Numerical Gear Vibration Simulation in the Presence of Localized and Distributed Defects

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***Abstract*— The early detection of gears and gearbox faults is the most important step in preventive maintenance. In this field, the modeling is the key phase to analyze theoretically the gears vibration behavior. Many models of gears have been developed in the literature in order to detect the existence of defects and their effect on the gear vibratory signal. The objective of this study is to simulate a two degrees of freedom system for which the time varying mesh stiffness is the main source of excitation. The effect of tooth defection on the vibration signature is analyzed using the wavelets transform. A comparative study with classical methods like cepstral and spectral analysis was conducted.**

***Keywords*— detection, faults, maintenance, vibration, modeling, gears, stiffness, wavelets.**

I. INTRODUCTION

Gear and gearbox systems are widely used in the majority of industrial applications to transmit power and motion with constant ratio. In spite of their importance in the industry, gears may have failure limiting their ordinary nominal life. So it is necessary to diagnose these mechanical components at the opportune time to avoid an undesirable stop and to ensure the machine availability [1]. Nowadays, many techniques of fault detection have been proposed by researchers. These techniques are either defined in the time, time-frequency domain or based on statistical approaches. Spectral analysis allows decomposing a signal into its complex basic constituents to represent the amplitude of a signal based on the frequency. Some gear defects manifest their presence by an amplitude modulation and frequency meshing signal. It is also possible using the demodulating amplitude and frequency methods to detect the presence of defect [2]. The time-frequency analysis is also used to identify the location of damage in the gear transmission system based on the Wigner-Ville distribution (WVD) and the continuous wavelets transform (CWT) [3-4]. Statistical indicators like Kurtosis, Root Mean Square (RMS) and Crest Factor (FC) are also used to examine the effect of defect on its vibratory signal. The RMS measures the mean energy of the signal; the FC value measures the maximum amplitude value between the extreme ones of the signal. Kurtosis is defined as the ratio between the central moment of order 4 and the square of central moment of order 2. The basic idea of those statistical indicators is that any occurrence of a fault causes a significant modification of the statistical characteristics of the signal [5-6]. Cepstrum of a time signal, defined as the spectrum of the logarithm of the time signal and used to separate the impulse response for excitations, is widely used in the literature in rotating machine diagnosis [7-8]. In this work, two degrees of freedom model is conducted to simulate spur gear in healthy case and in the presence of localized and distributed defect. Many analysis methods are used like time, frequency, cepstral analysis and wavelets transformation (WT) in order to assess the effect of defect on the vibratory signal therefore to examine the effect on the system vibration and spectrum amplitude.

II. GEAR –PAIR MODEL

The model of gear-pair examined in this study is illustrated in the Fig.1 [9]. It consists of a spur gear-pair with known effective masses m_n (n=1, 2), moments of inertia I_n and base radii R_n. Both gears are subjected to moments T_n. The system has two degrees of freedom. The mesh stiffness variation is the main source of excitation.

Neglecting the friction and the lateral displacements of both gears, the system is described by two rotation degrees of freedom $\theta_1(t)$ and $\theta_2(t)$. The motion equations are given by:

$$\begin{cases} I_1\dfrac{d^2\theta_1}{dt^2}+R_1C(t)(R_1\dfrac{d\theta_1}{dt}-R_2\dfrac{d\theta_2}{dt})+R_1k(t)(R_1\theta_1-R_2\theta_2)=T_1 \\ I_2\dfrac{d^2\theta_2}{dt^2}-R_2C(t)(R_1\dfrac{d\theta_1}{dt}-R_2\dfrac{d\theta_2}{dt})-R_2k(t)(R_1\theta_1-R_2\theta_2)=-T_2 \end{cases} \quad (1)$$

Where $C(t)(R_1\dfrac{d\theta_1}{dt}-R_2\dfrac{d\theta_2}{dt})$ is the damping force, $k(t)(R_1\theta_1(t)-R_2\theta_2(t))$ is the mesh elastic force and k(t) is the time varying mesh stiffness.

The mesh stiffness illustrates the elastic contact between the two conjugated teeth during the meshing. It was a subject of several studies trying to examine also the influence of defects on the stiffness [10-11-12]. The stiffness of one tooth is calculated from the deflection due to bending, filet foundation and Hertzian contact [10]. In order to simplify the model, the variation in slot of the meshing stiffness is approximated by the following function:

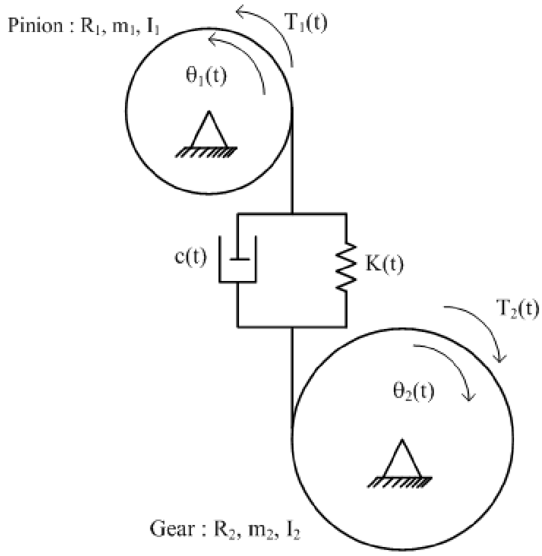

Fig. 1 Mechanical model

$$k(t)=\begin{cases} K_2 & for\ 0 \le t \le (\varepsilon-1)T_m \\ K_1 & \text{for } (\varepsilon-1)T_m \le t \le T_m \end{cases} \quad (2)$$

With T_m is the meshing period and ε is the contact ratio. K_1 and K_2 denote respectively the stiffness when the contact concerns single tooth pair or double ones. Equation (1) can be rewritten in a non-dimensional form as:

$$M_e \frac{d^2x}{dt^2}+c(t)\frac{dx}{dt}+k(t)x=F(t) \quad (3)$$

With $x=R_1\theta_1-R_2\theta_2$, $M_e=\dfrac{I_1I_2}{R^2{}_1I_2+R^2{}_2I_1}$

and $F(t)=M_e\left(\dfrac{T_1R_1}{I_1}+\dfrac{T_2R_2}{I_2}\right)$

Where x called the dynamic transmission error (DTE), Me is the equivalent mass and F(t) is the force transmitted through the gear-pair. The equation (3) can be written as:

$$\frac{d^2x}{dt^2}+2\xi\omega_0\frac{dx}{dt}+K(t)x=\frac{T_1R_1}{I_1}+\frac{T_2R_2}{I_2} \quad (4)$$

Where $\omega_0=\sqrt{k_m/M_e}$, k_m is the mean of k(t).

In equation (4), the only parameter varying with time is the stiffness function k(t). The other parameters are the geometrical characteristics of the gears and they are constant. The alteration of the gear system functioning is manifested either by degradation of the tooth due to repetitive contact pressures or the initial presence of malfunctions. These different types of defects influence, very significantly, the stiffness function k(t). This work proposes to model the presence of located defects or distributed one through the modification of the function k(t). Thereafter, the solution of equation (4) is obtained using Matlab.

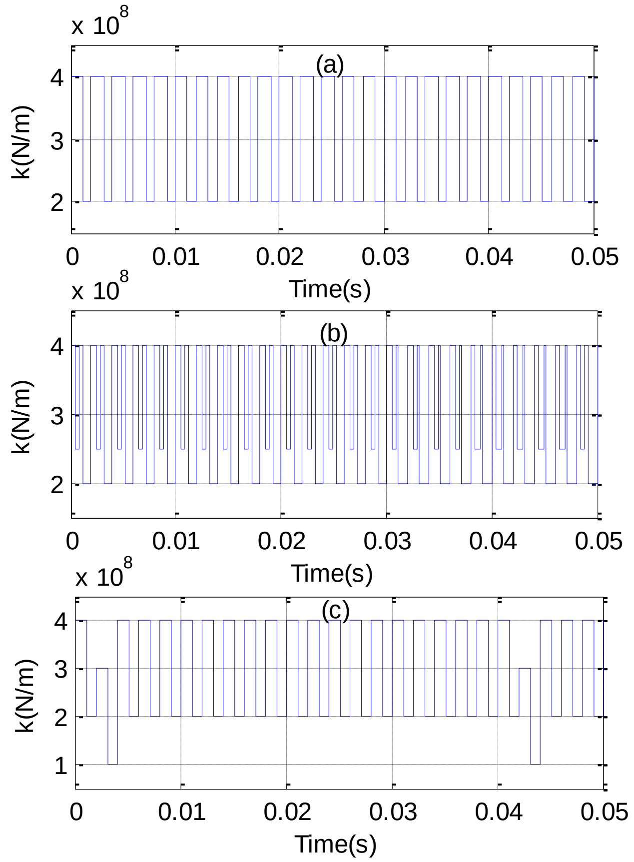

Fig. 1 Time varying mesh stiffness for: (a) healthy case, (b) distributed defect, (c) local defect pinion tooth defect

III. RESULTS AND DISCUSSIONS

The model presented in figure 1 was simulated using Simulink considering equation (4) with pinion rotation speed equal to 1500 tr/min and the parameters illustrated in table I.

TABLE I. PARAMETER OF THE SPUR GEAR (AS USED IN [13])

	Pinion		Wheel
Teeth numbers	20		40
Inertia moments (kg.m²)	0.00026		0.0045
Base circle (m)	0.05		0.11
Module(m)		0.003	
Pressure angle		A=20°	
Contact ratio		1.6	
Teeth width (m)		0.023	
Stiffness (N/m)		K_1=2 10^8 K_2=4 10^8	
Torque T_1(N.m)	150		

The tooth damage is modeled by a loss in the mesh stiffness. This modification will be as great as damage is important [11-12]. For a distributed damage, there's in every meshing a defective tooth giving a drop in the rigidity. However, when the defect is located on pinion tooth or on the wheel tooth or on both, the local damage is represented by a loss in mesh stiffness at the defected gear (pinion or wheel or both) rotation frequency (Figure 2).

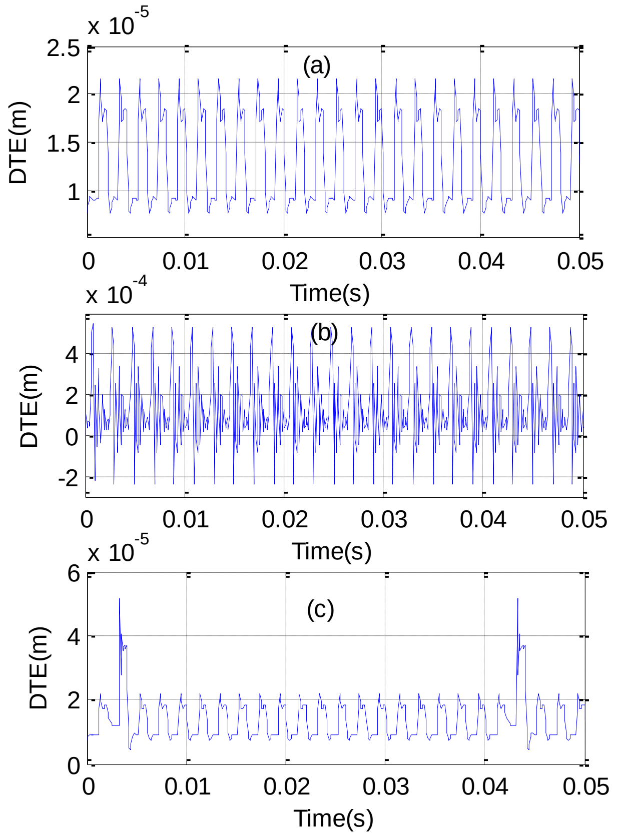

Fig. 3 DTE: (a) healthy case, (b) distributed defect, (c) local defect at pinion tooth

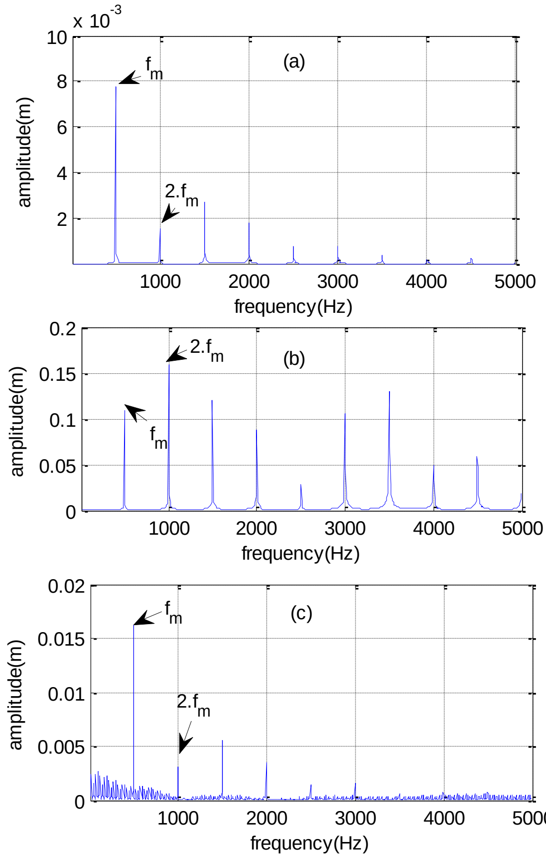

Fig. 4 Spectrum: (a) healthy case, (b) distributed defect, (c) local defect at pinion tooth

Figure 3 shows that in presence of distributed defect the DTE increases while for the localized defect many impulsions appears at the frequency corresponding to defect gear (pinion or wheel or both) rotation frequency. In the healthy case, when a tooth of the driving gear engages the driven wheel, the variation of the contact force provides a periodic vibration at the meshing frequency. The spectrum in this case is composed by the meshing frequency f_m and its harmonics as shown in figure 4-a. In the presence of distributed defects, a shock occurs at the passage of each tooth. Consequently the spectrum consists of a comb lines having a frequency corresponding to the meshing frequency, but with a much higher magnitude comparing with healthy case, as reflected in figure 4-b. On the other hand, for a local defect, the corresponding spectrum, figure 4-c, shows that the effect of defect on vibration signal is the amplitude modulation around the meshing frequency f_m and a comb lines whose pitch corresponds to the pinion rotation frequency f_p ranging up to high frequencies.

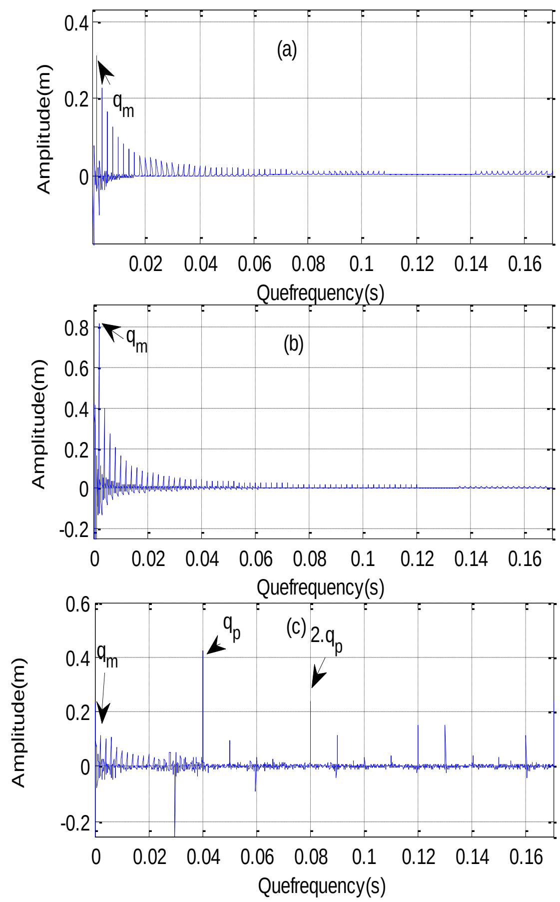

Fig. 5 Cepstrum:(a) healthy case, (b) distributed defect, (c) local defect at pinion tooth

The cepstral analysis allows extracting the fault period as shown in figure 5. In healthy case, the cepstrum is constituted by composant at the quefrequency q_m=0.002s corresponding to meshing quefrequency and its harmonics.

The same remarks steel available for a distributed damage (case (b)) but the magnitude increases. For a local defect, cepstral analysis allow to extract the local fault period q_p= 0.04s corresponding to pinion rotation quefrequency as shown in figure5-c.

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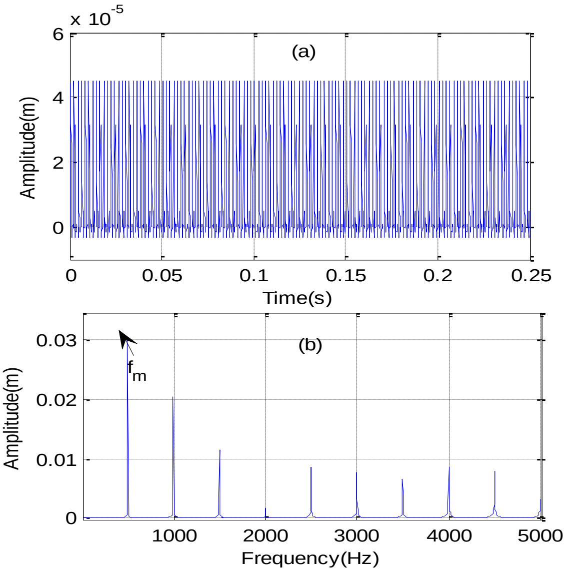

Fig. 6 (a) residual signal, (b) residual signal spectrum in the case of distributed deffect

Also another way for analyzing the gear vibratory behavior is to use the residual signal which corresponds to the signal in the presence of defect minus the signal for the healthy case. The fault is extracted based on the spectral analysis of its residual signal as shown in the figures 6 and 7.

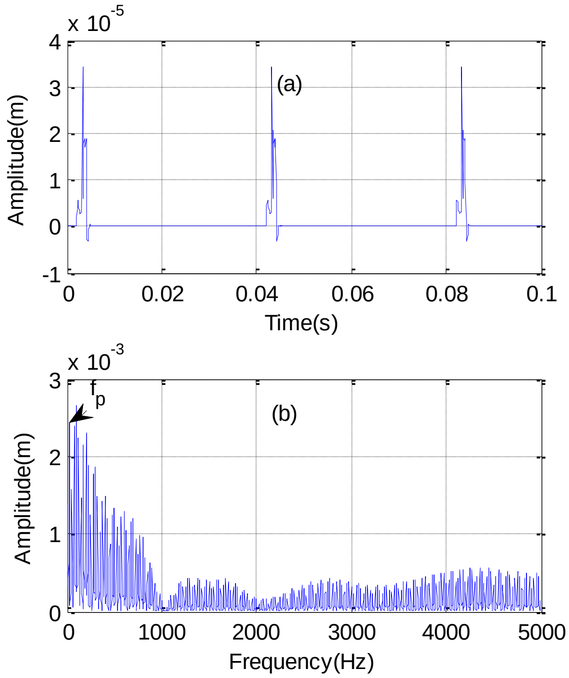

Fig. 7 (a) residual signal, (b) residual signal spectrum In the case of loacl defect

Wavelet transform (WT) is a time-scale representation technique which expresses a signal into a localized two-dimensional time functions and scale (pseudo-frequency), it can be categorized into two main types: continuous wavelet transform (CWT) and discrete wavelet transform (DWT) [14-15-16]. In the figures (8, 9, 10, 11, 12), the fault detection is done by the WT. On the one hand, the CWT is used with Morlet mother wavelet and gives us the representation (time-frequency-coefficients) for the three cases: healthy, local and distributed damages as illustrated in figures (8, 9, 10). For a distribute damage the coefficients values increase comparing with those of healthy case. Also, the CWT lets to extract the local fault that's represented by periodic shocks at every 0.04s as represented in figure 10. In addition the CWT gives a time-frequency representation. On the other hand, in the DWT the signals are decomposed into a hierarchical structure of details and approximations at limited levels. The mother wavelet used is Daubechies (db1) at level 4. The figure 12 represents the Discrete Wavelet Decomposition in the presence of the local fault; it shows also a periodic shocks with the period 0.04s as shown in the figure 11, too this decomposition led to a good fault extraction.

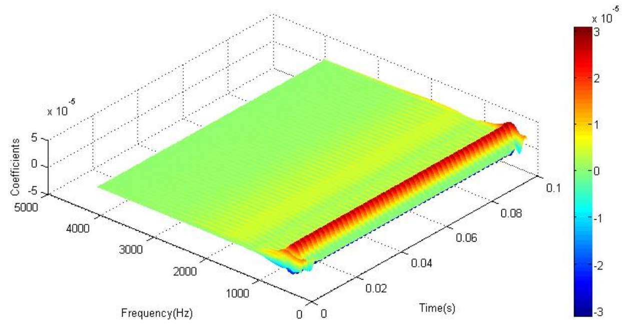

Fig. 8 CWT of the DTE in the healthy case

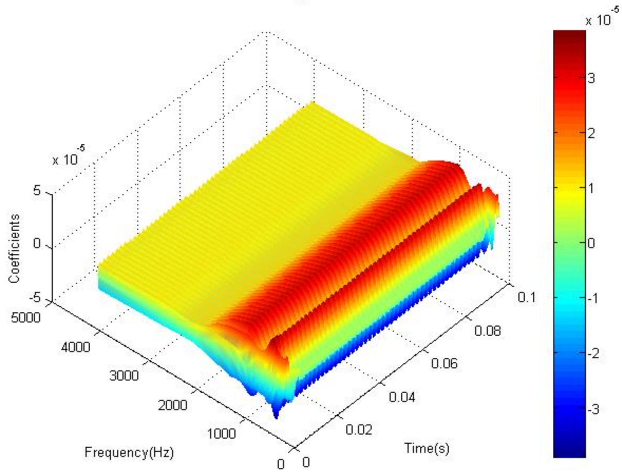

Fig. 9 CWT of the DTE in the case of distributed defect

The results obtained in this work show that for the first signal kind, the three methods (spectral, cepstraland WT analysis) allow identifying gear faults but the WT gives excellent results relatively to the cepstral and spectral analysis. In fact WT method gives more informations in time and frequency domain simultaneously due to the CWT. It lets to analyse the low and high frequencies separately. Moreover, the spectral analysis presents some limitations in local fault detection then it is difficult to extract the fault frequency because of the amplitude modulation caused by the fault. The cepstral analysis does this task but with less information respectively to the WT. When using residuals signals, the spectral analysis was sufficient.

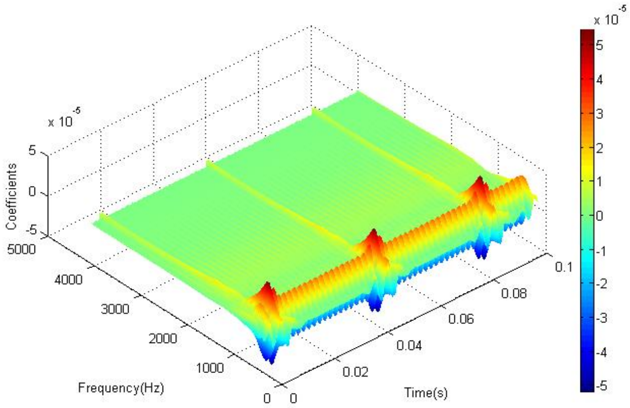

Fig. 10 CWT of the DTE in the case of local fault

IV. CONCLUSION

This study was devoted to simulate two degrees of freedom model of gear and time varying mesh stiffness in the presence of local and distribute teeth defect. There are two ways to treat the gear vibration behavior by treating the resulting defected gear vibration signals or the residuals signals. A comparative study is made between the three analysis methods: spectral, cepstral and WT analysis for fault detection based on the gear vibration signals.

As perspectives, six degrees of freedom model of gear was developed and bench test is designed and under construction for a comparative study between simulation and experimental results.

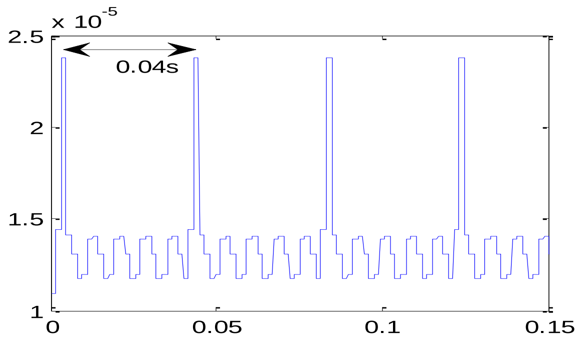

Fig. 11 The forth Approximation of the DWT decomposition

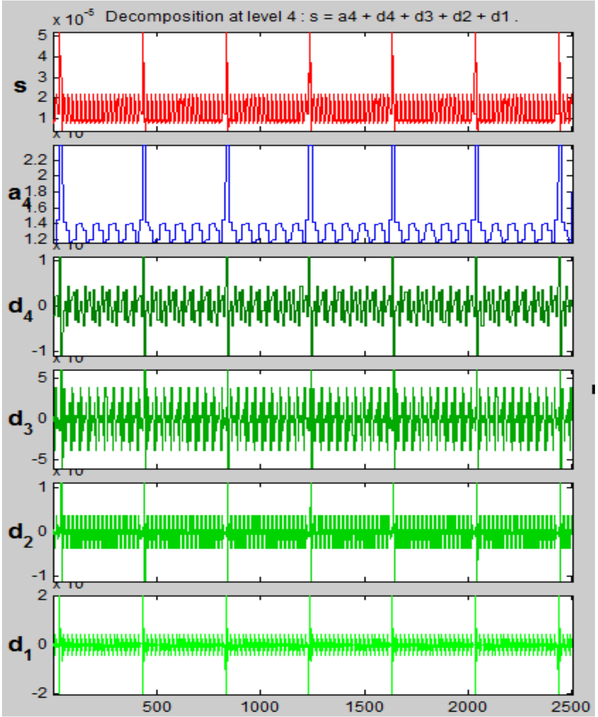

Fig. 12 Discrete wavelet decomposition at level 4

References

[1] A .Boulenger and C.Pachaud,analyse vibratoire en maintenance surveillance et diagnostic des machines in gestion industrielle,3éme édition.

[2] H.Chen and M.J.Zuo, " Demodulation of Gear Vibration Signals for Fault Detection,"IEEE,2009

[3] A.A.Mouloud, C. Djamel, "Application de la transformée en ondelette à l'analyse des signaux vibratoires d'un système d'engrenage en vue d'un diagnostic précoce," 4th International. Conference on Computer Integrated Manufacturing CIP'2007,2007.

[4] G.M.Rakotorazafindrazato, "Méthodes numériques pour la caractérisation vibratoire de structures complexes," Thèse, Institut Supérieur de Technologie d'Antananarivo , MADAGASCAR, 2010.

[5] C. Benchaabane, A. Djebala, N. Ouelaa et S.Guenfoud, "Diagnostic Vibratoire des Défauts d'Engrenages Basé sur les Indicateurs Scalaires".

[6] J.Yin, W.Wang,Z.Man,S.Khoo , "Statistical modeling of gear vibration signals and its application to detecting and diagnosing gear faults," Information Sciences.

[7] M.El Badaoui, F.Guillet, J.Danière, "Surveillance des systemes complexes à engrenages par l'analyse cepstrale synchrone,"Traitement du signal, 1999.

[8] El Badaoui, F. Guillet, J. Danièere , ''New applications of the real cepstrum to gear signals including definition of a robust fault indicator,'' Science Direct, Mechanical .Systems and Signal Processing 18 (2004) 1031–1046,2004.

[9] Mina ER-RAOUDI, Mohammed DIANY, Hicham AISSAOUI, ''Gear tooth faults detection using numerical vibration simulation,'' International symposium on aircraft materials ACMA2014, Morocco, 2014.

[10] F.Chaari, W.Baccar, '' M.S.Abbes and M.Haddar, spalling or tooth breakage on gear mesh stiffness and dynamic response of a one-stage spur gear transmission ,'' ScienceDirect, European Journal of Mechanics A/Solids 27 (2008) 691–705.

[11] F. Chaari , R. Zimroz, W. Bartelmus, T. Fakhfakh, M. Haddar, '' Modelling of local damages in spur gears and effects on dynamics response in presence of varying load conditions''.

[12] F.Chaari, T.Fakhfakh et M. Haddar, ''Simulation numérique du comportement dynamique d'une transmission par engrenages en présence de défauts de dentures,'' Mécanique & Industries, 2006.

[13] M.T. Khabou , N.Bouchaala,F.Chaari,T.Fakhfakh,M.Haddar, ''Study of a spur gear dynamic behavior in transient regime,'' Mechanical Systems and Signal Processing,2011.

[14] J. Rafiee , P.W. Tse , A. Harifi , M.H. Sadeghi , '' A novel technique for selecting mother wavelet function using an intelligent fault diagnosis system,'' Expert Systems with Applications,2009.

[15] G.M.Rakoto Razafindrazato,O.Riou, J.Felix, ''detection de défaut sur motoréducteur à engrenage en utilisant la transformée en ondelettes,'' Revue sciences et maintenance,2012.

[16] S.G.Mallat, ''A Theory for Multiresolution Signal decomposition the wavelet representation,'' IEEE transactions on pattern analysis and machine intelligence, 1989.