Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 2 Issue 5, May - 2015

More Effective Optimum Synthesis of Path Generating Four-Bar Mechanisms

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***Abstract*—The one-phase synthesis method using evolutionary or heuristic optimization algorithms can successfully solve the dimensional synthesis problems of path generating four-bar mechanisms discussed in the literature, where the input angles are usually used as design variables and the constraint of the rotational sequence of the input angles is necessary. However, there is still room for improvement in solution reliability and accuracy. To understand the influence of the input angles used as design variables, a new one-phase synthesis method without using the input angles as design variables is presented where the square deviation of the nearest distance between the target point and the coupler curve is used as the error metric and the differential evolution is used to solve the optimum synthesis problems. A scale-rotation-translation transformation technique, which can transform the coupler curve of the initial guess nearer to the desired curve in a similar scaling, orientation and location, is proposed. Findings show that the new one-phase synthesis method can improve solution accuracy markedly. Moreover, the scale-rotation-translation transformation technique for the initial guess is effective to improve solution reliability, accuracy or efficiency.**

Keywords—evolutionary algorithm; dimensional synthesis; mechanism optimization; path generation; scale-rotation-translation transformation

I. INTRODUCTION

The dimensional synthesis problem of the path generation of planar mechanisms (herein termed path synthesis problem) is to determine the dimensions of a definite mechanism whose coupler point can be used to trace a desired path or target points. The continuous path may be represented by a sequence of discrete points. The methods for path synthesis problems discussed in the literature may be divided into two categories. The first category is the direct synthesis method without utilizing atlas database [1-20]; the second category may be regarded as the indirect synthesis method using (computerized) atlas database. The direct synthesis method may be further divided into two subcategories. The first subcategory is one-phase synthesis [1-14] and the second subcategory is two-phase synthesis [15-20] for the direct synthesis method. The two-phase synthesis method first handles the shape synthesis and then handles the scale-rotation-translation synthesis. In the traditional one-phase synthesis method, the problem may be considered a mechanism optimization with the goal of minimizing an objective function. The most common objective function is the sum of the square of the distance between the desired point and the corresponding coupler point (herein termed the square deviation of the distance). The technique for the traditional one-phase optimum synthesis where the square deviation of the distance is used as the objective function and the input angles of the crank serve as design variables is based on the position information of the desired points. In other words, the technique for the optimum synthesis not only attempts to simultaneously satisfy the shape, size, location and orientation information of the desired path but also to obtain the input angles of the crank, i.e., the time information regarding the desired points. The number of design variables for such a technique increases with the increment in the number of the desired points. For example, there are a total of 34 design variables for the path synthesis of a four-bar linkage with 25 desired points and without prescribed timing. However, there are 25 design variables for the input angles of the crank corresponding to the 25 desired points.

The population-based heuristic optimization methods are widely used since they are simple, effective and easy to implement for solving complicated real-world optimization problems. In addition, there is no need for further demands pertaining to the gradient of the objective function. Furthermore, if the objective function of an optimization task is not expressed as an explicit function of the design variables, too complicated to manipulate or non-differentiable, the population-based heuristic optimization methods are more suitable to solve such an optimization task than the traditional deterministic optimization methods. The population-based heuristic optimization methods have two important groups: evolutionary algorithms and swarm intelligence based algorithms. As discussed in [6-12,14], the traditional one-phase synthesis method using evolutionary algorithms or swarm intelligence based algorithms can successfully solve difficult path synthesis problems. However, the obtained values of the objective function are scattered and the standard deviation is not small for a certain repeated runs, which can be seen from [12]. The mean value of the objective function can be much higher than the best value of the objective function for a certain repeated runs [12,14]. Therefore, there is still room for improvements in the mean value and the standard deviation (considered as reliability in this study), the best value (considered as accuracy in this study) or the number of evaluations of the objection function (considered as efficiency in this study) of the one-phase synthesis method using evolutionary algorithms or swarm intelligence algorithms. Besides the reason of the employed optimization algorithm, the above-mentioned scattered results can be attributed to the two main reasons described below. One is the use of the input angles as design variables and the associated constraint of the rotation sequence of the input angles. The other is the constraint of Grashof's condition.

To the author's best knowledge, the influence of the input angles used as design variables and the associated constraint of the rotation sequence of the input angles on the solution quality has not been reported in the literature. In this study, to investigate the influence of the input angles on the solution quality, a new one-phase synthesis method using the sum of the square of the nearest distance between the target point and the coupler curve (herein termed the square deviation of the nearest distance) as the error metric is proposed for the path synthesis problems without prescribed timing. The use of the nearest distance can avoid the need of the input angles as design variables and also avoid the constraint of the rotation sequence of the input angles for path synthesis problems without prescribed timing. For the path synthesis problems with prescribed timing, the new one-phase synthesis method and the traditional one-phase synthesis method are the same, since the input angles of the crank is specified. Moreover, in this study, to improve the solution quality of the proposed one-phase synthesis method for closed paths, a scale-rotation-translation transformation technique, which is originally used in the second synthesis of the two-phase synthesis method, is executed for the initial guess upon initialization. The scale-rotation-translation transformation technique can transform the coupler curves of the initial guess nearer to the desired curve in a similar scaling, orientation and location in order to execute the goal of minimizing the square deviation of the neatest distance.

To handle the constraint of Grashof's condition, we adopt Deb's heuristic constrained handling method [21] which uses a tournament selection (survivor selection) operator. Three heuristic rules are shown as follows. First, if one solution is feasible and the other is infeasible, then the feasible solution is selected. Secondly, if both the solutions are feasible, then the solution with the better value of the objective function is selected. Lastly, if both the solutions are infeasible, then the solution with the least constraint violation is selected. The last rule can be neglected with the help of the scheme [6,7] that the mechanism lengths are reassigned until the Grashof condition is satisfied during initialization. The most well-known heuristic optimization method using the survivor selection is the differential evolution (DE) [22], and the DE algorithm is used to solve the optimum path synthesis problems in this study. The effectiveness of the proposed one-phase synthesis method without or with the scale-rotation-translation transformation technique is demonstrated using the comparisons with the current best results discussed in the literature for several representative path synthesis problems of four-bar mechanisms with or without prescribed timing based on the best value, the mean value and the standard deviation of the objective function out of 50 repeated runs.

II. PROBLEM FORMULATION

Figure 1 depicts a kinematic stick diagram and all the geometric parameters for the four-bar mechanism.

The sum of the square of the nearest distance (or the distance error) between the target point and the corresponding coupler point is proposed as the error metric (f_{obj}) for path synthesis problems without (or with) prescribed timing. The path synthesis problem is considered a mechanism optimization with the goal of minimizing the error metric. Minimize

$$f_{obj} = \sum_{i=1}^{N}[(X^i - X_d^i)^2 + (Y^i - Y_d^i)^2] \quad (1)$$

where (X_d^i, Y_d^i) is the prescribed coordinates of the *i*-th target point, and (X^i, Y^i) is the coordinates of the corresponding *i*-th coupler point for path synthesis problems with prescribed timing, or the coordinates of the corresponding coupler point nearest to the *i*-th target point for path synthesis problems without prescribed timing; *N* is the number of the target points.

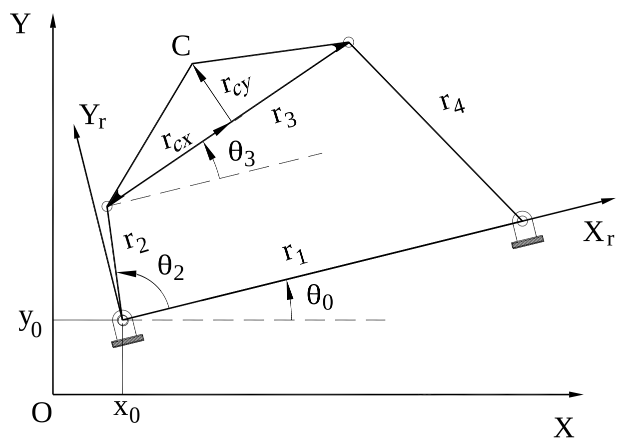

Fig. 1. *Four-bar mechanism in the global coordinate system.*

The position equation of the coupler point is expressed by

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} r_2\cos\theta_2 + r_{cx}\cos\theta_3 - r_{cy}\sin\theta_3 \\ r_2\sin\theta_2 + r_{cx}\sin\theta_3 + r_{cy}\cos\theta_3 \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad (2)$$

There are nine design variables r_1, r_2, r_3, r_4, r_{cx}, r_{cy}, x_0, y_0 and θ_0 to be optimized for path synthesis problems without prescribed timing, which does not need input angles as design variables. In addition to the nine design variables, the input angles, θ_2^1, corresponding to the first position may be considered as design variables for path synthesis problems with prescribed timing in this study. Thus, the design variable vector can be expressed as: $\mathbf{X}_p = \{x_1, x_2, \ldots, x_n\}$ (*n* is the number of the design variables). The coordinates of the coupler point for path synthesis problems with prescribed timing can be obtained from Eq. (2), where angle θ_3 can be solved by the Freudenstein equation [23]. In order to use Eq. (2) and the Freudenstein equation to determine the coordinates of the coupler point nearest to the target point for path synthesis problems without prescribed timing, the corresponding input angle θ_2 must be solved first. The input angle corresponding to the extrema of the variation of the distance between a target point and coupler points is determined by the following equation, which is derived by differentiating the distance with respect to θ_2.

$$(X - X_d)X' + (Y - Y_d)Y' = 0 \quad (3)$$

where

$$X' = -[r_{cx}\sin(\theta_3+\theta_0) + r_{cy}\cos(\theta_3+\theta_0)]\theta_3' - r_2\sin(\theta_2+\theta_0) \quad (4)$$

$$Y' = [r_{cx}\cos(\theta_3+\theta_0) - r_{cy}\sin(\theta_3+\theta_0)]\theta_3' + r_2\cos(\theta_2+\theta_0) \quad (5)$$

$$\theta_3' = \frac{d\theta_3}{d\theta_2} = \frac{r_2[r_3\sin(\theta_3-\theta_2) + r_1\sin\theta_2]}{r_3[r_2\sin(\theta_3-\theta_2) - r_1\sin\theta_3]} \quad (6)$$

Equation (3) can be solved using the bisection method and the maxima of distances between a target point and coupler points can be excluded when the following inequality is satisfied.

$$X'^2 + Y'^2 + (X - X_d)X'' + (Y - Y_d)Y'' \le 0 \quad (7)$$

In this study, the following two constraints are considered:

1) the design variables are within the specified ranges.

2) the Grashof condition is satisfied to allow for the entire turn of at least one link. It may be expressed by

$$2[\max(r_1,r_2,r_3,r_4) + \min(r_1,r_2,r_3,r_4)] < (r_1+r_2+r_3+r_4) \quad (8)$$

The value of a design variable is randomly generated using a uniform distribution during initialization. If the value obtained by the mutation operation of DE is not within the prescribed range, we use a simple method which sets the violating value to be the middle of the violated bound and the corresponding value of the parent individual [21]. During initialization the variables of mechanism lengths are reassigned until the Grashof condition is satisfied. The constraint of Grashof's condition can be handled using Deb's heuristic constrained handling method, as introduced in Section 1.

III. DE ALGORITHM

DE is a well-known evolutionary algorithm for real parameter optimization. The initial design variable vector is randomly generated using a uniform distribution. After initialization, a loop of evolutionary operations containing mutation, crossover and selection is implemented.

A. Mutation

A donor vector $\mathbf{V} = \{v_1, v_2, \ldots, v_n\}$ corresponding to each target (parent) vector $\mathbf{X}_p$ for the next generation is obtained according to the mutation operation. The three most frequently used mutation strategies are listed in the following:

1) DE/rand/1

$$\mathbf{V} = \mathbf{X}_{r1} + F(\mathbf{X}_{r2} - \mathbf{X}_{r3}) \quad (9)$$

2) DE/rand/1

$$\mathbf{V} = \mathbf{X}_{best} + F(\mathbf{X}_{r1} - \mathbf{X}_{r2}) \quad (10)$$

3) DE/current-to-best/1

$$\mathbf{V} = \mathbf{X}_p + F(\mathbf{X}_{best} - \mathbf{X}_p) + F(\mathbf{X}_{r1} - \mathbf{X}_{r2}) \quad (11)$$

where the DE/x/y/z with x: object to be disturbed, y: number of differential vectors, z: crossover way (not shown); the indices $r1$, $r2$ and $r3$ are distinct integers uniformly chosen from [1, N_p] (N_p is the population size); and a scaling factor F is a user supplied constant in the range (0,1+).

B. Crossover

After mutation, a binomial crossover operation forms the trial (new) vector $\mathbf{X}_{p,new} = \{u_1, u_2, \ldots, u_n\}$

$$u_i = \begin{cases} v_i & \text{if} \quad \text{rand}(0,1) \le C_r \quad \text{or} \quad i = i_r \\ x_i & \text{otherwise} \end{cases} \quad (12)$$

where rand(0,1) is a uniform random number on the interval [0,1]; i_r is a randomly chosen integer from [1, *n*], which ensures that the trial vector gets at least one component from the donor vector; C_r is the crossover ratio.

C. Selection

The superior $\mathbf{X}_{p,new}$ or $\mathbf{X}_P$ is selected as the offspring vector and then used as the parent vector in the next generation.

IV. SCALE-ROTATION-TRANSLATION TRANSFORMATION FOR A CLOSED PAT

In this study, a scale-rotation-translation transformation technique using the property of a polygonal curve for a closed path is introduced to transform the coupler curve of the initial population nearer to the desired curve in a similar scaling, orientation and location in order to execute the goal of minimizing the square deviation of the neatest distance. The proposed error metric combined with the geometric transformation technique for a closed path may lead to an improvement on solution quality. There are two geometric-based approaches proposed by Smaili and Diab [17] and Buśkiewicz et al. [18] individually for the second synthesis of the two-phase synthesis method for a closed path. Buśkiewicz et al. [18] uses a geometric-based procedure (translation first, following by rotation and finally scaling) after shape optimization to complete the second synthesis, which uses the properties of the centroid, the direction of the major principal axis and the perimeter of a polygonal curve to translate, rotate and scale the mechanism to the desired configuration, respectively. If translation is performed before rotation, then a second translation process should be performed after rotation because a pure rotation of the mechanism about the pivot causes rotation and translation of the coupler curve. In other words, rotation synthesis should be performed before translation synthesis so as to avoid a second translation process [17,24].

First, a scale transformation is performed, one may have the scale for a closed path

$$s = \frac{\bar{S}_d}{\bar{S}} \tag{13}$$

and

$$\mathbf{X} = [\, sr_1, sr_2, sr_3, sr_4, sr_{cx}, sr_{cy}, x_0, y_0, \theta_0, \theta_2^1 \,] \tag{14}$$

where $\bar{S}_d$ is the perimeter of polygonal curve $\bar{C}_d$ connecting the target points, and $\bar{S}$ is the perimeter of polygonal curve $\bar{C}$ connecting coupler points. In this study, a polygonal curve $\bar{C}$ connecting 180 coupler points from $\theta_2 = 0$ to 2π with an increment of $2\pi/180$ are used to represent the coupler curve.

Secondly, a rotation transformation is performed and the variable θ_0 is determined by

$$\theta_0 = \begin{cases} \theta_p - \tilde{\theta}_p & if\,(I_y - I_x)(\tilde{I}_y - \tilde{I}_x) \ge 0 \\ \theta_p - (\tilde{\theta}_p + \pi/2) & if\,(I_y - I_x)(\tilde{I}_y - \tilde{I}_x) < 0 \end{cases} \tag{15}$$

and

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2I_{xy}}{I_y - I_x}\right) \qquad \tilde{\theta}_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tilde{I}_{xy}}{\tilde{I}_y - \tilde{I}_x}\right) \tag{16}$$

where θ_p and $\tilde{\theta}_p$ are the angles defining a principal axis for polygonal curves $\bar{C}_d$ and $\bar{C}$ after scale transformation, respectively; and $I_x\,(\tilde{I}_x)$, $I_y\,(\tilde{I}_y)$ and $I_{xy}\,(\tilde{I}_{xy})$ are the second moment with respect to the centroidal axes for polygonal curve $\bar{C}_d$ ($\bar{C}$) after scale transformation.

Lastly, a translation transformation is performed and the variables x_0 and y_0 are determined by

$$x_0 = x_c - \tilde{x}_c \qquad y_0 = y_c - \tilde{y}_c \tag{17}$$

V. IMPLEMENTATION OF THE PROPOSED NEW ONE-PHASE SYNTHESIS METHOD

The procedure for the proposed new one-phase synthesis method for the closed path is described below.

1) *Compute the perimeter, the orientation of the principal axis and the centroidal position of the polygonal curve connecting the desired points.*

2) *Initialization: Randomly generate $\mathbf{X}_p = \{x_1, x_2, \ldots, x_n\}$, where the design variables x_i in each $\mathbf{X}_P$ is given by $x_i = \min(x_i) + \mathrm{rand}(0,1)\,(\max(x_i) - \min(x_i))$. For any $\mathbf{X}_P$, the variables of mechanism lengths are reassigned until Grashof's condition is satisfied.*

3) *Perform the scale-rotation-translation transformation for the initial population.*

4) *Calculate the fitness value of $\mathbf{X}_P$ and identify $\mathbf{X}_{best}$ thus far, if necessary.*

5) *Compute the donor vector $\mathbf{V} = \{v_1, v_2, \ldots, v_n\}$ using a mutation strategy.*

6) *Check the boundary constraints for each v_i. Let the violating value to be the middle of the violated bound and the corresponding value of the parent individual.*

7) *For any $\mathbf{X}_P$, compute the trial (new) vector $\mathbf{X}_{p,new} = \{u_1, u_2, \ldots, u_n\}$ using a binomial crossover operation. If $\mathbf{X}_{p,new}$ is superior to $\mathbf{X}_P$, $\mathbf{X}_{p,new}$ is donated to $\mathbf{X}_P$. Otherwise, $\mathbf{X}_P$ remains unchanged.*

8) *Calculate the fitness value of $\mathbf{X}_P$ and identify $\mathbf{X}_{best}$ thus far.*

9) If the termination criteria are satisfied, the optimal solution is obtained. Otherwise, return to step 5.

VI. RESULTS AND DISCUSSION

The effectiveness of the new one-phase synthesis method or the scale-rotation-translation transformation technique for a closed path on solution accuracy (the best value of the objective function) or reliability (the mean value and the standard deviation of the objective function) at a certain number of evaluations of the objective function is demonstrated using 50 repeated runs in four representative path synthesis problems with and without prescribed timing. The user-supplied parameters for the DE algorithm are as follows: 100 individuals (population size) for problems 1 and 2 and 200 individuals for problems 3, 4 and 5, 1000 generations, scaling factor *F* = 0.4, crossover ratio C_r = 0.9. In this study, the results obtained using DE/rand/1 is mainly shown because DE/rand/1 can usually obtain more reliable solutions, except that the DE/best/1 or DE/current-to-best/1 can obtain better accuracy. Because the synthesis problem solved does not belong to a real-time computing problem, the execution time for obtaining optimum results is not shown.

A. *Problem 1*

The synthesis problem with six target points arranged in an vertical straight line and without prescribed timing has been studied by Acharyya and Mandal [6], Lin [7], Cabrera et al. [8] and Ortiz et al. [12].

Target points: C_d^i =[(20,20),(20,25),(20,30),(20,35),(20,40),(20,45)].

Limits of the variables: $r_1, r_2, r_3, r_4 \in$ [5,60]; r_{cx}, r_{cy}, $x_0, y_0 \in$ [-60,60]. $\theta_0, \theta_2^i \in [0, 2\pi]$.

The number (N^o) of evaluations of the objective function, the synthesis design variables and the best value (Best), the mean value (Mean) and the standard deviation (SDev) of the objective function obtained using the proposed error metric are shown in Table 1, together with the synthesis solutions obtained by Cabrera *et al.* [8] (using modified DE) and Ortiz *et al.* [12] (using new DE with auto-adaptive control parameters). For three mutation strategies, DE/rand/1 obtains the best reliability and the worst accuracy; conversely, DE/best/1 obtains the best accuracy and the worst reliability. It can be seen that the best value of the objective function obtained using the proposed error metric (the square deviation of the nearest distance) with 9 design variables and the DE/best/1 algorithm is of the order of 10^{-7}, in contrast with the order of 10^{-4} obtained using the traditional error metric (the square deviation of the distance) with 15 design variables discussed in the literature. Therefore, the proposed new one-phase synthesis method can improve the solution accuracy markedly. The input angles θ_2^i ($i = 1,\ldots,6$) of the crank of the present solutions corresponding to the coupler point closest to the target point are shown in Tables 2 and 3.

TABLE I. SYNTHESIS RESULTS FOR PROBLEM 1

	Present without SRT		**Cabrera et al. [8]**	**Ortiz et al. [12]**
	DE/rand/1	**DE/best/1**		
N_p	100	100	100	100
N^o	100,000	100,000	100,000	100,000
r_1	59.9911	59.6515	31.7883	54.7158
r_2	20.6505	9.01654	8.20465	18.7310
r_3	58.6736	21.5863	24.9321	31.2231
r_4	54.4804	47.1033	31.3859	42.2237
r_{cx}	-45.0791	-3.04271	34.1937	27.2987
r_{cy}	59.1503	-60	14.4157	31.6505
x_0	-33.7171	-31.0643	-6.3665	43.0709
y_0	32.8054	47.8284	56.8368	27.4171
θ_0	3.01266	1.17629	4.01596	5.97746
Best	1.6312E-5	**4.2315E-7**	2.057E-4	2.371E-4
Mean	0.02719	0.19172	–	**0.0015**
SDev	0.06381	0.48485	–	**0.0011**

TABLE II. INPUT ANGLES OF THE CRANK OF THE PRESENT SOLUTION USING DE/RAND/1 FOR PROBLEM 1

θ_2^1	θ_2^2	θ_2^3
0.34378	0.258498	0.175281
θ_2^4	θ_2^5	θ_2^6
0.0928071	0.00971469	6.20767

TABLE III. INPUT ANGLES OF THE CRANK OF THE PRESENT SOLUTION USING DE/BEST/1 FOR PROBLEM 1

θ_2^1	θ_2^2	θ_2^3

2.89748	2.70872	2.53433
θ_2^4	θ_2^5	θ_2^6
2.36911	2.21038	2.05625

B. *Problem2*

The synthesis problem with 10 target points arranged in an ellipse and without prescribed timing has been studied by Acharyya and Mandal [6], Lin [7], Cabrera et al.[8] , Ortiz et al. [12].

Target points: [(20,10),(17.66,15.142),(11.736,17.878),(5,16.928),(0.60307,12.736),(0.60307,7.2638),(5,3.0718),(11.736,2.1215),(17.66,4.8577),(20,10)].

Limits of the variables: $r_1, r_2, r_3, r_4 \in$ [5,80]; r_{cx}, r_{cy}, $x_0, y_0 \in$ [-80,80]. $\theta_0, \theta_2^i \in [0, 2\pi]$.

The number of evaluations of the objective function, the synthesis design variables and the best value, the mean value and the standard deviation of the objective function obtained using the proposed one-phase synthesis method are shown in Table 4, together with the synthesis solutions obtained by Cabrera *et al.* [8] and Ortiz *et al.* [12]. The differences in accuracy and reliability among the three DE mutation strategies are small. It can be seen from Table 4 that the best value of the objective function obtained using the proposed error metric with 9 design variables and the DE/rand/1 algorithm is of the order of 10^{-4}, in contrast with the order of 10^{-3} to 10^{-2} obtained using the traditional error metric with 19 design variables discussed in the literature. Therefore, the proposed new one-phase synthesis method can improve the solution accuracy markedly again. In contrast to the solution accuracy, the solution reliability for the proposed new one-phase synthesis method without SRT is poor. However, with the help of the proposed the scale-rotation-translation transformation technique, the values of Mean and SDev for the proposed new one-phase synthesis method can be reduced to be the order of 10^{-4} from the order of 10^{0}. It can be also seen that in solution accuracy and reliability the proposed new one-phase synthesis method with SRT is much superior to Ortiz *et al.* [12]. The input angles of the crank of the present solution corresponding to the coupler point closest to the target point are shown in Table 5.

TABLE IV. SYNTHESIS RESULTS FOR PROBLEM 2

	Present		**Cabrera et al. [8]**	**Ortiz et al. [12]**
	DE/rand/1 without SRT	**DE/rand/1 with SRT**		
N_p	100	100	100	100
N^o	100,000	100,000	100,000	100,000
r_1	79.9742	79.9985	79.5161	65.4288
r_2	8.248	8.72535	9.72397	8.01639
r_3	51.2634	51.8801	45.8425	47.2217
r_4	42.3971	43.3619	51.4328	44.1366
r_{cx}	-10.5296	-6.63658	8.21392	−11.5709
r_{cy}	2.73185	8.2734	−2.95396	−1.90491
x_0	6.50569	15.9712	2.02111	10.6354
y_0	-0.206904	18.5445	13.2166	−1.67548
θ_0	4.15041	1.34791	5.59694	3.86733
Best	4.7487E-4	**4.5551E-4**	0.0047	0.01910
Mean	1.08718	**6.2704E-4**	–	0.04371
SDev	5.06295	**2.6405E-4**	–	0.0266

TABLE V. INPUT ANGLES OF THE CRANK OF THE PRESENT SOLUTION WITH SRT FOR PROBLEM 2

θ_2^1	θ_2^2	θ_2^3	θ_2^4	θ_2^5
5.04915	5.73644	0.13974	0.832578	1.53611
θ_2^6	θ_2^7	θ_2^8	θ_2^9	θ_2^{10}
2.24424	2.96272	3.66871	4.36455	5.04915

C. *Problem 3*

This is a very representative path synthesis problem with 18 target points and prescribed timing.

Target points: [(0.5,1.1),(0.4,1.1),(0.3,1.1),(0.2,1.0),(0.1,0.9),(0.05,0.75),(0.02,0.6),(0.0,0.5),(0.0,0.4),(0.03,0.3),(0.1,0.25),(0.15,0.2),(0.2,0.3),(0.3,0.4),(0.4,0.5),(0.5,0.7),(0.6,0.9),(0.6,1.0)].

Prescribed timing: $\theta_2^i = \theta_2^1 + \frac{\pi}{9}(i-1)$, i = 2 to 18.

Limits of the variables: $r_1, r_2, r_3, r_4 \in$ [0,10]; r_{cx}, r_{cy}, $x_0, y_0 \in$ [-10,10]. $\theta_0, \theta_2^1 \in [0, 2\pi]$.

The problem has been studied by many researchers and the current best results are obtained by Penunuri et al. [9]. The difficulty of the problem consists in the quite uneven allocation of the target points. In other words, the curve connecting the target points is not smooth. The difficulty can also be seen from the study that Penunuri et al. [9] used DE/rand/1 algorithm with 200 individual and 11,817 generations to obtain the best value of the objective function, i.e., Best = 9.088E-3. To further improve the accuracy, they use 200 individuals and 30,000 to obtain Best = 9.06E-3. Moreover, by making a third refinement of the search space (the first two Best values are obtained by reducing the search space two times), they obtained Best= 9.03E-3, as reported in [9]. Note that for the path synthesis problems with prescribed timing, the new one-phase synthesis method and the traditional one-phase synthesis method are the same and that the two-phase synthesis method cannot be applied to the path synthesis problems with prescribed timing.

The number of evaluations of the objective function, the synthesis design variables and the best value, the mean value and the standard deviation of the objective function obtained using the proposed one-phase synthesis method are shown in Table 6, together with the synthesis solutions obtained by Penunuri et al. [9]. It can be seen that the best value of the objective function obtained using the proposed one-phase synthesis method without SRT and with 1000 generations is higher about 10.6% than that obtained by Penunuri et al. [9]. To obtain more an accurate solution, we use 2000 generations and obtain the almost same accuracy with [9]. In contrast with the number of evaluations of the objective function, i.e., 400,000 for proposed one-phase synthesis method without SRT and 2,363,400 for [9], the solution efficiency of the proposed one-phase synthesis method without SRT is superior to that of [9]. This work and [9] use the same error metric and the DE/rand/1 algorithm; however, the difference of the solution efficiency is considerable. This might be attributed to the scaling factor, the crossover ratio and the handling of the boundary constraints. They used a random number for the scaling factor and 0.3 for the crossover ratio.

It also can be seen from Table 6 that the solution reliability (Mean and SDev) of the proposed one-phase synthesis method can be improved using the scale-rotation-translation transformation technique. It is worth noting that the solution efficiency (N^o) of the proposed one-phase synthesis method is also improved using the scale-rotation-translation transformation technique for the almost same accuracy (0.009030 and 0.009029). In fact, the Best value is achieved at 511 generations and therefore the number of evaluations of the objective function is only 102,200 for DE/rand/1 with SRT.

By the way, the 18-point path synthesis problem with prescribed timing has been studied by Lin [7], but the sixth target point in [7] is (0.005,0.75). The solutions obtained by Lin have been checked using the SolidWorks® 2D sketch [7]. Therefore, the controversy of differences in the values of the objective function discussed by Penunuri et al. [9] arises from the two different sixth target points of (0.05,0.75) and (0.005,0.75), not rounding errors.

TABLE VI. SYNTHESIS RESULTS FOR PROBLEM 3

	Present			**Penunuri *et al.* [9]**
	DE/rand/1 without SRT	**DE/rand/1 without SRT**	**DE/rand/1 with SRT**	
N_p	200	200	200	200
N^o	200,000	400,000	200,000	2,363,400
r_1	10	1.05419	1.05394	1.08913
r_2	0.330348	0.42356	0.423875	0.42259
r_3	0.499572	0.916063	0.91425	0.96444
r_4	9.84945	0.597558	0.598918	0.58781
r_{cx}	0.151927	0.371632	0.370598	0.39137
r_{cy}	-0.322595	0.401029	0.399345	0.42950
x_0	0.543914	0.268935	0.267654	0.27892
y_0	0.735323	0.152628	0.15465	0.11673
θ_0	3.31805	0.287893	0.284824	0.32195
θ_2^1	3.5043	0.889959	0.891555	0.86323
Best	0.010054	0.009030	**0.009029**	0.009088
Mean	0.031193	0.02618	**0.010618**	–
SDev	0.013479	0.057998	**0.003785**	–

D. *Problem 4*

Problem 4 is the same with problem 3 but without prescribed timing. The problem was studied by Penunuri et al. [9].

The number of evaluations of the objective function, the synthesis design variables and the best value, the mean value and the standard deviation of the objective function obtained using the proposed one-phase synthesis method are shown in Table 7, together with the synthesis solutions obtained by Penunuri et al. [9]. It can be seen that the best value of the objective function obtained using the proposed one-phase synthesis method without SRT and with 1000 generations is higher about 11.0% than that obtained by Penunuri et al. [9]. To obtain more an accurate solution, we use 2000 generations and obtain Best = 3.327E-3, about 10.0% smaller than that obtained in [9]. Moreover, in contrast with the number of evaluations of the objective function, the solution efficiency of the proposed one-phase synthesis method without SRT is superior to that of [9]. It also can be seen from Table 7 that the solution accuracy, reliability and efficiency of the proposed one-phase synthesis method for this problem can be improved using the scale-rotation-translation transformation technique. The input angles of the crank of the present solution corresponding to the coupler point closest to the target point are shown in Table 8.

TABLE VII. SYNTHESIS RESULTS FOR PROBLEM 4

	Present			Penunuri *et al.* [9]
	DE/rand/1without SRT	**DE/rand/1 without SRT**	**DE/rand/1 with SRT**	
N_p	200	200	200	200
N^o	200,000	400,000	200,000	–
r_1	5.4444	5.3384	1.38529	2.27468
r_2	0.430105	0.473576	0.282635	0.44667
r_3	0.785115	7.38872	1.41628	2.18422
r_4	5.75102	3.67361	0.361258	0.72409
r_{cx}	-0.090849	5.75533	-0.155436	1.02937
r_{cy}	-0.709063	-4.76842	1.25277	0.82440
x_0	0.834245	-1.4135	1.02798	0.22922
y_0	0.693761	-6.60996	-0.276996	-0.63525
θ_0	3.11589	1.54934	0.329418	0.58183
Best	0.004106	0.003327	**0.003144**	0.003698
Mean	0.066653	0.006354	**0.004132**	–
SDev	0.048330	0.005422	**0.001788**	–

TABLE VIII. INPUT ANGLES OF THE CRANK OF THE PRESENT SOLUTION WITH SRT FOR PROBLEM 4

θ_2^1	θ_2^2	θ_2^3	θ_2^4	θ_2^5
0.943284	1.30598	1.63217	2.06968	2.47907
θ_2^6	θ_2^7	θ_2^8	θ_2^9	θ_2^{10}
2.89978	3.28561	3.54144	3.80919	4.14291
θ_2^{11}	θ_2^{12}	θ_2^{13}	θ_2^{14}	θ_2^{15}
4.58207	4.78697	5.08908	5.40299	5.6364
θ_2^{16}	θ_2^{17}	θ_2^{18}		
5.97166	0.0969069	0.3988		

VII. CONCLUSIONS

The proposed new one-phase synthesis method can improve the solution accuracy markedly for the synthesis problems of open or closed paths. With the help of the scale-rotation-translation transformation technique, the proposed new one-phase synthesis method can improve the solution reliability, even accuracy or efficiency for the synthesis problems of closed paths. The improvement of the solution reliability for the synthesis problems of open paths should be a further work.

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