Study of passengers’ flow at underground stations

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Abstract—In this study has been developed a methodology for processes simulation in an underground station by using a direct-event approach. The passenger flow was investigated with multi-level model in four stages: entrance of the metro station, entrance-hall, validating machines, escalators and stairs. The research has been conducted by using fully licensed software Arena Enterprise Suite Academic Rockwell, version 14. Developed basic simulation model of an underground station, which is studied in different inflow of passengers and parameters of technological processes. The simulation model is verified and validated for 9 main stations of the Sofia subway. There were 219 experimental simulations, which consists equation to determine the total time the passenger in the system - from its entry into the metro to his arrival on the platform. With the help of the equation is obtained an estimate of the total time in the system, without the need to simulate different scenario in the metro. The determination of the total time in the system is necessary in synchronous interval of movement of the ground public transport to points of intermodal connections in underground stations. The model can be used for simulation and analysis of the technological processes in underground stations and also for projections in the design of the underground stations.

Keywords: underground station, simulation modelling, ARENA, queue theory, passenger flows, correlation and regression analysis

I. INTRODUCTION

An underground network system represents an ecological and high-speed rail transport which takes an important role helping public transport satisfy any needs for transportation in cities. Underground stations are places where incoming stream of passengers should be served quickly. The simulations of processes in underground stations are important for making management decisions related to transportation. Simulation modeling gives very useful information for exploitation of an underground station which could not be observed or gained otherwise such as an average time for passengers waiting at underground stations, an average time of available subsystems. This allows us to suggest technological decisions to improve service. Simulation modelling allows us to examine and analyze many technological situations in different time intervals, with different incoming and outgoing streams of passengers and different technological time for service, with different usage of capacity and capabilities of an underground system, which are very hard to be examined in real time.

Underground stations are an interesting subject for simulation modelling and that is the reason why some authors have conducted studies in this area.

A queuing network analytical model of station is created in [1] for calculating subway station capacity, which is built by M/G/C/C state dependent queuing network and discrete time Markov chain. The case study of Beijing Station in Beijing subway line 2 is implemented to verify the validity and practicability of the proposed methods by comparison with simulation. In [2] is elaborated a simulation model of the rail network including a group of four consecutive stations for simulation the vehicle operating and compute special system performance parameters. In [3] a simulation model for streams of passengers has been designed for metro stations. In [4] presented a methodology for managing metro and analytical management model in macro perspective to solve the problem of calculating the number of passengers in the processes of collecting and scattering the specific frequencies of the other passengers, and supply management methodology the station. The presented methodology in the study of modeling underground station (stairs, walkways, doors and vestibule escalators) with unidirectional inflow as M / G / C / C. The arrival of each passenger in the underground station is an independent variable. Therefore adopted Poison law of distribution of frequency “k”. Time for the operation of the modeled elements, such as doors, escalators and Anter has exponential distribution, but can be modeled as a M / M / C / C model.

Principal states of systems for mass service have been developed in [5, 6, and 7].

A detailed simulation of the processes in the entrance-hall, validating machines and subway leading to platforms, has not been conducted in the studies mentioned above.

The aim of this study is to develop a methodology of simulation modeling for technological processes in an underground station.
II. MATHEMATICAL FORMULATION

A. A presentation of a underground station as a queue theory

An underground station is presented by queuing theory as a multi-level open system without priority with four consecutive servicing devices which have their own characteristics.

The stages of the multi-level model are:

- 1st stage: Entrance of the underground station. At this stage, passengers’ waiting in the entrance-hall has been observed. When the entering a underground station is accomplished from two directions entrance-halls are two. Because of that in the next stages servicing devices are examined for both directions separately.
- 2nd stage: Entrance-hall. Here, processes of servicing passengers from the entrance hall to the validating machines have been observed. The focus is on ticket offices and ticket machines.
- 3rd stage: Validating machines. Passengers’ going through validating machines is observed closely at this stage.
- 4th stage: Escalators and stairs. Passengers’ transition from validating machines to a platform.

In the defined multi-level system, there are not any buffers and that’s why it could be seen as compounded of separate single-level systems taking into account the transformations of streams of passengers between them. When the incoming stream of passengers is Poison, and the time of service is exponential and the system is without failures, then the outgoing stream of passengers is also Poison’s. In the study, the system M/M/S has been taken for stages 1, 2 and 3, e.g. Poison’s incoming stream of passengers, exponential time for service, and multi-channel system with a number of channels S. When a metro station is only one entrance then for the first level the system is M/M/1. In the 4th stage the system has been examined by M/D/1 for escalators, e.g. Poison’s incoming stream of passengers, constant time for service, and one channel, and M/M/1 for stairs.

The intensity of a stream of passengers per hour (pass./h) \( \lambda \), coming in the metro station is formed by a stream of passengers which enter the entrance-hall from the two entrances of the metro station.

\[ \lambda = \lambda_1 + \lambda_2, \text{pass./h} \]  

\[ \lambda_1 = \gamma_m \lambda, \text{pass./h} \]  

\[ \lambda_2 = (1-\gamma_m) \lambda, \text{pass./h} \]

where: \( \gamma_m \) is the coefficient which shows the relative part of a stream of passengers coming in a underground station from an entrance 1.

The intensity of the stream of passengers \( \lambda_{1,u} \) from all entrances which goes directly to the validating machines is:

\[ \lambda_{1,u} = (1-\alpha_m) \lambda, \text{pass./h} \]  

where: \( \alpha_m \) is a coefficient which shows the relative part of a stream of passengers that goes directly to the validating machines.

The intensity of the stream of passengers from all entrances which goes to buy tickets from ticket offices or ticket machines is:

\[ \lambda_t = \beta_m \lambda_{1,u}, \text{pass./h} \]

where: \( \beta_m \) is the coefficient which shows the relative part of the whole stream of passengers that goes to ticket offices and ticket machines with a focus on the part that goes to ticket offices only.

To avoid detention at an entrance of an underground station the condition must be met:

\[ \lambda_t < \lambda_c; \lambda_2 < \lambda_c \]

where: \( \lambda_c \) is the limit intensity of the incoming stream of passengers where there would be observed a passengers waiting at an entrance of an underground station.

\[ \lambda_c = p_m F_m, \text{pass./h} \]

where: \( p_m \) is the coefficient showing the optimal number of passengers per \( m^2 \) when conditions of comfort and safety are met, pass./m\(^2\); \( p_m =7\) pass./m\(^2\); \( F_m \) is the area that could be used by passengers freely (without stepping on any restrict lines). For example, the underground station Sofia’s University “St.Kliment Ohridski” has \( \lambda_c = 3350 \) pass./h for common platform; the Metro Station “G.M.Dimitrov” has \( \lambda_c =1675 \) pass./h per a platform.

B. Simulation models whit ARENA discrete event simulation tool

The system for imitation modeling Arena allows us to shape dynamic model for heterogeneous processes which could be optimized [8, 9, 10, 11, and 13]. Modeling is conducted by using the modeling language SIMAN and an animation system. It has been used blocks for modeling which connect to each other in accordance to dependences as well as operations in the studied system. The Arena building blocks used are Create, Waiting, Assign, Signal, Split, Hold, Delay and Dispose. Modules are divided in two categories: flowchart modules and data modules. Flowchart modules describes the dynamical processes of movement and changes in the module. Data modules are defining the characteristics of the various objects like entities, resources and queues.

The main modules included in the simulation models for underground stations are CREATE, DECIDE, PROCESS, DISPOSE, etc.

The CREATE module is the generator of passengers, which can simulate passengers to enter into the system. In this module, the initial creating
times, the max number of passengers, the time between arrivals and the basic unit of time can be set. This module have name in simulate model: ENTRANCE 1, 2.

The DECIDE module is used to determines the percentage of passengers who have more than one choice. This module is used to select the transport document, the selection device for the purchase of transport document, choice of direction, and for the selection of a device for transfer.

The PROCESS module can implement the process of passengers waiting for tells, ticket machines, validators’ machines and the check-in process. Besides, the waiting queue length and waiting time reflected in the report ultimately are also run in the module. Therefore, this module is the core of the simulation. In this module, the queuing rules, the resources and delay types, etc. This module have name in simulate model: TELL; TICKET MACHINE; VALIDATOR; ESCALATORS; STAIRS.

The function of DISPOSE module is to exit of the simulation environment for passenger entities. In this simulation, the module is set in two positions: one position is used for receiving the passengers who miss the check-in time; the other one is used for receiving passengers who get on the platform, to finish the simulation process. This module have name in simulate model: “PLATFORM 1, 2, etc.

In fig.1 is shown the main simulation model for underground station in ARENA software.

This model is made up with the following parameters:
- Average intensity of service validator - 3s/pass.;
- Average intensity of service ticket machines - 3s/pass.;
- Average total duration of the transfer 25 s/pass. (Pedestrian transfer - 10 s/pass.; escalator or stairs - 15 s/pass.);
- Intensity of inflow of passengers - 2000 pass/h.
- Percentage distribution of passengers’ flow with subscription transport document and a single transport document in the metro, α = 0,7 (70% are postpaid transport document);
- The percentage distribution of passengers’ flow supply tickets between ticket office and ticket machine in the underground, β = 0,5, (50% of the passengers who must obtain a ticket using cash registers).
- Ratio of inflow to enter the underground station of the two inputs (input 1-62% entrance 2-38%).

III. APROBATION OF THE METHODOLOGY

Imagine methodology is applied to research in basic simulation model of a underground station, as well as studies of the main stations of the Sofia subway.

A. Variants of simulation

With a main simulation model simulations have been conducted with variations of inflow of 2000 pass./h to 4000 pass./h in increments of 200 pass./h. For each variant of incoming passengers have been tested variants of transfer time passenger (walking motion and moving escalator or stairs to reach the platform). Composed options are an amendment to transfer time from 25 to 35 s. On Figure 2 shows the change of the total time in the system at different inflow and different transfer time, and Figure 3 shows the change of the total time in the system at different inflow of passengers.

Fig.1. Main simulation model in ARENA

![Main simulation model in ARENA](image)

This model is made up with the following parameters:
- Average intensity of service validator - 3s/ pass.;

Fig.2. Total time in the system. Results of the simulations at different inflow

![Total time in the system. Results of the simulations at different inflow](image)
account the percentage distribution of passengers' flow supply tickets between ticket office and ticket machine in the metro,%; $T_m$ is common minimum (by simulation) duration in the system (since the entry of passengers in the metro station until the arrival of the platform, min. Total duration of the system is determined by the main simulation simulation model for given initial parameters and $T_m = 0.51$ min.

The sign $\pm$ in formula (9) "depends on the type of transport document the inflow of passengers. If $\alpha > 50%$ sign is "+" Otherwise, the sign is "-". For each of the coefficients and $k_t$, $k_a$, $k_\lambda$ were tested regressions, as is carried out correlation and regression analysis.

$$k_t = f(t_t); \ k_a = f(\alpha); \ k_\lambda = f(\lambda)$$ (9)

Approximations are made in the following regression models: linear, polynomial of the second degree, power, exponential, logarithmic. The models were checked for statistical significance of equation Fisher criterion (F test) and statistical significance of the coefficients of the equation of the Student criterion (t - test) [12]. Table 1 shows a comparison of the coefficient of determination for approximating functions. In all embodiments the factor has a high value, indicating the presence of a functional relationship.

<table>
<thead>
<tr>
<th>Coefficient of determination ($R^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
</tr>
<tr>
<td>$k_t = f(t_t)$</td>
</tr>
<tr>
<td>$k_a = f(\alpha_m)$</td>
</tr>
<tr>
<td>$k_\lambda = f(\lambda)$</td>
</tr>
</tbody>
</table>

The results in Table 1 indicate that a polynomial dependencies studied factors and parameters. Odds $k_\lambda$ suitable logarithmic dependence due to the coefficient values to the parameters. Below are shown the obtained correlations.

$$k_t = 0.79 + 0.766 \cdot t_t + 0.321 \cdot t_t^2, \ R^2 = 0.99$$ (10)

$$k_a = 13.32 - 31.76 \cdot \alpha + 19.73 \cdot \alpha^2, \ R^2 = 0.96$$ (11)

$$k_\lambda = -1.303 + 0.001 \cdot \lambda - 1.538 \cdot \lambda^{-7}, \ R^2 = 0.99$$ (12)

$$k_\lambda = -10.813 + 1.556 \cdot \ln \lambda, \ R^2 = 0.99$$ (13)

Figure 4 and Figure 5 shows the results for total time in the system obtained equation simulation model and experimentation mental observations main stations of the Sofia underground.

B. Study of the technological processes in the departure of passengers

The survey equations based on generalizations model simulations of underground station. The equation describes all time intervals of the phases through which the passenger door to the station platform. With the help of the equation gets rough idea of the total time in the system, without the need to simulate different scenario in the underground. Determining the total time in the system is necessary for synchronous interval movement of ground public transport in the points of intermodal connections in subway stations. Optimal synchronization between the intervals in intermodal connections helps reduce the total average travel time in the city, leading to increased satisfaction of transport of passengers in their daily trips in the city.

The equation has the following expression:

$$T_s = k_t \cdot k_a \cdot k_\lambda \cdot \beta \cdot T_m \pm T_m, \ min$$ (8)

where: $k_t$ is the coefficient that takes into account the duration of the transfer (walking, using the stairs and / or escalator); $k_a$ is the coefficient, which account for changes in the distribution of passenger fitted with subscription transport document; $k_\lambda$ is the coefficient that takes into account the change of the total inflow in the metro; $t_t$ is the duration of transfer passengers (walking, using stairs and / or escalator), in the metro, min.; $\alpha$ is the percentage distribution of passengers' flow with subscription transport document and a single transport document in the underground,%; $\lambda$ is the total inbound passenger traffic Underground station for an hour, pass./h; $\beta$ is the coefficient, which takes into
Experimental observations of the total time in the system are performed by the timing observations at different time intervals corresponding with the size of the input stream. The maximum total time in the system is a underground station “VASIL LEVSKI”. This is due to the fact that it is a single-input is the greatest during the transfer.

Figure 6 shows comparison in the parameters of the basic model (2000 pass./h) for three variants of determining the total time in the system.

C. Validation of the research results

The simulation models are approximate imitations of real-world systems. Due to that, a model should be verified and validated for its application.

To validation the simulations conducted by the main simulation model and the corresponding experimental calculations equation "(8)" and determining the deviation indicator is used the simple standard deviation and variation, [12].

\[
\delta = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} \tag{14}
\]

\[
\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{15}
\]

where: \(\delta\) is standard deviation, \(\bar{x}\) is the average of the surveyed data, \(i\) is the number of observations.

The standard error of the mean shall be calculated by the formula:

\[
S = \frac{\delta}{\sqrt{n}} \tag{16}
\]

To compare the results of the simulation model and experimental calculations is used validation by mean absolute error, mean relative error and standard error of the estimate. These indicators were selected as the criteria for validation.

The mean absolute error is given by the formula:

\[
S_{MAE} = \frac{\sum_{i=1}^{n}|x_i - \bar{x}'|}{n} \tag{17}
\]
The indicator shows the sample verification of the absolute values of the differences between simulation and the corresponding observations. The mean absolute relative error represents the average variation of the estimates obtained in the result of the modeling to the actual values. It is determined by formula:

\[
S_{MARE} = \frac{1}{n} \sum_{i=1}^{n} \frac{|x_i - \hat{x}_i|}{x_i}
\]  

(18)

The standard error of the estimate is a measure of the accuracy of predictions.

\[
S_{est} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \hat{x}_i)^2}{n-2}}
\]  

(19)

where: \(x_i\) is the actual value by simulation or experiment and \(\hat{x}_i\) is the forecast value by equation.

Table 2 shows the statistical indicators for the main simulation model and equation "(8)." It can see that the variance is small and the results of simulations and experimental calculations equation "(8)" validated. The values of the statistical tools have small differences for simulation model to those of equations in the equation.

Table 3 shows a comparison of statistical indicators of the simulations conducted for 9 main underground stations of the Sofia Underground parameters in the basic pattern (2000 pass./h). Below are the indicators in experimental timing observations of the same underground stations.

**TABLE 2. Statistics as main simulation model**

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Simulation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case number</td>
<td>219</td>
<td>219</td>
</tr>
<tr>
<td>Mean</td>
<td>0.48</td>
<td>0.47</td>
</tr>
<tr>
<td>Std. Error of Mean</td>
<td>0.029</td>
<td>0.027</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.434</td>
<td>0.407</td>
</tr>
<tr>
<td>Variation</td>
<td>0.188</td>
<td>0.166</td>
</tr>
</tbody>
</table>

**TABLE 3. Statistical indicators for experimental research**

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Equation</th>
<th>Experiments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case number</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Mean</td>
<td>0.837</td>
<td>0.814</td>
</tr>
<tr>
<td>Std. Error of Mean</td>
<td>0.535</td>
<td>0.524</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.321</td>
<td>0.314</td>
</tr>
<tr>
<td>Variation</td>
<td>0.103</td>
<td>0.099</td>
</tr>
</tbody>
</table>

In table 4 are shown comparisons of the simulation and equation "(8)" and of the experiment and equation "(8)" by the criteria mean absolute error; mean absolute relative error; standard error of the estimate.

**TABLE 4. Comparison for examined cases**

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Case number</th>
<th>S_{MAE}</th>
<th>S_{MARE}</th>
<th>S_{est}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation-Equation</td>
<td>219</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Experiment-Equation</td>
<td>36</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Small differences in the results of the statistical tools prove verification of the simulation model to the real conditions in underground stations. The results show that the deviation is small and conducted experimental timing observations and calculations equation "(8)" validated. The comparison of the simulation, equation and experiment indicate that the developed simulation model and equation are accurate and credible.

The simulation results gained in this study should be useful for enhancing the current situation. It is also hoped that the developed model could be used as a reference source for the underground stations and in other countries.

**IV. CONCLUSIONS**

The study has shown the following results:

- A methodology for presenting a metro station as a multi-level system has been developed.
- A multi-level system is examined as a compounded of separate single-level systems with Poison’s incoming stream of passengers, exponential time for service and s number of channels (M/M/s).
- The decomposition of levels is consistent with passengers’ going through and servicing them by the system: entrance, entrance-hall (ticket offices and ticket machines), validators, escalators and stairs.
- A general simulation model to explore the technological parameters for departing passengers to the underground station. The model is applied to study the 9 main stations of the Sofia subway.
- Based on the performed experimental simulations was worked out an equation for determining the total time in the system. Regressions were obtained for the coefficients of the equation.
- The equation can be used for preliminary studies and analyzes for design, emergency synchronization links between subway and different types of public transport.
- The simulation models of underground stations have been developed with Arena software.
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