Eighth Degree Spline Solution for Seventh Order Boundary Value Problems

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***Abstract*—In this communication we developed a numerical method by constructing an eighth degree spline function using Bickley's method and apply it to solve the linear seventh order differential equation with the specified boundary conditions. We consider three different problems with different boundary conditions to illustrate the efficiency and implementation of the method. The results reveal that the method is very effective, straightforward, and simple.**

Keywords—Seventh order boundary value problems; Cubic spline functions; Numerical results.

I. INTRODUCTION

In many fields the boundary value problems of ordinary differential equations play an important role. These problems occur very frequently in various fields of science and engineering such as mechanics, electro hydro dynamics, quantum physics and theory of thermal expansions. Bickley [1] has considered the use of cubic spline for solving second order two point boundary value problems. The essential feature of his analysis is that it leads to the solution of a set of linear equations whose matrix coefficients are of upper Heisenberg form. Fifth-order boundary value problems generally arise in mathematical modeling of viscoelastic flows[3, 4].Caglar et al.[2] solved fifth order BVPs by collocation method with sixth degree B-splines. Kasi viswanadham and Murali Krishna[5] developed a finite element method involving Galarkin method with quintic B-splines as basis functions to solve fifth order BVPs. A conventional approach for the solution of fifth order boundary value problems using sixth degree spline functions has been given by Parcha Kalyani et al. [6].Siddiqi and Ghazala [9] presented the solution of fifth order boundary value problems using non polynomial spline technique. The dynamo action in some stars may be modeled by sixth order boundary problems, which arise in astrophysics. The theory of seventh order boundary value problems is seldom in the numerical analysis literature; generally arise in modeling induction motors with two rotor circuits. The solution to these type of problems is given by Siddiqi Ghazala and Muzammal [10]. Siddiqi Ghazala and Muzammal [8] presented a method based on the solution of seventh order boundary value problems using variational iteration technique. Siddiqi.S.S. and Muzammal [7] presented the solutions of seventh-order linear boundary value problems by variation of parameters methods.

In this article we develop numerical method to solve seventh order boundary value problems using eighth degree spline functions.

II. CUBIC SPLINE BICKELY'S METHOD

Suppose the interval $[x_0,x_n]$ is divided in to n sub intervals with knots $x_0, x_1, x_2, \ldots, x_n$, starting at x_0 the function u(x) in the interval $[x_0, x_1]$ is represented by a cubic spline in the form

$$S(x)= a + b(x - x_0) + c(x - x_0)^2 + d(x - x_0)^3 \quad (1)$$

For the next interval $[x_0, x_1]$ the spline function S(x) is supposed in the form

$$S(x) = S(x) \text{ on } [x_0,x_1] + d_1(x - x_1)^3$$

Proceeding in to the next interval $[x_2,x_3]$ we add term $d_2(x - x_2)^3$ and so on until we reach x_n. Thus the function S(x) is represented in the form

$$S(x)= a + b(x - x_0)+ c(x - x_0)^2 +\sum_{i=0}^{n-1} d_i(x - x_i)^3 \quad (2)$$

Differentiating (2) we get

$$S'(x)= b + 2c(x - x_0)+ 3\sum_{i=0}^{n-1} d_i(x - x_i)^2 \quad (3)$$

$$S''(x)= 2c + 6\sum_{i=0}^{n-1} d_i(x - x_i) \quad (4)$$

a. The two point second order boundary value problems

We consider the linear differential equation

$$P(x)u''+q(x)u'+r(x)u=v(x) \quad (5)$$

With the boundary conditions

$$\alpha_0 u+\beta_0 u=\gamma_0 \text{ at } x=x_0, \text{ at } x=x_0,\ \alpha_n u + \beta_n u= \gamma_n \text{at } x=x_n \quad (6)$$

The number of coefficients in (2) is (n+3), the satisfaction of the differential equation by the spline function at (n+1) nodes gives (n+1) equations in the (n+3) unknowns. Also the boundary conditions (6) give two more equations in the unknowns. Thus we get (n+3) equations in (n+3) unknowns a, b, c, $d_0, d_1, d_2, \ldots, d_{n-1}$.

After determining these unknowns we substitute them in (2) and thus we get the cubic spline approximation of u(x). Putting x= $x_0, x_1, x_2, x_3, \ldots, x_n$ in the spline function thus determined, we get the solution at the nodes. The system of equations to be satisfied by the constants a, b, c, $d_0, d_1, d_2, \ldots, d_{n-1}$, is derived below.

Substituting (2), (3), (4) in (5) at x=$x_m$we get

$a r_m + b[r_m(x_m - x_0) + q_m] + c[r_m(x_m - x_0)^2 + 2q_m(x_m - x_0) + 2p_m] + \sum_{i=0}^{m-1} d_i \; [r_m(x_m - x_0)^3 + 3q_m(x_m - x_i)^2 + 6p_m(x_m - x_i)] = S_m$ (7)

where$p_m = p(x_m)$, $q_m = q(x_m)$, $r_m = r(x_m)$ and $S_m = S(x_m)$

Applying boundary conditions (6), we get

$\alpha_0 a + \beta_0 b = \gamma 0$; $\alpha_n a + [\alpha_n(x_n - x_0) - \beta_n]b + [\alpha_n(x_n - x_0)^2 - 2\beta_n(x_n - x_0)]c + \sum_{m=0}^{n-1}[\alpha_n(x_n - x_0)^3] - 3\beta_n[\alpha_n(x_n -)^2]d_m = \gamma_n$ (8)

If these equations are taken in order (8), (7) with m= n, n-1, .,0, the coefficient matrix of unknowns d_{n-1}, d_{n-2}, d_{n-3}, . . . , d_1 , d_0, c, b, a is the Hessenberg form namely an upper triangle with a single lower sub diagonal. The forward elimination is then simple with only one multiplier at each step and the back substitution is correspondingly easy.

III. Construction of Eighth degree spline

Suppose the interval $[x_0, x_n]$ is divided into n subintervals with grid points x_0, x_1, x_2, x_3, . . ., x_n. Starting at x_0, the function y(x) in the interval $[x_0, x_1]$ is represented by eighth degree spline.

$$S(x) = a + b(x - x_0) + c(x - x_0)^2 + d(x - x_0)^3 + e(x - x_0)^4 + g(x - x_0)^5 + h(x - x_0)^6 + j(x - x_0)^7 + k_0(x - x_0)^8$$

proceeding to the next interval $[x_1, x_2]$, we add a term $k_1(x - x_1)^8$, proceeding in to the next interval $[x_2, x_3]$ we add another term $k_2(x - x_2)^8$ and so on until we reach x_n. Thus the function y(x) is represented in the form

$$S(x) = a + b(x - x_0) + c(x - x_0)^2 + d(x - x_0)^3 + e(x - x_0)^4 + g(x - x_0)^5 + h(x - x_0)^6 + j(x - x_0)^7 + \sum_{i=0}^{n-1} k_i(x_n - x_0)^8 \quad (9)$$

It can be shown that S(x) and its first seven derivatives are continuous across nodes.

a. Method of obtaining the solution of seventh order boundary value problems using eighth degree spline function

Consider the linear seventh order differential equation

$$y^{(7)}(x) + f(x)y(x) = r(x) \quad (10)$$

with the boundary conditions

$y(x_0) = \alpha$, $y(x_n) = \beta$, $y'(x_0) = \alpha'$, $y'(x_n) = \beta'$,

$y''(x_0) = \alpha''$, $y''(x_n) = \beta''$, $y^{(3)}(x_0) = \alpha'''$ (11)

From (11), and taking spline approximation in (10) at x=x_i for i= 0, 1,2,3,4,.,n we get (n+8) equations in (n+8) unknowns a,b,c,d,e,g,h,j,k_0, k_1, k_2, . . ., k_{n}. After determining these unknowns we substitute them in (9) and thus we get the eighth degree spline approximation of y(x). Putting x= x_1, x_2, x_3, . . ., x_n in the spline function thus determined we get the solution at the grid points. The system of equations to be satisfied by the coefficientsa,b,c,d,e,g,h,j,k_0, k_1, k_2, . . . ,k_{n-1}, is derived below.

From (9) we get

$$S^{(7)}(x) = 5040j + 40320\sum_{i=0}^{n-1} k_i(x - x_i) \quad (12)$$

Substituting (9) and (12) in the differential equation (10) at x= x_m we get

$S_m = a f_m + b f_m(x_m - x_0) + c f_m(x_m - x_0)^2 + d f_m(x_m - x_0)^3 + e f_m(x_m - x_0)^4 + g f_m(x_m - x_0)^5 + h f_m(x_m - x_0)^6 + j[f_m(x_m - x_0)^7 + 5040] + \sum_{i=0}^{n-1} k_i [f_m(x_m - x_0)^8 + 40320(x_m - x_i)] r_m$ m=0,1,2,…,n (13)

where $f_m = f(x_m)$, $r_m = r(x_m)$ and $S_m = S(x_m)$.

Since S(x) approximates y(x), from (9) and from the boundary conditions (11) we obtain

$a = \alpha$, (14)

$a + b(x - x_0) + c(x - x_0)^2 + d(x - x_0)^3 + e(x - x_0)^4 + g(x - x_0)^5 + h(x - x_0)^6 + j(x - x_0)^7$

$+\sum_{i=0}^{n-1} k_i(x - x_i)^8 = \beta$ (15)

$b = \alpha'$, (16)

$b + 2c(x - x_0) + 3d(x - x_0)^2 + 4e(x - x_0)^3 + 5g(x - x_0)^4 + 6h(x - x_0)^5 + 7j(x - x_0) + 8\sum_{i=0}^{n-1} k_i(x - x_i) = \beta'$ (17)

$2c = \alpha''$ (18)

$2c + 6d(x - x_0) + 12e(x - x_0)^2 + 20g(x - x_0)^3 + 30h(x - x_0)^4 + 42j(x - x_0)^5 + 56\sum_{i=0}^{n-1} k_i(x - x_i)^6 = \beta''$ (19)

$6d = \alpha'''$ (20)

From (13)-(20) we have (n+8) equations, if these equations are taken in the order (15), (17), and (19) with m= n,n-1, .,0, (20), (18), (16) and (14) the coefficient matrix of the unknowns,k_n,k_{n-1}, . . ., k_1, k_0, j, h, g, e, d, c, b, a will be an upper triangular matrix with two lower sub diagonals.

The forward elimination is then simple with only two multipliers at each step, and back substitution is correspondingly easy.

IV. Numerical illustrations

In this section we consider three linear boundary value problems. Their numerical solution and absolute errors are given at different step lengths. The approximate solution, exact solutions and absolute errors at the grid points are summarized in tabular form. Further the approximate solution and exact solution have been shown graphically. The comparison of maximum absolute errors at different step lengths has been presented in tabular form.

Example 1: Consider the linear non homogeneous seventh order boundary value problem with constant coefficients.

$$u^{(7)}(x) = -u(x) - e^x(35x + 12x + 2x^2), \; 0 \le x \le 1 \quad (21)$$

with the boundary conditions

$u(0)= 0,\ u'(0) = 1,\ u^{(2)}(0) = 0,\ u^{(3)}(0)= -3$

$u(1) = 0,\ u'(1) = -e,\ u^{(2)}(1) = -4e$ (22)

The exact solution is $u(x) = x\,(1 - x)e^{x}$.

We find the solution of (21) - (22) by taking step lengths h = 0.2 and h = 0.1 at equal sub intervals.

Solution with h= 0.2

The eighth degree spline S(x) which approximates y(x) is given by

$S(x)= a + b(x-x_0) + c(x-x_0)^2 + d(x-x_0)^3 + e(x-x_0)^4 + g(x-x_0)^5 + h(x-x_0)^6 + j(x-x_0)^7 +\sum_{i=0}^{4} k_i(x-x_i)^8$ (23)

where $x_0= 0,\ x_1= 0.2,\ x_2= 0.4,\ x_3= 0.6,\ x_4= 0.8,\ x_5 =1$. We have 13 unknowns a, b, c, d, e, g, h, j, k_0, k_1, k_2, k_3, $k_4$and 13 conditions to satisfied by these unknowns are

$S(x_0)= 0,\ S(x_5)= 0,\ S'(x_0) = 1,\ S'(x_5) = -e,\ S^{(2)}(x_0) = 0,\ S^{(2)}(x_5) = -4e,\ S^{(3)}(x_0)= -3,$

$S^{(7)}(x_i)=-S(x_i)-e^{x_i}(35x_i+12x_i+2x_i^2),0\le x\le 1$ (24)

Since $S(x_0)= 0,\ S'(x_0) = 1,\ S''(x_0) = 0,\ S^{(3)}(x_0)=-3,$

it follows that a= 0, b= 1, c= 0 and d=-0.5,

hence the spline S(x) reduces to the form

$S(x)= (x-x_0) - 0.5(x-x_0)^3 + e(x-x_0)^4 +$

$g(x-x_0)^5 + h(x-x_0{}^6 + j(x-x_0)^7 +$

$\sum_{i=0}^{4} k_i(x-x_i)^8$ (25)

From (25)

$S'(x)= 1 - 1.5(x-x_0)^2 + 4e(x-x_0)^3 + 5g(x-x_0)^4 + 6h(x-x_0)^5 + 7j(x-x_0)^6 +8\sum_{i=0}^{4} k_i(x-x_i)^7$ (26)

$S''(x)= - 3(x-x_0) + 12e(x-x_0)^2 + 20g(x-x_0)^3 + 30h(x-x_0)^4 + 42j(x-x_0)^5 +56\sum_{i=0}^{4} k_i(x-x_i)^6$ (27)

$S^{(3)}(x)= - 3 + 24e(x-x_0) + 60g(x-x_0)^2 + 120h(x-x_0)^3 + 210j(x-x_0)^4 +336\sum_{i=0}^{4} k_i(x-x_i)^5$ (28)

and the seventh derivative is

$S^{(7)}(x)= 5040j +40320\sum_{i=0}^{4} k_i(x-x_i)$ (29)

Substituting equation (29) and (23) in (21) we get the following system of equations.

From equation

$u^{(7)}(x_0) = -u(x_0)- e^{x}{}_0(35x_0 + 12x_0 + 2x_0^2),$

we get j= -0.006944444444

From the remaining conditions we get the following system of equations

$e(0.2)^4 + g(0.2)^5 + h(0.2)^6 + 8064.00000256K_0= -10.8960055106560$ (30)

$e(0.4)^4 + g(0.4)^5 + h(0.4)^6 +16128.0006553600k_0+ 8064.00000256K_1= -25.1719954918140$ (31)

$e(0.6)^4 + g(0.6)^5 + h(0.6)^6 + 24192.0167961600k_0+ 16128.0006553600k_1+8064.00000256K_2= -43.6971445129846$ (32)

$e(0.8)^4 + g(0.8)^5 + h(0.8)^6 + 32256.16777721600k_0 + 24192.0167961600k_1 + 16128.0006553600k_2 + 8064.00000256K_3=-67.6506527150140$ (33)

$e+g+h+40321k_0+32256.16777721600k_1+24192.0167961600k_2 + 16128.0006553600k_3 + 8064.00000256K_4 = -98.6888651502728$ (34)

$e + g + h + k_0+ k_1(0.8)^8+ k_2(0.6)^8 +k_3(0.4)^8+k_4(0.2)^8= -0.49305555555556$ (35)

$4e+5g+6h+ 8k_0 + 8k_1(0.8)^7 + 8k_2(0.6)^7+ 8k_3(0.4)^7+ k_4(0.4)^7= -2.16967071734825$ (36)

$12e + 20g + 30h + 56k_0+ 56k_1(0.8)^6+ 56k_2(0.6)^6 +56k_3(0.4)^6 +56k_4(0.2)^6=-7.5814606477138$ (37)

Solving the above set of equations we get the following values

$e=-0.333273629599774\ k_1= -0.000418053692099$

$g=-0.125168664521974\ k_2= -0.00522567927666$

$h=-0.033182790473916\ k_3= -0.000662319571231$

$k_0=-0.001351119803040\ K_4=0.000856903012248.$

Substituting these values in equation (23) we get the spline approximation S(x) of u(x).The values of S(x), u(x) and the corresponding absolute errors at x_1, x_2, x_3, x_4 have been given in the Table I and the comparison has been shown in fig.I.

Table I: Approximate solution S(x), exact solution u(x) and absolute error of example 1 with h=0.2

x	S(x)	u(x)	Absolute error
0.0	0.000000000	0.00000000	0.0000000000
0.2	0.195424490	0.19542444	-5.000000E-08
0.4	0.358038320	0.35803792	-4.016999E-07
0.6	0.437309130	0.43730851	-6.2200000-07
0.8	0.356086800	0.35608654	-2.538000E-07
1.0	0.000000000	0.00000000	0.0000000000

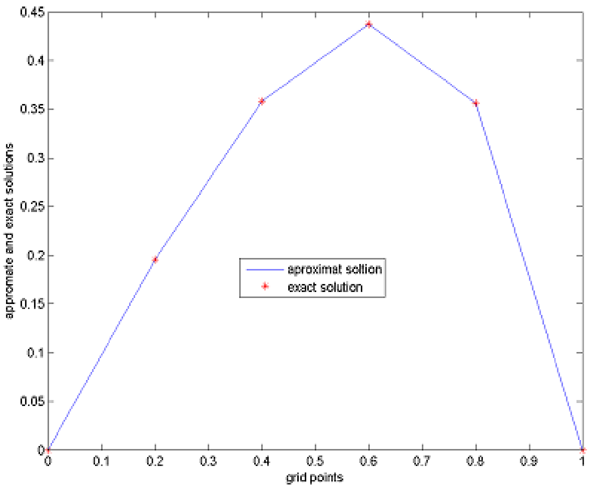

Figure I: Comparison of approximate solution and exact solution for example 1 with h = 0.2

Solution with h= 0.1

Since h= 0.1 we suppose the grid points $x_0 x_1$, x_2, x_3, x_4, $x_5 x_6 x_7$, x_8, x_9, x_{10},where x_0= 0, x_1= 0.1, x_2= 0.2, x_3= 0.3, x_4= 0.4, x_5 =0.5, x_6= 0.6, x_7= 0.7, x_8= 0.8, x_9= 0.9, x_{10}=1.

From equation (9) eighth degree spline S(x) which approximate s u(x) becomes

$$S(x)= (x-x_0) - 0.5(x-x_0)^3 + e(x-x_0)^4 + g(x-x_0)^5 + h(x-x_0)^6 + j(x-x_0)^7 + \sum_{i=0}^{9} k_i (x-x_i)^8 \quad (38)$$

Proceeding as in the above case, we get the following values

e =-0.33439375 k_3= -0.00022969

g=-0.12241914 k_4= -0.00026126

h=-0.03497064 k_5= -0.00029721

j=-0.006944444 k_6= -0.00033812

k_0=-0.00127201 k_7=-0.00038463

k_1= -0.00017772 k_8= -0.00043747

k_2= -0.00020199 k_9 = -0.00037555

Substituting these values in equation (38) we get the spline approximation s(x) of u(x).the values of s(x), u(x) and the corresponding absolute errors at x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} have been given in the table II and the comparison has been shown in fig II.

Table II: Approximate solution S(x), exact solution u(x) and absolute error of example 1 with h=0.1

x	S(x)	u(x)	Absolute error
0.0	0.00000000	0.00000000	0.00000000
0.1	0.09946530	0.09946538	7.9999E-08
0.2	0.19542346	0.19542444	9.8000E-07
0.3	0.28346683	0.28347034	2.3657E-05
0.4	0.3580304	0.35803792	7.4400E-06
0.5	0.41216902	0.41218031	1.1289E-05
0.6	0.43729505	0.43730851	1.3459E-05
0.7	0.42287363	0.42288806	1.4430E-05
0.8	0.35606617	0.35608654	2.0369E-05
0.9	0.22131651	0.22136428	4.7770E-05
1.0	0.00000000	0.00000000	0.00000000

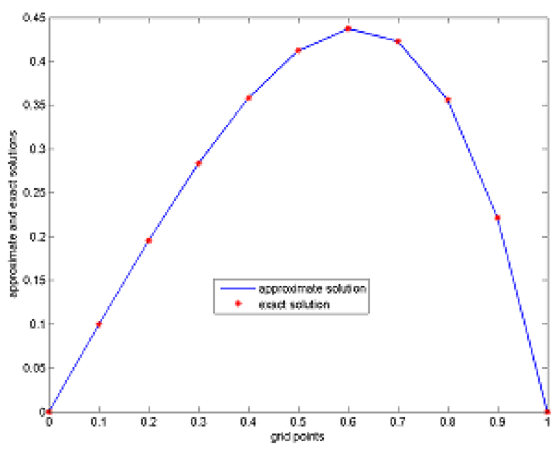

Figure II: Comparison of approximate solution and exact solution for example 1 with h = 0.1

Example 2: Consider non- homogeneous linear seventh order boundary value problem with variable coefficients

$$u^{(7)}(x) = x u(x) + e^{x}(x^2-2x-6),\ 0 \le x \le 1 \quad (39)$$

Subject to the boundary conditions

$$u(0)= 1,\ u'(0)=0,\ u''(0)= -1,\ u^{(3)}(0) = -3,\ u(1)= 0,$$

$$u'(1) = -e,\ u''(1) = -2e \quad (40)$$

The exact solution is $u(x) = (1-x)e^{x}$.

We find the solution of boundary value problems (39-40) by taking the step lengths h= 0.2 and h= 0.1 at equal sub intervals.

Solution when h= 0.2

Since h=0.2 we suppose the grid points x_0= 0, x_1 = 0.2, x_2=0.4, x_3= 0.6, x_4= 0.8, x_5= 1

From equation (9) eighth degree spline S(x) which approximates u(x).

$$S(x)= a + b(x-x_0) + c(x-x_0)^2 + d(x-x_0)^3 + e(x-x_0)^4 + g(x-x_0)^5 + h(x-x_0)^6 + j(x-x_0)^7 + \sum_{i=0}^{4} k_i (x-x_i)^8 \quad (41)$$

From S(x) and boundary conditions we get the following values.

a = 1, b= 0, c= -0.5, d = -0.333333

with these values(41) reduces to the form

$$S(x)= 1- 0.5(x-x_0)^2 - 0.333333(x-x_0)^3 + e(x-x_0)^4 + g(x-x_0)^5 + h(x-x_0)^6 + j(x-x_0)^7 + \sum_{i=0}^{4} k_i (x-x_i)^8 \quad (42)$$

Solving the system of equations obtained from the boundary conditions, we get the following values

e= -0.12498874, k_1= -0.00005591

g= -0.03336552, k_2= -0.00004894

h= -0.00691664, k_3= -0.00008057

j= -0.001190476, k_4 = -0.0001085

k_0=-0.00019502

Substituting these values in equation (42) we get the spline approximation S(x) of u(x). The values of S(x), u(x) and the corresponding absolute errors at x_1, x_2, x_3, $x_4$have been given in the Table III and the comparison has been shown in Fig III.

Table III: Approximate solution S(x), exact solution u(x) and absolute error of example 2 with h =0.1

X	S(x)	u(x)	Absolute error
0.0	1.000000000	1.000000000	0.00000000
0.2	0.977122218	0.977122206	-1.2000E-08
0.4	0.895094904	0.895094818	-8.6000E-08
0.6	0.728847686	0.728847501	-1.8500E-06
0.8	0.445108390	0.445108185	-2.0500E-06
1.0	3.00030E-08	0.000000000	0.00000000

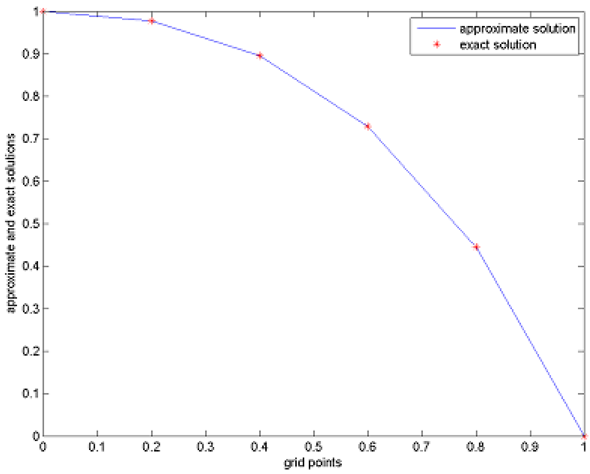

Figure III: Comparison of approximate solution and exact solution for example 2 with h = 0.2

Solution when h= 0.2

Since h=0.1 we suppose the grid points x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} where x_0= 0, x_1= 0.1 x_2=0.2, x_3= 0.3 x_4=0.4, x_5=0.5, x_6=0.6, x_7=0.7, x_8=0.8, x_9=0.9, x_{10} =1.

From equation (9) eighth degree spline S(x) which approximates u(x) becomes

$$S(x)= a + b(x-x_0) + c(x-x_0)^2 + d(x-x_0)^3 + e(x-x_0)^4 + g(x-x_0)^5 + h(x-x_0)^6 + j(x-x_0)^7 + \sum_{i=0}^{9} k_i(x-x_0)^8 \quad (43)$$

From S(x) and boundary conditions we get the following values.

e = -0.1249252519 k_3= -0.0000215572

g = -0.0335857836 k_4 = -0.0000219390

h= -0.0067363055 k_5= -0.0000223603

j= -0.00119047619 k_6= -0.00002279981

k_0= -0.0001839143 k_7= -0.0000148585

k1= -0.000119182 k_8= -0.0000409173

k_2= 0.0000282878 k_9= -0.0000163853

Table IV: Approximate solution S(x), exact solution u(x) and absolute error of example 2 with h=0.1

x	S(x)	u(x)	Absolute error
0.0	1.0000000000	1.0000000000	0.00000000
0.1	0.9946538340	0.9946538260	-7.99990E-09
0.2	0.9771222853	0.9771222065	-7.87990E-08
0.3	0.9449013980	0.9449011650	-2.32699E-07
0.4	0.8950952040	0.8950948185	-3.85500E-07
0.5	0.8243610150	0.8243606350	-4.10000E-07
0.6	0.7288476240	0.7288475201	-1.03900E-07
0.7	0.6041254247	0.6041258122	3.87499E-07
0.8	0.4451073729	0.4451081856	8.12699E-07
0.9	0.2459595550	0.2459603111	7.56099E-07
1.0	0.00000000000	0.0000000000	0.000000000

Substituting these values in equation (43) we get the spline approximation S(x) of u(x).The values of S(x), u(x) and the corresponding absolute errors at x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} have been given in the Table IV and the comparison has been shown in Fig IV.

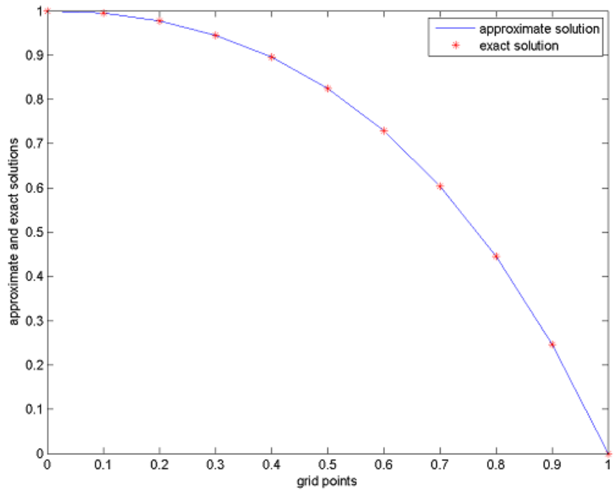

Figure IV: Comparison of approximate solution and exact solution for example 2 with h=0.1

Example 3: Consider the linear non homogeneous seventh order boundary value problem with constant coefficients.

$$u^{(7)}(x)=u(x)-7e^{x}, 0\leq x\leq 1 \quad (44)$$

u (0)= 1, u'(0)=0 u''(0)= -2, u (1)= 0, u'(1)= -e,

u''(1)= -2e

The exact solution of the problem is u(x) = (1-x)e^x.

We find the solution of equation (44) by taking the step lengths h= 0.2 and h= 0.1 at equal sub intervals.

Solution when h= 0.2

Since h=0.2 we suppose the grid points x_0= 0, x_1= 0.2, x_2=0.4, x_3= 0.6, x_4= 0.8 and x_5= 1.

From (43) and the boundary conditions, we get the following values.

e = -0.125041789711 k_1= -0.000049886445

g= -0.033277012926 k_2 = -0.000062416383

h= -0.006956224931 k_3= -0.0000780491328

j= -0.001190476190 k_4= -0.0000975445581

k_0= -0.000195026928

The values of S(x), u(x) and the corresponding absolute errors at x_1, x_2, x_3, x_4 has been given in the Table V and the comparison has been shown in Fig V.

Table V: Approximate solution S(x), exact solution u(x) and absolute error of example 3 with h=0.2

x	S(x)	u(x)	Absolute error
0.0	1.000000000000	1.000000000000	0.000000000
0.2	0.977122183500	0.9771222065280	2.297153E -08
0.4	0.895094482456	0.895094818584	3.361270E -07
0.6	0.728846499626	0.728847520156	1.020529E -06
0.8	0.445106918662	0.445108185698	1.267040E -06
1.0	0.000000000000	0.000000000000	0.0000000000

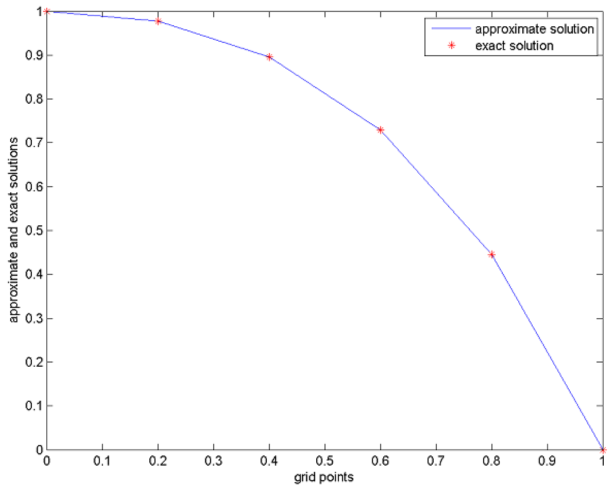

Figure V: Comparison of S(x) and u(x) for example 3 with h=0.2

Solution when h= 0.1

Since h=0.1 we suppose the grid points

x_0= 0, x_1 = 0.1, x_2=0.2, x_3= 0.3, x_4= 0.4, x_5= 0.5,

x_6= 0.6, x_7= 0.7, x_8= 0.8, x_9= 0.9, x_{10}= 1.

Proceeding as in the above case, we get the following values

a= 1 b= 0,

c= -0.5 d= -0.333333

e =-0.125002950953 k_4= -0.000031111763

g= -0.033331216571 k_5= -0.000034793064

h= -0.006942412469 k_6= -0.000038904570

k_0= -0.00018391433 k_7= -0.000043496054

k_1= 0.000022225173 k_8= -0.000048623000

k_2= -0.00002486579 k_9= -0.000054347255

k_3= -0.00002781602.

The values of S(x), u(x) and the corresponding absolute errors at x_1, x_2, x_3, x_4 , x_5, x_6, x_7, x_8, x_9, x_{10} has been given in the Table VI and the comparison has been shown in Fig VI.

Table VI: Approximate solution S(x), exact solution u(x) and absolute error of example 3 with h=0.1

x	S(x)	u(x)	Absolute error
0.0	1.000000000	1.0000000000	0.0000000000
0.1	0.994653829	0.9946538267	3.048609E-09
0.2	0.977122229	0.9771222066	-2.27374E-08
0.3	0.944901238	0.9449011653	-7.02438E-08
0.4	0.895094984	0.8950948186	-1.65563E-07
0.5	0.824360958	0.8243606354	-3.19223E-07
0.6	0.728848077	0.7288475202	-5.57016E-07
1.0	0.604126722	0.6041258122	-9.09862E-07
	0.000000000	0.0000000000	0.000000000

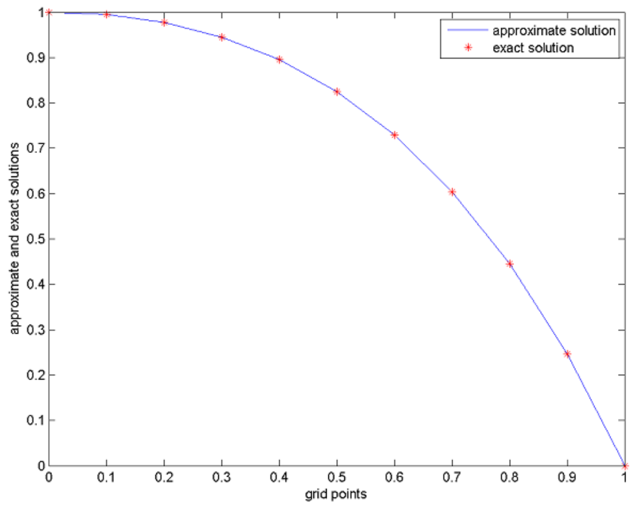

Figure VI: Comparison of approximate solution and exact solution for example 3 with h=0.1

IV. CONCLUSION

We developed the numerical method to obtain the solution of seventh order boundary value problems using eighth degree spline approximation. It has been employed on three examples at different step lengths. At h=0.2 and h=0.1 the maximum absolute error for example1, 2 and 3 are $7.99\times10^{-8}$and -5.0000×10^{-8} , -1.2×10^{-8} and $-7.9999\times10^{-9}$and 1.267040E -06 and -9.0986257E-07 respectively. It is observed that there is a good agreement with the exact solution. It is also noted that the approximate solution is more close to the exact solution when h is small.

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