{x_i - \mu_{xi}}{\sigma_{xi}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

Where $i = 1,2, \ldots\ldots\ldots\ldots\ldots..n$

The limit state in terms of reduced variates is given by;

$$g(\sigma_{xi}x'_i + \mu_{xi}, \sigma_{x2}x'_2 + \mu_{x2} \ldots\ldots\ldots\ldots.\sigma_{xn}x'_n \\ + \mu_{xn} \ldots\ldots \qquad (4)$$

Where, μ and σ are the means and standard deviations of the design variables. The distance D, from a point $x'_i = (x'_1, x'_2, x'_{3\ldots\ldots\ldots\ldots}, x'_n)$ on the failure boundary $(g) = x'_i$ to the origin of x'_i space is given by;

$$D = \sqrt{x'^2_1 + x'^2_2 + \cdots \ldots\ldots\ldots\ldots\ldots.. + x'^2_n} \ldots \qquad (5)$$

Equation 4 and 5 can be solved by transforming it into vector gradient

$$G'x' = -\frac{G'GD}{\sqrt{(G'G)}} = -(G'G)^{\frac{1}{2}}D \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (6)$$

The minimum distance from the origin describing the variable space to the line representing the failure surface equals β and equation (7) becomes;

$$D = -\frac{G'x'}{\sqrt{(G'G)}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (7)$$

$$\beta = -\frac{G^{*'}x^{*'}}{\sqrt{(G'G)}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (8)$$

Where $G^{*'}$ is the gradient vector at the most probable failure point $(x^*_1, x^*_2, x^*_{3,\ldots\ldots\ldots\ldots,} x^*_n)$ and the value of safety index, β is the measure of the safety of any given design under uncertainties in the decision variables. Therefore equation (8) can be represented in scalar form as;

$$\beta = \frac{\sum_i x^{*i}\left(\frac{\partial g}{\partial x'_i}\right)}{\sqrt{\sum_i\left(\frac{\partial g}{\partial x'_i}\right)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$evaluated\ at\ design\ point$

Equation (9) can be truncated at first order linear term and simplified to;

$$\beta = \frac{\mu_g}{\sigma_g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

III. BS 8110 (1997) requirements for two-way solid slab

Section 3.5 of the Code [17], gives in depth details of the analysis and design of solid slab.

1) Structural analysis

Moment: A two – way spanning slab is a slab with reinforcement in both directions. The reinforcement parallel to the shorter dimensions are designed first and acts as main reinforcement while the reinforcement parallel to the longer side are designed later and distributes onto the shorter side reinforcements. It is recommended that none of the reinforcement sizes should be less than 12 mm in diameter; except top distribution bars, which may be 10 mm in diameter. A slab is designed in both directions generally when:

a) The longer sides exceed 4.5 m in domestic buildings;

b) The slab is heavily loaded, and the shorter span exceeds 4.0 m;

c) In the opinion of the design engineer, it is more economical to design the slab in two directions and

d) When deflection may be excessive, if the slab is designed in one direction.

The analysis of a two-way spanning slab is governed by section 3.5.3 of the code [17]. There are basically two types of two-way spanning Slabs as follows:

Types 1 are those without restraints at the edges. Types 2 are those with restraints at the edges. The moment of a two-way slab is given by:

$$M = \beta w l^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Where,

β = moment coefficient obtained from tables 3.13 and 3.14 of the BS 8110 code.

w= load on the slab

l = Span of slab (shorter span)

In design of slabs, the serviceability limit stale of cracking is controlled by bar spacing rules. The span effective depth ratio and the cover are important factors in the selection of slab thickness.

2) Limit state of deflection

In design, deflection is usually controlled by limiting the ratio of the span to the effective depth of the slab cross section. The following steps are usually followed to check the deflection requirements.

a. Select the basic span/depth ratio from table 3.10 of the code, namely, 7 for cantilever slabs, 20 for simply supported slabs and 26 for continuous slabs

b. The basic span/depth ratio is now multiplied by a modification factor obtained from table 3.11 of the code [17], to allow for the effect of the tension reinforcement.

c. The product obtained from step b, above can be multiplied where necessary by a modification factor, obtained from table 3.12 in [17] to allow for the effect of the compression reinforcement.

With respect to [17] deflection requirements are met when the actual span/effective depth ratio does not exceed that obtained from step (c) above.

3) Limit state of cracking

In design, crack width is usually controlled by limiting the spacing of the reinforcement in a typical slab cross-section. The requirement is that the clear spacing between bars should not exceed 3d or 750 mm whichever is less, where d is the effective depth of the slab. This requirement applied to both main and distribution bars. However, the institution of structural Engineers manual [4] gives separate but more restrictive recommendation for main bars and distribution bars spacing should not exceed 3d or 300mm; distribution bar spacing should not exceed 3d or 400mm.

If the above requirement is met, no further check on bar spacing is necessary, provided that at least;

(a) For f_y = 250N/mm^2, h<250mm; (where h is the overall depth of the slab);

(b) For f_y = 460 N/mm^2: h < 200m;

(c) The steel ratio, ρ = (As/bd) < 0 3%;

(As = minimum recommended area of steel)

If none of the above three conditions are met then the main bar spacing should be limited as given in clause 3. 12. 11.2 3 of [17]

4) Minimum area of reinforcement

The area of reinforcement in each direction should not be less than.

(a) 0.13% of bh for f_y = 460N/mm^2

(b) 0.24% of bh for f_y = 250N/mm^2

In cases where the control of shrinkage and temperature cracking is important, the minimum areas in (a) and (b) above should be increased to 0.25% and 0.3% respectively.

5) Concrete cover for durability

Table 3.4 in [17] gives the nominal covers to meet the durability requirements for slabs and other structural members. Nominal cover is defined in [17] Clause 3.3.1.1, namely, the nominal cover is the design depth of concrete cover to all reinforcement, including links.

6) Performance function

The concept of limit state is to help to define the mode of failure of a structure [18]. A limit state function is defined as the boundary between the desired and undesired performance of a structure. For design of the two-way slab, the performance functions are derived from the various mode of bending such as bending and deflection.

a. For bending, the performance function is given by;

$$R_n = 0.95 f_y z A_s \;\ldots\ldots \qquad (12)$$

The slab is examined for the limit state exceeding the capacity in bending. The limit state would be:

$$g(f_y, f_{cu}, A_s, w, d, b) = 0.95 f_y z A_s \;\ldots\ldots\text{(13)}$$

Where,

$M = \beta w l^2$

A_s= area of reinforcement

f_y= characteristics yield strength of steel

f_{cu}= concrete strength

z= lever arm = $d(0.5 + \sqrt{\left(0.25 - \frac{k}{0.9}\right)}$

k=$\frac{M}{f_{cu} b d^2}$

d= effective depth of slab.

The limit state is evaluated using the mean, standard deviation, bias factor and the coefficient of variation of the random variables.

b. Deflection

The code specifies:

Limiting deflection α $\frac{Allowable\ span}{effective\ depth} =$

$$effective\ \ depth\ \ ratio \times m.f_{TS} \times m.f_{CS}$$

Actual deflection α $\frac{Actual\ \ span}{effective\ \ depth}$

But,

$$M = \beta w l^2$$

$$m.f_{TS} = 0.55 + \frac{\left(477 - {}^{2}\!/_{3}\, f_y \frac{As}{A_{sp}}\right)}{120\left(0.9 + \frac{M}{bd^2}\right)} \ldots. \qquad (14)$$

Therefore, the limit state in deflection is given by:

$$g\left(f_y, w, b_v, d, L, A_s, A_{sp}\right) = \frac{lx}{m.f \times eff.depth\ ratio} - \frac{L}{d} \ldots(15)$$

The stochastic models for the basic variables in the different limit state are calculated from equation 16-18.

$$COV = \frac{S(X)}{E(X)} \ldots\ldots\ldots\ldots\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(16)}$$

$$E(X) = N \times \lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \text{(17)}$$

$$S(X) = COV. \times E(X) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(18)}$$

Where,

COV = Coefficient of variation of the basic variables, S(X) = Standard deviation of the basic variables, E(X) = Mean of the basic variables, λ= Bias factor of the basic variables

N = Nominal values of the basic variables obtained from the deterministic analysis of the two-way slab.

The coefficient of variation and the bias factor are computed accordingly.

IV Results and discussion

1) Design of two-way slab based on BS8110 (1997)

The panels below were designed as two-way slab and results presented in table I.

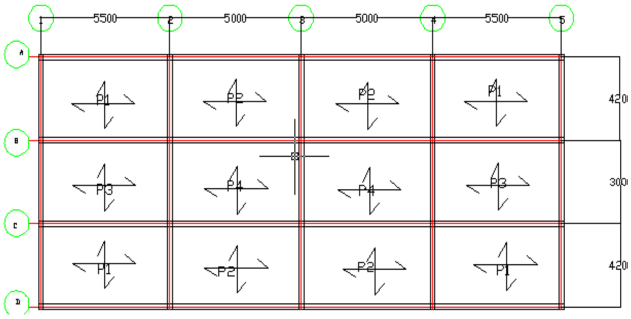

Fig. 1: Plan of an office complex.

2) Reliability based design using FORM

The reliability analysis and design was carried out using the first order reliability method (FORM) and with a simple subroutine created for the limit state and then linked with FORM5 (reliability software).

3) Comparison between BS 8110 (1997) and reliability based design of two-way slab

The comparison between the deterministic and reliability design of the two-way solid floor are presented in table I.

TABLE I: Comparison between deterministic design and Reliability design for P4.

Description/Equations		**DET. DESIGN $1.4G_K+1.6\ Q_K$**	**Safety Index, β using FORM**	**RELIABILITY DESIGN**	**Safety Index, β using FORM**	**Remark**
Depth, h (mm)		150	-	150	-	-
Effective depth, d (mm)		119	-	119	-	-
$A_{s,}$ Area of steel prov. (mm^2)						
Mid-span	Short span	377	3.75	339	3.22	Satisfactory
Cont. edge		377	**2.27***	452	3.20	*
Mid-span	Long span	377	3.97	314	3.05	Satisfactory
Cont. edge		377	5.12	262	3.33	Satisfactory
Bars used		Y12 @300c/c	-	Y10,Y12,Y10,Y10	-	-

2.27* implies unsatisfactory safety index for the continuous edge

TABLE II: Variation of safety index with live-dead load ratio

Qk/Gk	0.25	0.5	0.75	1	1.25	1.5	1.75	2
β	4.08	3.11	2.22	1.39	0.613	-0.119	-0.814	-1.48

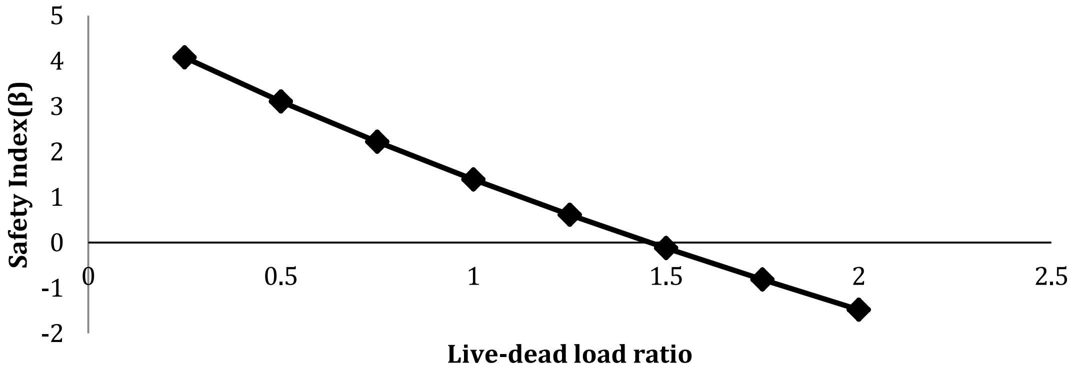

Fig. 2: Graph of safety index against live-dead load ratio

TABLBE III: Variation of safety index with depth of slab

h(mm)	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
β	0.375	2.57	3.87	4.77	5.45	5.98	6.39	6.70	6.95

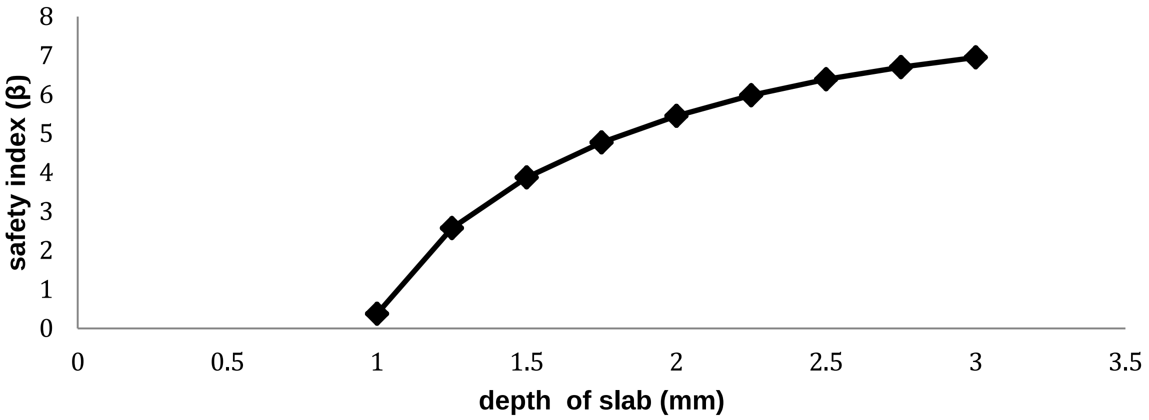

Fig. 3: Graph of safety index against live-dead load ratio

TABLE IV: variation of safety index with length of slab

Length(m)	2.5	3	3.5	4	4.5	5	5.5	6	7
β	7.41	6.54	5.45	4.31	3.17	2.02	0.754	-0.207	-1.95

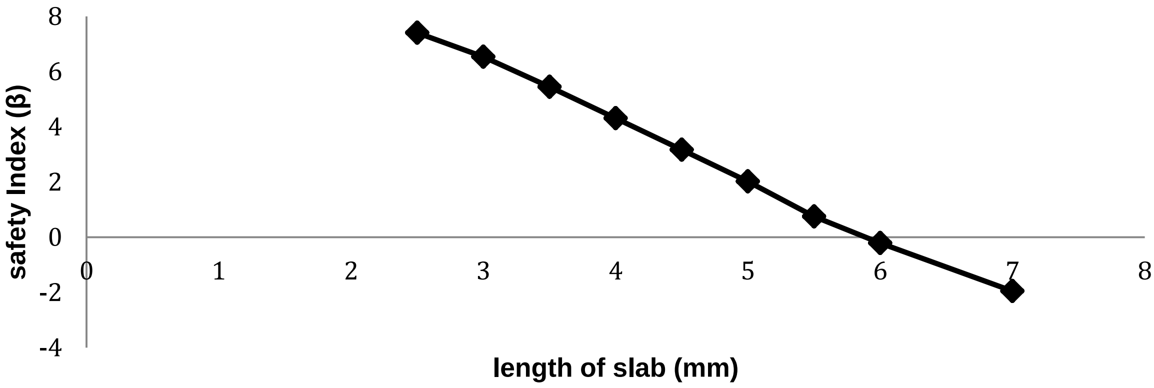

Fig. 4: Graph of safety index against live-dead load ratio

4) Discussion of results

The deterministic design of the two-way slab following the BS8110 (1997) code's requirements yielded an area of reinforcement, A_s=377 mm^2 and depth , h = 150 mm for all panels with an unsatisfactory area of reinforcement along the shorter span (continuous edge only), while the reliability analysis on the other hand, yielded satisfactory area of reinforcement in the order of 339, 452, 339 and 262 mm^2 and depth, h = 150 mm for short span and long span respectively with average safety index, β=3.20 in bending which satisfies the JCSS (2000) code's requirement for safety index based on different consequences of failure. This revealed the weakness in ultimate limit state (ULS) design of the two-way slab along the shorter span (continuous edge only).

A resistance model factor of 1.2 obtained from the ratio of reliability area to deterministic area was proposed for correcting the weakness of the ULS design of the two-way slab along the shorter span (continuous edge only).

Fig. 2 shows the reduction of safety index with increase in live-dead load ratio.

Fig. 3 shows that reliability index increases with increase in the depth of slab but becomes unsafe at depth of 100 mm and uneconomical at depth between 225 mm to 300 mm.

Fig. 4 revealed that reliability index decreases with increase in slab length.

V. Conclusions and recommendations

1) Conclusions

- Reliability analysis yields area of reinforcement capable of resisting all load effect on the structure at the same time enhancing the structural safety of the two-way slab. It also yields an economical design with a predetermined safety level and failure probability.
- The results of the sample run from FORM5 on the slab panels designed in this research can resist load effect comfortably using the optimized values of the resistance which also gives lower probability of failure.

2) Recommendations

- The use of FORM is very essential in all engineering designs to pre-determine the safety index of structural elements. Therefore, it is recommended that the use of FORM be encouraged in design to check whether a structural element like two-way slab satisfies the code's safety requirements or not.

Acknowledgement

Authors are very grateful to Civil Engineering laboratory, University of Ilorin and other parties who had helped by providing data and useful information as well as giving various thought in this research.

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