

# Numerical Simulation In Geomechanics On Local Nonlinearity Of Contact

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**Abstract**—In the current practice of numerical analyses of structure problems, it is often necessary to model contacts between two solids (concrete-rock contact surface, soil-structure contact surface, etc.). The problem is to model the behavior of a discontinuity surface between two solids 1 and 2. The model is composed of a series of two node contact elements Therefore a program using the finite element method was elaborated. It uses nine (09) nodes quadratic rectangular element for modeling the solids and a contact element with two nodes (using springs with an important stiffness) for modeling the interface. The program allows the determination of the final state of the contact with the determination of there strain and stress states.

**Keywords**—Contact - Discontinuity - Friction - sliding.- Interface – structure

## I. INTRODUCTION

The phenomena of contact [1] are omnipresent in various sectors of mechanical engineering, the civil engineering or in the industrial methods. They appear for example, during the assemblies by bolts or in the bearings or in the interactions ground structure. For their calculation, they present several difficulties, inherent in their non-linear and irregular characters, making their resolution analytical often impossible. To overcome this difficulty, the studies were directed towards the numerical modelling of the behaviour of discontinuity, by taking of account the conditions local of the contact (friction, slipping and separated nodes) [2]. These studies are of particular interest to make it possible to follow on the one hand the evolution of the mechanical characteristics of the solids in contact: deformation, degradation, ruptures. and in addition distribution of the constraints of contact.

An approach of the problem consists in representing each discontinuity by finite elements of contact.

By taking of account real geometry of discontinuity and its mechanical characteristics, in order to determine the final state in any point of discontinuity starting from a given initial state and according to the conditions of loading and the laws of contact

## II. NUMERICAL MODEL

The resolution of the problem [2] consists in modelling the behaviour of discontinuity surface  $S_c$  between two solids  $S_1$  and  $S_2$  which can slip with friction, to fall apart or return in contact. The model is composed of a series of standard element of contact arises, connecting a point of the  $S_1$  solid to a point of the solid  $S_2$  (1-2, 3-4, 5-6, etc), (see figure 1). These elements are compatible with the elements of the solid mass used to model  $S_1$  and  $S_2$

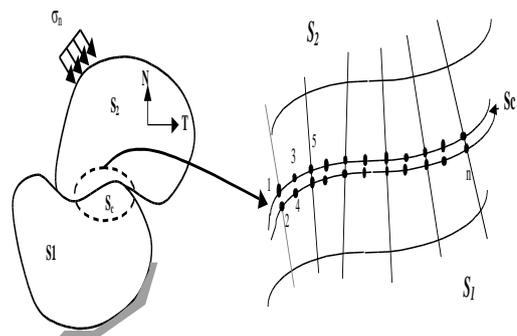


Figure 1 two bodies in contact

### 2.1 Elements of contact

On the two points of the element (see figure 2), the forces of contact in the local frame:

$$F_{X2} = -F_{X1} = F_T \quad \text{et} \quad F_{Y2} = -F_{Y1} = F_N \quad (1)$$

Displacements are noted  $(u_1, v_1)$  for node 1 and  $(u_2, v_2)$  for node 2 in this frame. The matrix of stiffness' of the element of contact [3] in the local frame of reference :

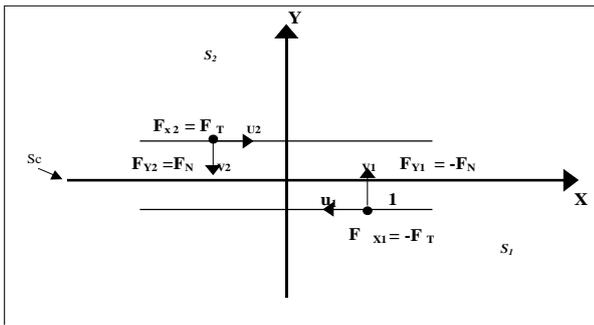


Figure 2 two nodes contact element

$$\begin{Bmatrix} Fx_1 \\ Fy_1 \\ Fx_2 \\ Fy_2 \end{Bmatrix} = K \cdot \begin{bmatrix} T & 0 & -T & 0 \\ 0 & N & 0 & -N \\ \text{Sym} & T & 0 & 0 \\ 0 & 0 & N & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \quad (2)$$

$T = 1$  or  $0$ , according to whether the tangential spring exists or not.

$N = 1$  or  $0$ , according to whether the normal spring exists or not.

$K$ : is the factor of penalization [4] which is equal to

$$K = 10^9 \cdot \frac{E_1}{E_2} \quad (3)$$

Where  $E_1, E_2$ : Young modules of the solids  $S_1$  and  $S_2$

The study is made in plane elasticity, then the field of displacement will be defined by displacements  $U$  and  $V$  in the Cartesian frame of reference. The solids will be modelled by quadratic rectangular elements with (09) nodes.

These elements have a quadratic variation along dimensioned vertical and horizontal of the rectangular element [3].

The function of interpolation  $\varphi(x, y)$  will be provided by polynomial  $\varphi(x, y)$  of the 2<sup>nd</sup> order expressed in  $x$  and  $y$  and presenting nine (09) coefficients:

$$\varphi(x,y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 x^2 + \alpha_5 y^2 + \alpha_6 xy + \alpha_7 x^2 y + \alpha_8 xy^2 + \alpha_9 x^2 y^2 \quad (4)$$

### III. LAW OF CONTACT AND FRICTION

The different criteria [4] are expressed in terms of constraint:

#### 3-1 Tensile strength criterion

$$\sigma_n \leq R_T \quad (5)$$

$\sigma_n$ : Normal constraint with discontinuity

$R_T$ : Tensile strength of discontinuity

#### 3-2 Coulomb's friction criterion

$$|\sigma_T| \leq |\mu \cdot \sigma_n| \quad (6)$$

$\sigma_T$ : Tangential constraint with discontinuity.

$\mu$ : Coefficient of friction.

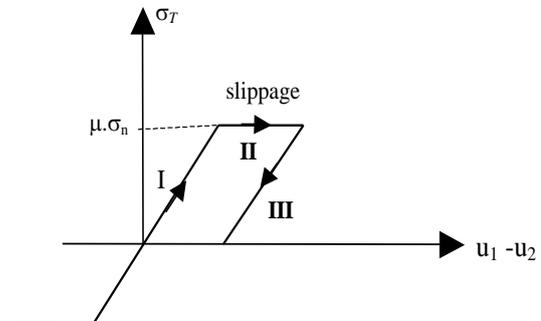
### 3-3 Opening Criterion

$$(v_1 - v_2) > e \quad (7)$$

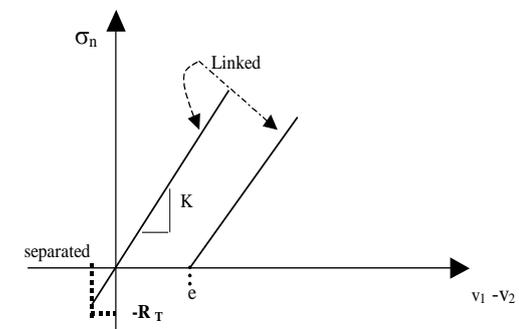
$e$ : is the initial opening of the discontinuity

$v_1, v_2$ : Displacement vertical of the solids  $S_1$  and  $S_2$

These different criteria are represented in figure 4.



(a)



(b)

Figure 4: a) Behaviour in the tangential direction

(b) Behaviour in the normal direction

The method consists [4, 5] in imposing or removing high tangential  $k_T$  and /or normal  $k_N$  stiffness Between the nodes into contact element. At the end of each iteration, one calculates the constraints of contact in the elements of contact, and one will check if the criteria are satisfied. If it is not the case, tangential  $K_T$  and / or normal rigidity  $k_N$  will be modified. Consequently:

- If the criterion (4-1) is not satisfied,  $K_T, k_N$  are set to zero, the two points are separated.

- If the criterion (4-2) is not satisfied, slippage occurs and  $K_T$  only is set to zero

- If the criterion (4-3) is not satisfied, (i.e. the two solids interpenetrate ), both  $K_T$  and  $k_N$  are set ( or reset) to very large value : the two points are linked or relined together

### IV. RESULTS AND DISCUSSIONS

The description given herein corresponds to the method implemented on fem code [2,3], which is an incremental method for each iteration step, ie the displacements calculated at each iteration step are added to the displacements of the previous iteration of

the local increment under consideration. At the end iteration step these relation allow to calculate the part of the contact constraints due to the spring and to check if criteria are satisfied. If it is not the case the tangential and / or the normal stiffness is modified.

#### 4.1 Behaviour of a rock fissured

It is of a fissured rock, rectangular section, thickness the unit, height  $H = 4$  cm and length  $L = 12$  cm. It is subjected in its loose lead to a Load  $P = 100$  kg (figure 5).

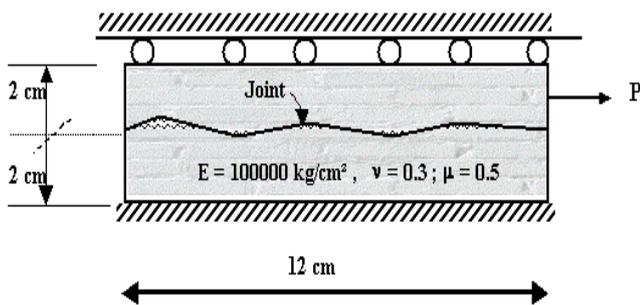


Figure 5 beam fissured

The model of calculation comprises 54 nodes, 08 *Quadratic element* and 09 elements of contact, having a tensile strength null (see figure 6). These elements are linked at the beginning.

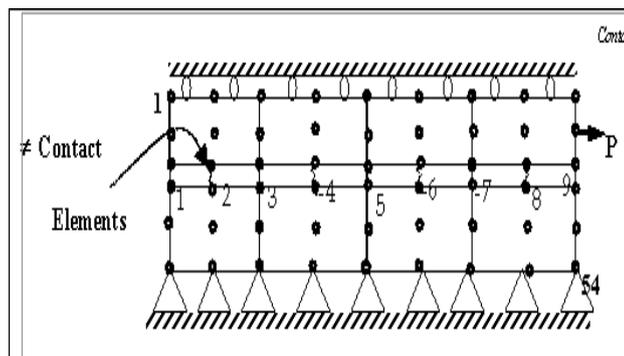


FIGURE 6 MESH OF BEAM The variations of the contact stresses of  $\sigma_n$  and  $\sigma_t$  according to tangential displacement  $U$  are shown in figure 7.

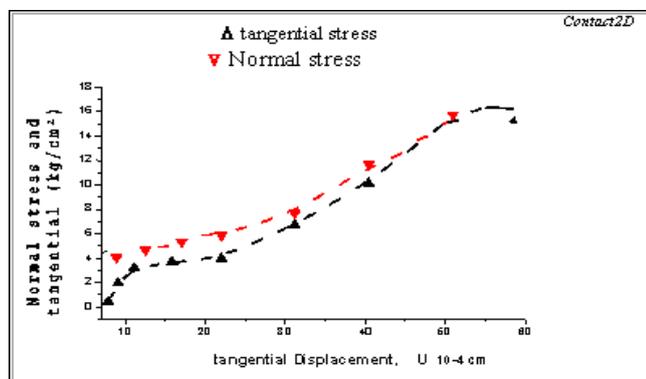


Figure 7 Variation of the normal constraints and tangential according to displacement  $U$

The evolution of shear stresses  $\tau_{xy}$  along the piece are shown in figure 8. The points noted by the

symbol  $\neq$  represent the elements of contact found in a state of separation, while the points noted by symbol  $\equiv$  represent the elements of contact found in a state of slip. One can also notice the concentration of the constraints  $\tau_{xy}$  along the interface. The arrows indicate the extension of the zones of deformation in the piece.

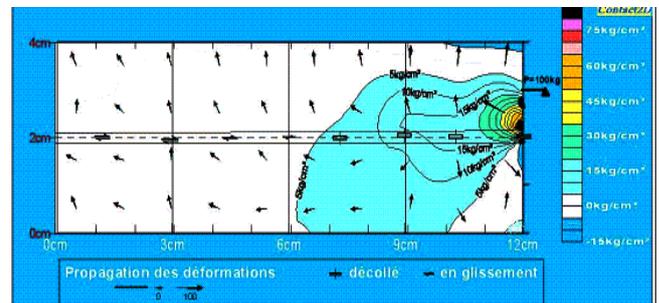


Figure 8 Evolution of the contact stress  $\tau_{xy}$

The contact considered in this example has surface roughness that is small compared with the macroscopic contact area and has well defined normal and tangent direction to the contact surface

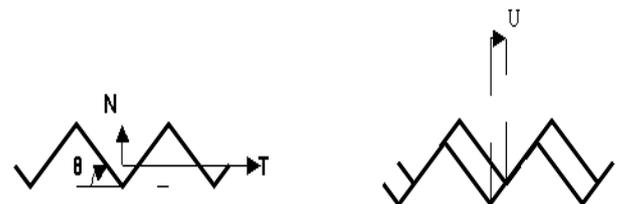


Figure 9 irregularity Surface

We considered an initial slope of the joint  $\theta$  between the two pieces and by keeping the same conditions as previously. The modification made on the field of displacements can be judged starting from figure 10.

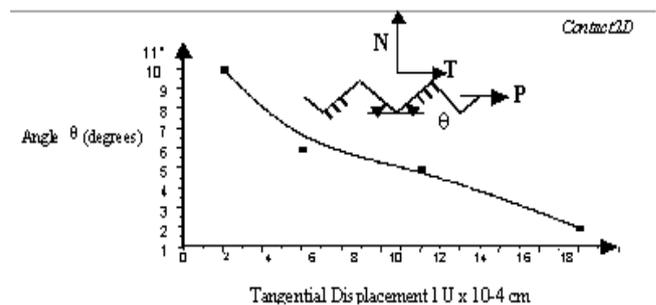


Figure 10 Variation of displacement tangential  $u$

#### V. CONCLUSION

This study was devoted to the development of a digital model for the analysis of the elastic behaviour of a composite beam subjected to stresses of contact.

That required the determination of the stresses of contact between the various layers in contact. The resolution of the equations of elasticity was carried out by the finite element method. This model makes it possible to study the influence of the parameters like: the state of the interface, the thickness of the joint and the property mechanics of the various layers.

The analysis of the results of the treated applications makes it possible to draw from them several conclusions of which most important are:

The conventional description of the contact which under hears the deformations of a semi-infinite medium is insufficient.

The treated example shows that balance in a contact is not only one problem of surface.

\* Stresses of contact and friction not depending only on materials in contact, but also on the state of surfaces in contact

\* Assimilation of a fissured beam as being a continuous body [12,17] pleasing to neglect the role of the interfaces and do not reflect the real behaviour of the beam.

The state of the interfaces has a great influence on the value and the distribution of the stresses.

The angle  $\Theta$  of the interfaces has a great influence on the value and the distribution of displacements

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