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Deflection Analysis of Spur Polymer Gear Teeth

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***Abstract*—The presented article describes an investigation regarding the deflection behaviour of polymeric gear transmission using numerical analyses (FEM) and a standardised procedure. A polymer gear pair was modelled and analysed using ABAQUS software and the numerical results were then compared with the analytical results according to the German norm VDI 2736. In the numerical analyses the gear deflection behaviour is determined using Young's material model and the hyper elastic Marlow model. The computational analyses have shown that the selection of the appropriate FE-model has a significant influence on the accuracy of the numerical results. The numerical analyses also indicated that an appropriate non-linear material model should be considered in the case of higher contact forces, and consequently large deflections.**

Keywords — Polymer gears, tooth deflection, numerical analysis, coating

I. Introduction

Many gear manufacturing companies are reducing costs, and one way of doing this is by replacing conventional steel material with engineering polymers. Since more and more gears are being manufactured out of polymers and being used in many complicated mechanical devices, the need to accurately predict their mechanical behaviour is crucial in their design.

Polymers were first used in gearing applications in the 1950s and have since developed into a large range of applications. Most of these tend to be in reduced motion control (low load, temperature, and speeds).

Optimization of a gear pair should be done before undergoing the expenses of tool manufacturing and testing them. It is quite normal that engineers have had access to a limited number of tools, or have their own experience in gear design. Most recognize the use of semi-analytical tools for the prediction of gear tooth strength as the standardised procedure according to ISO, VDI, and AGMA standards [1–3].

Deflection of the gear tooth is important for gears made from polymer materials. When large deflection of the gear tooth occurs we can observe disturbance in gear meshing. The procedure of calculating the deflection is described in VDI 2736 standard [3], where deflection of the spur gear tooth is influenced by a force, the tooth face width, and material stiffness, which is represented with a Young's modulus [4].

Thermoplastic polymers are widely used materials in the manufacturing of polymer gears. They have different stress-strain behaviours in tensile and compression regions, which lead to different values of a Young's modulus in both regions. Furthermore, polymers can show nonlinear behaviour in the elastic region – which cannot be described with a Young's modulus. Therefore, the influence of different stress-strain behaviour in tensile/compression region and a nonlinear behaviour in the elastic region should be described with an appropriate constitutive model [4].

In the presented paper a finite element method is used to analyse the tooth deflection of the polymer spur gear pair. The pinion and gear are modelled as engagement gears with external loading acting in the outer point of the single engagement. The numerical results are then compared to the standardised procedure as described in VDI 2736 standard [3].

II. Theoretical Background

Hyperelasticity is a non-linear elastic material model theory that is commonly used to represent the large-strain response of rubbers, and is often an available option in finite element software solutions and then quite easy to use with the appropriate FEM-analyses [4].

Hyperelastic material is a special case of a Cauchy elastic material. It is used when linear elastic models do not accurately describe the observed material behaviour. The most common example of this kind of material is rubber, whose stress-strain relationship can be defined as non-linearly elastic, isotropic, incompressible, and generally independent of strain rate. Hyperelasticity provides a means of modelling the stress-strain behaviour of such materials. The behaviour of unfilled, vulcanized elastomers often conforms closely to the hyperelastic ideal. Filled elastomers are also often modelled via the hyperelastic idealization [5,6].

A considerable amount of literature has been published on modelling of hyperelastic material. The choice of the suitable model depends on its application, corresponding variables, and its available data to determine the needed material parameters [7]. Modelling of hyperelastic materials is the selection of a proper strain energy function W, and the accurate determination of needed material parameters [8].

There are various forms of strain energy potentials for the modelling of incompressible and isotropic elastomers. However, only some of them describe the complete behaviour of these materials, especially for different loading conditions with experimental data [9–11]. Different models for describing the deformation behaviour of polymer materials were also analysed by Boyce and Arruda [12].

Most FE software packages support a large selection of different hyperelasticity models. For simulating our problem a Marlow model [13] was chosen as an appropriate model. The model is suitable when only one set of test data is available. In this case a strain energy potential is constructed that will fit the test data very accurately. Beside this, it will cover the approximate behaviour in other deformation modes [5].

The Marlow model is defined by providing uniaxial test data that define the deviatoric behaviour and, optionally, the volumetric behaviour if compressibility must be taken into account. Figure 1 shows the stress-strain response of treated material Delrin 100 NC010 and a comparison between the Marlow HE model and a linear elastic E model.

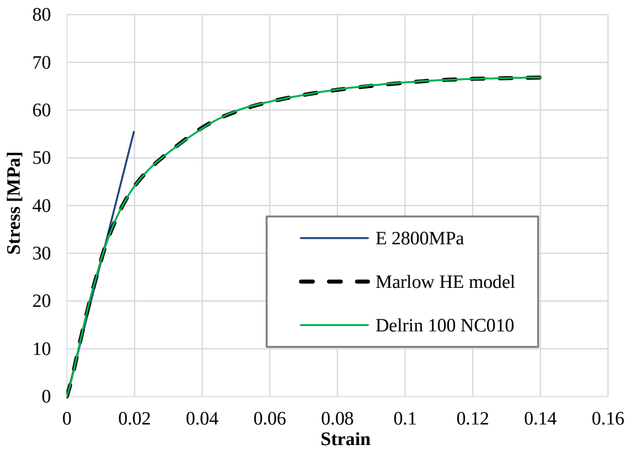

Fig. 1. *Calibration of material data with the Marlow model and a linear elastic constitutive model*

It can be seen from Figure 1 that the interpolation of stress-strain data with the Marlow model is satisfying and approximately linear for small and large strains. For intermediate strains in the range 0.01 to 0.04, a noticeable degree of nonlinearity may be observed for treated material Delrin 100 NC010. To minimize undesirable nonlinearity, enough data points should be specified in the intermediate strain range.

III. Practical Example

The standardised procedures as described in AGMA and ISO standards are usually focused on gears made of metallic materials. Therefore, it is assumed that no significant deformation occurs during gear operation. However, it is not the case in the operation of polymer gears where significant tooth deflections may occur, which can lead to a disturbance of the meshing gear flanks. The consequence of this is higher noise in the wear of teeth flanks.

In this study an accurate two-dimensional model of a gear pair was created using ABAQUS software. A summary of the gear geometry is given in Table 1.

Table 1: Data of gear pair

Parameter	**Pinion**	**Gear**
Module *m* [mm]	4	
Centre distance *a* [mm]	80	
Pressure angle α [°]	25	
Tooth width *b* [mm]	25	
Number of teeth *z* [-]	9	31
Torque [Nm]	10	34.444
Profile shift *x* [mm]	0.4871	- 0.4871
Addendum height h_{aP} [mm]	4.750	2.000
Dedendum height h_{fP} [mm]	3.051	6.949
Root radius profile ρ [mm]	1.24	2.48
Material	POM	42CrMo4
Young's Modulus *E* [MPa]	650	206000
Poisons number ν [-]	0.35	0.33

The computational procedure was combined from several steps. First, the gear deflection according to VDI 2736 [3] and ISO 6336 [2] standards was obtained where the values are calculated in six steps giving the six points of curve in Stress-Strain diagram [4].

In second step, FE-models of the treated gear are created for further numerical simulation. In our previous work [4], a five teeth FE-E model and five teeth FE-HE model (see Figure 2, left) were created. In both models, the normal force on gear flank F is acting at the outer point of single engagement of the meshing gears. The third (FE-E) and fourth (FE-HE) models were modelled taking into account the real contact of meshing gears (see Figure 2, right).

1. *Initial step:* Boundary conditions were applied to completely constrain the driving and driven gears.
2. *Approach:* A rotational constraint was removed from the driving gear, then a rotational boundary condition was activated, closing the 'gap' between interacting tooth flanks. The magnitude of rotation was equivalent to the theoretical backlash for the benchmark geometry.
3. *Loading:* A rotational constraint was removed from the driven gear. Opposing rotational moments were then applied on the driving and driven gears, essentially resisting each other.
4. *Rotation:* Rotational boundary conditions were applied about the drive shaft centres.

In the presented numerical simulations the static load of gear teeth is considered, which is based on the assumption that the number of cycles during the service life of the gear is very small. Furthermore, it is assumed that there is enough time for a strain recovery, which can lead to the neglecting of a permanent set effect of a polymer.

Fixed constraints were assumed at the centre of the hole. Other constraints are placed where the gear has been sliced [4].

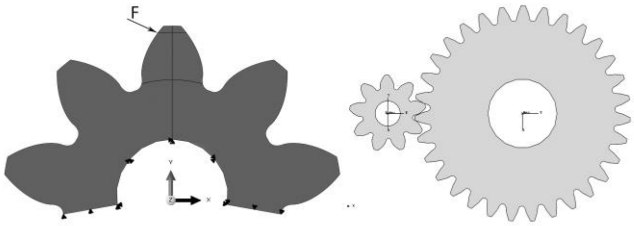

Fig. 2. *Numerical model with five teeth (left) and a numerical model of meshing gears (right).*

Two approaches have been taken into account in the numerical analyses: [i] the elastic Hook's law approach, and [ii] a hyperelastic approach. The first approach uses Young's modulus *E* (see Figure 3) and Poisson's ratio 0.35. The material is assumed to be isotropic. Hyperelastic material data for the second approach was also obtained from Figure 3 and then included in the used material model. The deviatory response was uniaxial and the volumetric response was ignored [4].

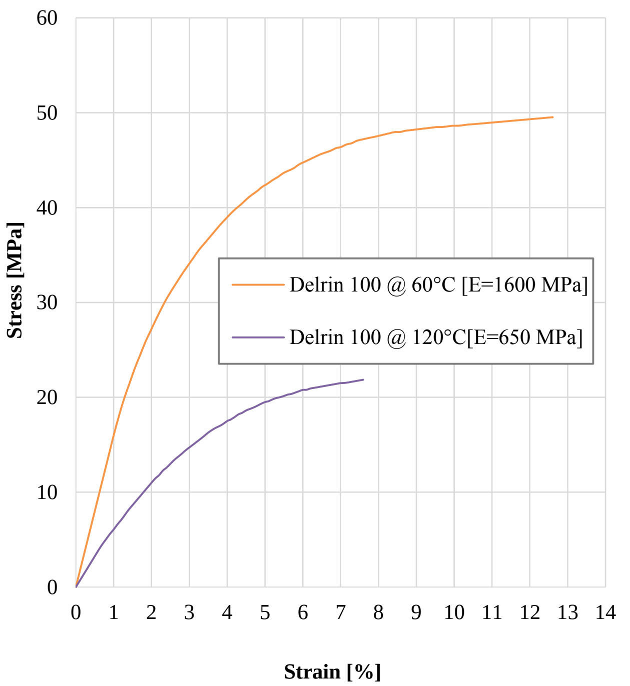

Fig. 3. *Stress strain behavior in tension for Delrin 100 NC10 at different temperatures [14]*

The results of the computational analyses and comparison to the results from VDI 2736 are presented in section 4. In the final step, force-deflection diagrams were created based on the FEM results.

IV. Results

Using numerical models as described above, computational simulations were initiated to test a variety of common operational conditions for polymer gears. Hyperelastic stress-strain relationships usually differ significantly for tension, compression, and shear modes of deformation. It should be noted that stresses in the hyperelastic model are within the elastic region for DuPont Delrin 100NC010. The stress at which material starts to deform plastically at 23°C is 71 MPa, and the corresponding strain is 27%, whereas a nominal strain at break point is 45% [4].

Figure 4 shows the convergence tests for the used numerical model where the number of finite elements for pinion has been taken as a variable. It can be seen from Figure 4 the significant improvement of FE results when the number of elements increased from beginning value 2000 to value 18000. There was no important difference in numerical results when 32000 finite elements were used.

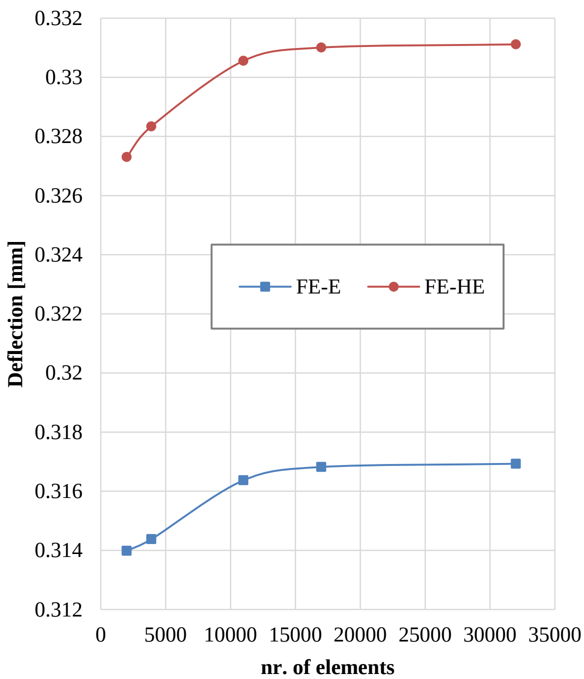

Fig. 4. *Convergence analyses of used numerical model*

Contact interactions between different parts play a key role when simulating assemblies, manufacturing processes, dynamic impact events, and other systems. Accurately capturing these interactions is essential for solving problems.

An investigation was made into determining the influence of the different contact methods available in the chosen FE software. It should be noted, that in the Lagrange multiplier method and the Penalty method a friction coefficient close to zero was chosen. Considering these values, very similar deflection values for Lagrange, Penalty, and Frictionless methods can be observed in Figure 5. When the Rough method is used, a deflection deviation of about 24%, in comparison to the other three methods, can be observed.

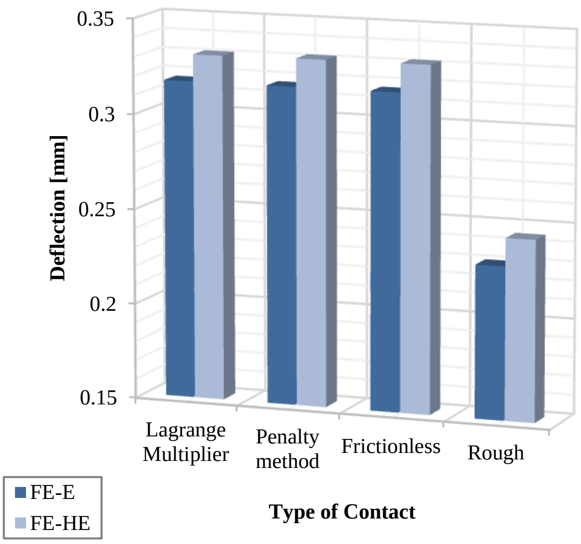

Fig. 5. *Deflection values for different contact methods*

Table 2 presents the numerical values for tooth deflection as shown in Figure 5. The large difference of the Rough model in comparison to the other stems from the different equation parameters used by it. In Abaqus, the “rough” model uses an infinite coefficient of friction (μ=∞). This means that any type of surface interaction is, and with it all relative sliding motion between two contacting surfaces, prevented.

TABLE 2: DEFLECTION VALUES IN [MM] FOR DIFFERENT CONTACT FORMULAS

Contact type	FE-E	FE-HE
Lagrange Multiplier	0.316929	0.331115
Penalty method	0.316929	0.331770
Frictionless	0.316929	0.331981
Rough	0.230328	0.245100

Rough friction is intended for non-intermittent contact; once surfaces close and undergo rough friction, they should remain closed. Convergence difficulties may arise in Abaqus/Standard if a closed contact interface with rough friction opens, especially if large shear stresses have developed. The rough friction model is typically used in conjunction with the no separation contact pressure-overclosure relationship for motions normal to the surfaces, which prohibits separation of the surfaces once they are closed [15].

The rough model in Figure 5 and Table 2 are included in the simulation as examples of which should not be used when simulating contact between gears. It is necessary, for contact simulations to work properly, to know the friction properties between surfaces; this applies to the Lagrange and penalty methods. If the friction coefficient is not known a Frictionless model should be used. For contact problems a good mesh will generally make the problem easier to converge.

In this work, the influence of the finite element type on the numerical results (deflection values) was also studied. As shown in Figure 6, a slightly larger tooth deflection was observed when quad elements are used. It can also be seen that linear and quadratic element type have similar deflections for both, triangular or quad shape of elements. Deviation between the appropriate linear model and the quadratic model is in the range of 1%. The numerical results are also summarised in Table 3.

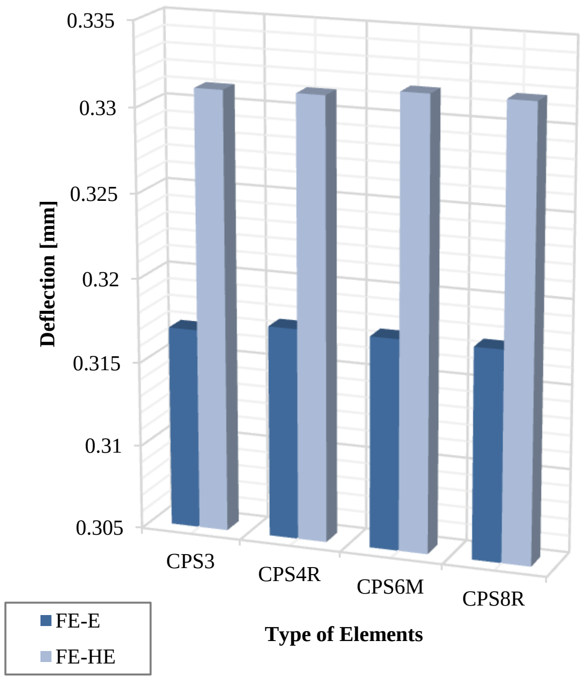

Fig. 6. *Influence of element type on tooth deflection*

TABLE 3: DEFLECTION VALUES FOR DIFFERENT ELEMENTS USED

Element type	FE-E	FE-HE	Note
CPS3	0.316929	0.331115	Linear / Tri
CPS4R	0.317580	0.331245	Linear / Quad
CPS6M	0.317584	0.331770	Quadratic / Tri
CPS8R	0.317580	0.331783	Quadratic / Quad

Figure 7 represents the gear tooth deflection for treated material Delrin 100 NC010 at 120°C for “force model” as described in [4]. It can be seen a relatively good correlation between linear elastic and hyperelastic model up to 0.15 mm of gear tooth deflection. From 0.15 mm the hyperelastic model gives larger deflections at the same force applied. If one compares the numerical results with the results from standard procedure VDI 2736, a little smaller deflection values have been observed in the numerical calculation.

A significant point in Figure 7 is the permissible tooth deflection (marked with a cross), which is calculated using standardised procedure VDI 2736. This point is defined as a limited tooth deflection for the functional operation of gear pair. In the case of higher deflection the disturbing operation of gear pair can be expected. It should be noted that the permissible tooth deflection is not materially constant and is only dependent on the gear size and shape of a gear tooth.

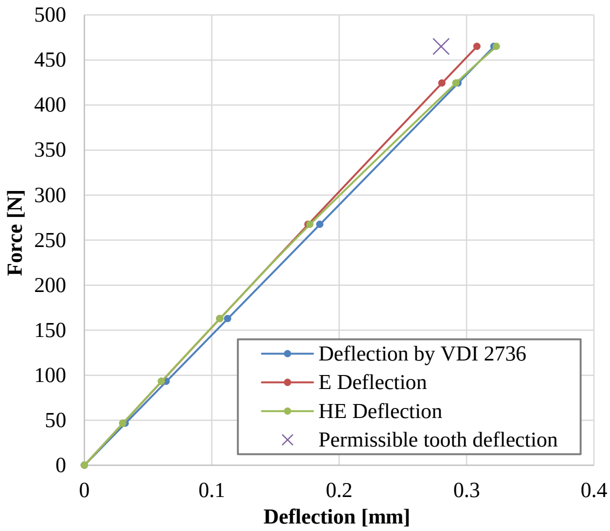

Fig. 7. *Deflection of the gear tooth for five tooth model at 120°C for the material Delrin 100 NC010*

Figure 8 shows the gear tooth deflection for real gear pair using contact FE-E and FE-HE models for the material Delrin 100 NC010 at 120°C. The HE model starts to deviate from the E model by a value around 0.17 mm and ends up with a larger deflection at the same force applied. A permissible tooth deflection value of 0.29 mm according to the VDI 2736 standard is reached when the normal force on gear tooth exceeds 421 N.

In Table 4 the peak deflection values previously calculated using all three methods (VDI norm, elastic model, and hyperelastic model) are presented. The results are presented for both, the force model and real contact model.

Table 5 shows matching between peak deflection using different methods. The best match was obtained using an the elastic contact method for an FEM analysis. The deviation to 100%, matching with standard VDI 2736, is due to numerical error when using FEM.

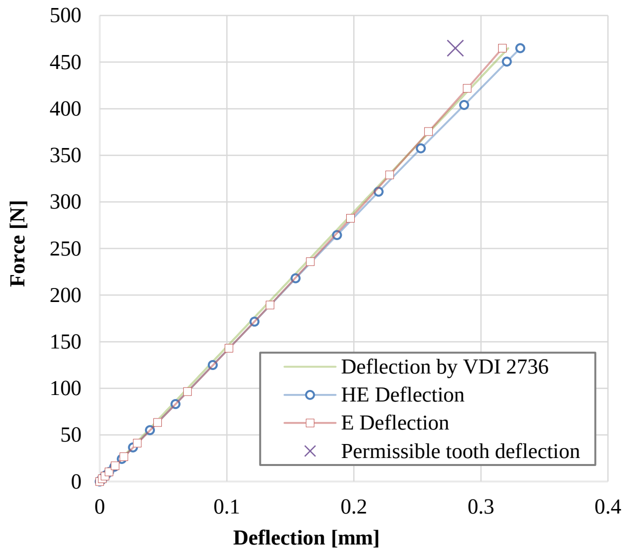

Fig. 8. *Deflection of the gear tooth for real gear pair model at 120°C for the material Delrin 100 NC010*

TABLE 4: DEFLECTION VALUES IN VDI, FORCE, AND REAL CONTACT MODEL

FE	Force model	Real contact model	VDI 2736	Unit
E	0.308184	0.316929	0.321524	mm
HE	0.323431	0.331115	/	mm

TABLE 5: DEFLECTION DEVIATION VALUES IN PERCENTAGES

FE	Force/Contact	Force/VDI	Contact/VDI
E	97.24	95.85	98.57
HE	97.68	/	/

V. CONCLUSION

The finite element analysis of gear tooth deflection of spur polymer gears is presented in this paper. Tooth deflection, as a consequence of the contact between gear teeth, is numerically studied using Young's elastic and Marlow hyperelastic models. The results are compared with a procedure described in standard VDI 2736, which is based on the linear elastic theory.

The assumptions made by the classical gear theory and inherited by most common gear-rating standards, specifically those of negligible tooth deflections and frictional effects, are not valid for dry-running non-metallic gears that have high friction coefficients.

The results of the numerical analysis showed that the appropriate finite element model should be used to get the appropriate stiffness of the treated gear, and consequently, comparable results with the standard VDI 2736. It has been determined that in the case of a real contact model we achieved better converge towards standard values if Young's elastic model is used. There is also acceptable correlation between the appropriate force model and real contact model.

The used Abaqus software offers four different methods to describe the contact between meshing gear flanks: Lagrange Multiplier, Penalty, Frictionless, and Rough. The numerical results have shown that the Rough model should be avoided when using contacts between gear teeth due to its prevention of slipping regardless of contact pressure. The frictionless model should be used when the coefficient of friction is not known. When the friction coefficient is known the Lagrange Multiplier method or Penalty method should be used.

During the numerical analyses the Penalty method has shown to be a more robust option and easier to converge since it has less parameters. On the other hand, the Lagrange Multiplier method introduces more DOF and is in general more complicated and needs longer computational time. The benefits of this method is the exact fulfilment of constraints. Hence, the results are closer to the reality. Also, dependence on specifically chosen parameters for the observed problem is not large and they can be obtained by trial and error method.

The use of different plain stress element types had shown no significant influence on the numerical results. As expected, due to more integration points of quad element versus linear element, quadratic elements show larger deflection values. In general, two-dimensional models are preferred in engineering practise because of modelling time as well as computational time remains affordable, especially if many different configurations have to be analysed during the initial design phase.

The computational results also showed that a deflection of the gear tooth is slightly smaller when linear elastic model is used instead of a standardized procedure according to VDI 2736. Special care should be focused on the determination of gear tooth deflection in the case when a nonlinear material is used. It is a consequence of the fact that in the region of higher loads a gear tooth deflection can be better described using the hyperelastic constitutive model instead of the linear elastic theory.

On the basis of the computational analysis presented in this paper, it can be concluded that combining numerical simulations with proper material parameters can lead to a better understanding of the gear tooth deflection of polymer gears. Further research work on polymer gears should be focused on the experimental determination of gear tooth deflection of the meshing gear pair.

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