Fixed Point Theorem for P-1 Compatible in Random Probabilistic Space

Manoj Shukla, Shailendra Kumar Patel[1] and R. P. Dubey[2]
Govt. Model Science College (autonomous), Jabalpur (M.P.) India
manojshukla2012@yahoo.com
[1]Bhilai Institute of Tech., Raipur (C.G) India
spskpatel@gmail.com
[2]Dr C.V. Raman Institute of Sci. and Techn. Kota Bilaspur (C.G) India
ravidubey1963@yahoo.co.in

***Abstract*—In this paper we have proved fixed point theorem in random probabilistic space for P-1 compatible mappings which is introduced by Sevet Kutukcu and Sushil Sharma.**

Keywords—Fuzzy Menger Space, P-1 Compatible mappings, Common fixed point.

Introduction and Preliminaries

Menger [5] in 1942 introduced the notation of the probabilistic metric space. The probabilistic generalization of metric space appears to be well adopted for the investigation of physical quantities and physiological thresholds. Schweizer and Sklar [7] studied this concept and then the important development of Menger space theory was due to Sehgal and Bharucha-Reid [8].Sessa [9] introduced weakly commuting maps in metric spaces. Jungck [2] enlarged this concept to compatible maps. The notion of compatible mapsin Menger spaces has been introduced by Mishra [6]. Cho [1] et al. and Sharma [10] gave fuzzy version of compatible maps and proved common fixed point theorems for compatible maps in fuzzy metric spaces. So many works have been done in fuzzy and menger space [3],[4] and [12]. Sevet Kutukcu and Sushil Sharma introduce the concept of compatible maps of type (P-1) and type (P-2), show that they are equivalent to compatible maps under certain conditions and prove a common fixed point theorem for such maps in Menger spaces. Rajesh Shrivastav, Vivek Patel and Vanita Ben Dhagat[11] have given the definition of fuzzy probabilistic metric space and proved fixed point theorem for such space.

We prove fixed point results for fuzzy probabilistic space with compatible P-1.

Definition 1. A binary operation *: $[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm if it satisfies the following conditions

(1)* is associative and commutative,

(2)* is continuous,

(3)$a*1 = a$ for all $a\in[0,1]$,

(4)$a*b \le c*d$ whenever $a \le c$ and $b \le d$, for each a, b, c, d $\in[0,1]$.

Two typical examples of continuous t-norm are

$a*b = a\,b$ and $a*b = \min(a,b)$.

Definition 2. Let (Ω, $\sum$) be a measurable space with $\sum$ a sigma algebra of subsets of Ω and *M* a non-empty subset of a metric space $X = (X, d)$. Let 2^M be the family of all non-empty subsets of *M* and *C*(*M*) the family of all nonempty closed subsets of *M*. A mapping $G: \Omega \to 2^M$ is called measurable if, for each open subset *U* of *M*,

$$G^{-1}(U)\in\sum, where\, G^{-1}(U)=\{w\in\Omega : G(w)\cap U\neq\phi\}.$$

Definition 3. A mapping $\xi : \Omega \to M$ is called a measurable selector of a measurable mapping $G : \Omega \to 2^M$ if ξ is measurable and $\xi(w) \in G(w)$ for each $w \in \Omega$.

Definition 4. A mapping $T : \Omega \times M \to X$ is said to be a random operator if, for each fixed $x \in M$, $T(., x) : \Omega \to X$ is measurable.

Definition 5. A measurable mapping $\xi : \Omega \to M$ is a random fixed point of a random operator $T : \Omega \times M \to X$ if $\xi(w) = T\left(w, \xi(w)\right)$ for each $w \in \Omega$.

Definition 5.1.1 A fuzzy probabilistic metric space (FPM space) is an ordered pair (X,F_α) consisting of a nonempty set X and a mapping F_α from XxX into the collections of all fuzzy distribution functions $F_\alpha\in R$ for all α. $\in$ [0,1]. For x, y $\in$ X we denote the fuzzy distribution function F_α (x,y) by $F_{\alpha(x,y)}$ and $F_{\alpha(x,y)}$ (u) is the value of $F_{\alpha(x,y)}$ at u in R.

The functions $F_{\alpha(x,y)}$ for all α. $\in$ [0,1] assumed to satisfy the following conditions:

(a) $F_{\alpha(x,y)}(u) = 1 \ \forall\, u > 0$ iff $x = y$,

(b) $F_{\alpha(x,y)}(0) = 0 \ \forall$ x , y in X,

(c) $F_{\alpha(x,y)} = F_{\alpha(y,x)} \ \forall$ x , y in X,

(d) If $F_{\alpha(x,y)}(u) = 1$ and $F_{\alpha(y,z)}(v) = 1 \Rightarrow F_{\alpha(x,z)}(u+v) = 1 \ \forall$ x , y ,z $\in$X and u, v > 0.

Definition 5.1.2 A commutative, associative and non-decreasing mapping t: $[0,1] \times [0,1]\to [0,1]$ is a t-norm if and only if $t(a,1)= a \ \forall a\in[0,1]$, $t(0,0)=0$ and $t(c,d) \ge t(a,b)$ for $c \ge a$, $d \ge b$.

Definition 5.3 A Fuzzy Menger space is a triplet (X,F_α,t), where (X,F_α) is a FPM-space, t is a t-norm and the generalized triangle inequality

$$F_{\alpha(x,z)}(u+v) \ge t\left(F_{\alpha(x,z)}(u), F_{\alpha(y,z)}(v)\right)$$

holds for all x, y, z in X u, v > 0 and α. ∈ [0,1].

The concept of neighborhoods in Fuzzy Menger space is introduced as

Definition 5.1.4 Let (X,F_{α},t) be a Fuzzy Menger space. If $x \in X$, $\varepsilon > 0$ and $\lambda \in (0,1)$, then (ε,λ) - neighborhood of x, called $U_x(\varepsilon,\lambda)$, is defined by

$$U_x(\varepsilon,\lambda) = \{y \in X: F_{\alpha(x,y)}(\varepsilon) > (1-\lambda)\}.$$

An (ε,λ)-topology in X is the topology induced by the family $\{U_x(\varepsilon,\lambda): x \in X, \varepsilon > 0, \alpha \in [0,1] \text{ and } \lambda \in (0,1)\}$ of neighborhood.

Remark: If t is continuous, then Fuzzy Menger space (X,F_{α},t) is a Housdroff space in (ε,λ)-topology.

Let (X,F_{α},t) be a complete Fuzzy Menger space and $A \subset X$. Then A is called a bounded set if

$$\liminf_{u\to\infty\; x,y\in A} F_{\alpha(x,y)}(u) = 1$$

Definition 5.1.5 A sequence $\{x_n\}$ in (X,F_{α},t) is said to be convergent to a point x in X if for every $\varepsilon>0$and $\lambda>0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $x_n \in U_x(\varepsilon,\lambda)$ $\forall n \geq N$ or equivalently $F_{\alpha}(x_n, x; \varepsilon) > 1-\lambda$ for all $n \geq N$ and $\alpha \in [0,1]$.

Definition 5.1.6 A sequence $\{x_n\}$ in (X,F_{α}, t) is said to be cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that for all $\alpha \in [0,1]$ $F_{\alpha}(x_n,x_m; \varepsilon) > 1-\lambda$ $\forall$ n, m ≥ N.

Definition 5.1.7 A Fuzzy Menger space (X,F_{α},t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all $\alpha \in [0,1]$.

Following lemmas is selected from [8] and [12] respectively in fuzzy menger space.

Lemma 1. Let $\{x_n\}$ be a sequence in a Menger space $(X, F_{\alpha}, *)$ with continuous t-norm $*$ and $t * t \geq t$. If there exists a constant $k \in (0, 1)$ such that

$$F_{\alpha(x_n,x_{n+1})}(kt) \geq F_{\alpha(x_{n-1},x_n)}(t)$$

for all t > 0 and n = 1, 2, . . .,then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2 ([12]). Let $(X, F_{\alpha}, *)$ be a Menger space. If there exists $k \in (0, 1)$ such that

$F_{\alpha(x,y)}(kt) \geq F_{\alpha(x,y)}(t)$ for all $x, y \in X$ and $t > 0$, then x = y.

Definition 6. Self maps A and B of a Menger space $(X, F_{\alpha},*)$ are said to be compatible of type (P) if $F_{\alpha(ABxn,BBxn)}(t) \to 1$ and $F_{\alpha(BAxn,AAxn)}(t) \to 1$ $\forall t > 0$, whenever$\{x_n\}$ is a sequence in X such that $Ax_n,Bx_n \to z$ for some $z \in X$ as $n \to \infty$.

Definition 7. Self maps A and B of a Menger space $(X, F_{\alpha}, *)$ are said to be compatible of type (P-1) if $F_{\alpha(ABxnBBxn)}(t) \to 1$ for all t > 0, whenever $\{x_n\}$ is a sequence in X such that $Ax_n,Bx_n \to z$ for some z in X as $n \to \infty$.

Main Results

Theorem 1. Let A,B, P, Q, S and T be self maps on a complete Menger space $(X, F_{\alpha}, *)$ with continuous t-norm $*$ and $t * t \geq t$, for all $t \in [0, 1]$, satisfying:

(1.1) $P(X) \subseteq ST(X)$, $Q(X) \subseteq AB(X)$,

(1.2) there exists a constant $k \in (0, 1)$ such that

$F_{\alpha(Px,Qy)}(kt) \geq F_{\alpha(ABx,Sty)}(t) * F_{\alpha(Px,ABx)}(t) * F_{\alpha(Qy,Sty)}(t) * F_{\alpha(Px,Sty)}(\beta t) * F_{\alpha(Qy,ABx)}((2-\beta)t)]$

$\forall x, y \in X$, $\beta \in (0, 2)$ and $t > 0$,

(1.3) AB = BA, ST = TS, PB = BP,QT = TQ,

(1.4) either P or AB is continuous,

(1.5) the pairs (P,AB) and (Q, ST) are compatible of type (P-1).

Then A,B, P, Q, S and T have a unique common fixed point.

Proof. Let x_0 be an arbitrary point of X. By (1.1) there exists $x_1, x_2 \in X$ such that

$Px_0 = STx_1 = y_0$ and $Qx_1 = ABx_1 = y_1$.

Inductively, we can construct sequences$\{x_n\}$ and $\{y_n\}$ in X such that

$Px_{2n} = STx_{2n+1} = y_{2n}$ and $Qx_{2n+1} = ABx_{2n+2} = y_{2n+1}$ for n = 0, 1, 2, . . .

Step 1. By taking $x = x_{2n}$, $y = x_{2n+1}$ for all $t > 0$ and $\beta = 1 - q$ with $q \in (0, 1)$ in (1.2), we have

$$F_{\alpha(Px_{2n},Qx_{2n+1})}(kt) = F_{\alpha(y_{2n},y_{2n+1})}(kt)$$
$$\geq F_{\alpha(y_{2n-1},y_{2n})}(t) * F_{\alpha(y_{2n},y_{2n-1})}(t) * F_{\alpha(y_{2n+1},y_{2n})}(t) * F_{\alpha(y_{2n},y_{2n})}((1-q)t)$$
$$* F_{\alpha(y_{2n+1},y_{2n-1})}((1+q)t)$$
$$\geq F_{\alpha(y_{2n-1},y_{2n})}(t) * F_{\alpha(y_{2n-1},y_{2n})}(t) * F_{\alpha(y_{2n},y_{2n+1})}(t) * 1$$
$$* F_{\alpha(y_{2n-1},y_{2n})}(t) * F_{\alpha(y_{2n},y_{2n+1})}(qt)$$
$$\geq F_{\alpha(y_{2n-1},y_{2n})}(t) * F_{\alpha(y_{2n},y_{2n+1})}(t) * F_{\alpha(y_{2n},y_{2n+1})}(qt).$$

Since t-norm is continuous, letting $q \to 1$, we have

$$\geq F_{\alpha(y_{2n},y_{2n+1})}(kt) * F_{\alpha(y_{2n-1},y_{2n})}(t) * F_{\alpha(y_{2n},y_{2n+1})}(t).$$

Similarly, we also have

$$\geq F_{\alpha(y_{2n+1},y_{2n+2})}(kt) * F_{\alpha(y_{2n},y_{2n+1})}(t) * F_{\alpha(y_{2n+1},y_{2n+2})}(t).$$

In general, for all n even or odd, we have

$$F_{\alpha(y_n,y_{n+1})}(kt) * F_{\alpha(y_{n-1},y_n)}(t) * F_{\alpha(y_n,y_{n+1})}(t).$$

Consequently, for p = 1, 2, . . ., it follows that,

$$F_{\alpha(y_n,y_{n+1})}(kt) * F_{\alpha(y_{n-1},y_n)}(t) * F_{\alpha(y_n,y_{n+1})}\left(\frac{t}{k^p}\right).$$

By nothing that $F_{\alpha(y_n,y_{n+1})}\left(\frac{t}{k^p}\right) \to 1 \quad as \quad p \to \infty$

we have

$$F_{\alpha(y_n,y_{n+1})}(kt) \geq F_{\alpha(y_{n-1},y_n)}(t)$$

for k∈(0, 1) all n ∈ N and t > 0. Hence, by Lemma1, $\{y_n\}$ is a Cauchy sequence in X. Since (X, F, ∗) is complete, it converges to a point z in X. Also its subsequences converge as follows: $\{Px_{2n}\} \to z$, $\{ABx_{2n}\} \to z$, $\{Qx_{2n+1}\} \to z$ and $\{STx_{2n+1}\} \to z$.

Case I. AB is continuous, and (P,AB) and (Q, ST) are compatible of type(P-1).

Since AB is continuous, $AB(AB)x_{2n} \to ABz$ and $(AB)Px_{2n} \to ABz$. Since (P,AB) is compatible of type (P-1), $PPx_{2n} \to ABz$.

Step 2. By taking $x = Px_{2n}$, $y = x_{2n+1}$ with β= 1 in (1.2), we have

$$F_{\alpha(PPx_{2n},Qx_{2n+1})}(kt) \geq F_{\alpha(ABPx_{2n},STx_{2n+1})}(t) * F_{\alpha(PPx_{2n},ABPx_{2n})}(t) * F_{\alpha(Qx_{2n+1},STx_{2n+1})}(t) * F_{\alpha(PPx_{2n},STx_{2n+1})}(t) * F_{\alpha(Qx_{2n+1},ABPx_{2n})}(t).$$

This implies that, as n → ∞

$$F_{\alpha(z,ABz)}(kt) \geq F_{\alpha(z,ABz)}(t) * F_{\alpha(ABz,ABz)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(z,ABz)}(t) * F_{\alpha(z,ABz)}(t)$$
$$= F_{\alpha(z,ABz)}(t) * 1 * 1 * F_{\alpha(z,ABz)}(t) * F_{\alpha(z,ABz)}(t) * F_{\alpha(z,ABz)}(t).$$

Thus, by Lemma 2, it follows that z = ABz.

Step 3. By taking x = z, $y = x_{2n+1}$ with β= 1 in (1.2), we have

$$F_{\alpha(Pz,Qx_{2n+1})}(kt) \geq F_{\alpha(ABz,STx_{2n+1})}(t) * F_{\alpha(Pz,ABz)}(t) * F_{\alpha(Qx_{2n+1},STx_{2n+1})}(t) * F_{\alpha(Pz,STx_{2n+1})}(t) * F_{\alpha(Qx_{2n+1},ABz)}(t).$$

This implies that, as n → ∞

$$F_{\alpha(z,Pz)}(kt) \geq F_{\alpha(z,z)}(t) * F_{\alpha(z,Pz)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(z,Pz)}(t) * F_{\alpha(z,z)}(t)$$
$$= 1 * F_{\alpha(z,Pz)}(t) * 1 * F_{\alpha(z,Pz)}(t) * 1$$
$$\geq F_\alpha(z,Pz)(t).$$

Thus, by Lemma 2, it follows that z = Pz. Therefore, z = ABz = Pz.

Step 4. By taking x = Bz, $y = x_{2n+1}$ with β = 1 in (1.2) and using (1.3), we have

$$F_{\alpha(P(Bz),Qx_{2n+1})}(kt) \geq F_{\alpha(AB(Bz),STx_{2n+1})}(t) * F_{\alpha(P(Bz),AB(Bz))}(t) * F_{\alpha(Qx_{2n+1},STx_{2n+1})}(t) * F_{\alpha(P(Bz),STx_{2n+1})}(t) * F_{\alpha(Qx_{2n+1},AB(Bz))}(t).$$

This implies that, as n → ∞

$$F_{\alpha(z,Bz)}(kt) \geq F_{\alpha(z,Bz)}(t) * F_{\alpha(Bz,Bz)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(z,Bz)}(t) * F_{\alpha(z,Bz)}(t)$$
$$= F_\alpha(z,Bz)(t) * 1 * 1 * F_{\alpha(z,Bz)}(t) * F_{\alpha(z,Bz)}(t)$$
$$\geq F_{\alpha(z,Bz)}(t).$$

Thus, by Lemma 2, it follows that z = Bz. Since z = ABz, we have z = Az.

Therefore, z = Az = Bz = Pz.

Step 5. Since P(X) ⊂ ST(X), there exists w є X such that z = Pz =STw. By taking $x = x_{2n}$, y = w with β = 1 in (1.2), we have

$$F_{\alpha(Px_{2n},Qw)}(kt) \geq F_{\alpha(ABx_{2n},STw)}(t) * F_{\alpha(Px_{2n},ABx_{2n})}(t) * F_{\alpha(Qw,STw)}(t) * F_{\alpha(Px_{2n},STw)}(t) * F_{\alpha(Qw,ABx_{2n})}(t)$$

which implies that, as n → ∞

$$F_{\alpha(z,Qw)}(kt) \geq F_{\alpha(z,z)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(z,Qw)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(z,Qw)}(t)$$
$$= 1 * 1 * F_{\alpha(z,Qw)}(t) * 1 * F_{\alpha(z,Qw)}(t)$$
$$\geq F_{\alpha(z,Qw)}(t).$$

Thus, by Lemma 2, we have z = Qw.

Hence, STw = z = Qw. Since (Q, ST) is compatible of type (P-1), we have Q(ST)w = ST(ST)w.

Thus, STz = Qz.

Step 6. By taking $x = x_{2n}$, y = z with β = 1 in (1.2) and using Step 5, we have

$$F_{\alpha(Px_{2n},Qz)}(kt) \geq F_{\alpha(ABx_{2n},STz)}(t) * F_{\alpha(Px_{2n},ABx_{2n})}(t) * F_{\alpha(Qz,STz)}(t) * F_{\alpha(Px_{2n},STz)}(t) * F_{\alpha(Qz,ABx_{2n})}(t)$$

which implies that, as n → ∞

$$F_{\alpha(z,Qz)}(kt) \geq F_{\alpha(z,Qz)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(Qz,Qz)}(t) * F_{\alpha(z,Qz)}(t) * F_{\alpha(z,Qz)}(t)$$
$$= F_{\alpha(z,Qz)}(t) * 1 * 1 * F_{\alpha(z,Qz)}(t) * 1 * F_{\alpha(z,Qz)}(t)$$
$$\geq F_{\alpha(z,Qz)}(t).$$

Thus, by Lemma 2, we have z = Qz. Since STz = Qz, we have z = STz.

Therefore, z = Az = Bz = Pz = Qz = STz.

Step 7. By taking $x = x_{2n}$, y = Tz with β = 1 in (1.2) and using (1.3), we have

$$F_{\alpha(Px_{2n},Q(Tz))}(kt) \geq F_{\alpha(ABx_{2n},ST(Tz))}(t) * F_{\alpha(Px_{2n},ABx_{2n})}(t) * F_{\alpha(Q(Tz),ST(Tz))}(t) * F_{\alpha(Px_{2n},ST(Tz))}(t) * F_{\alpha(Q(Tz)ABx_{2n})}(t)$$

which implies that, as $n \to \infty$

$$F_{\alpha(z,Tz)}(kt) \geq F_{\alpha(z,Tz)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(Tz,Tz)}(t) * F_{\alpha(z,Tz)}(t) * F_{\alpha(z,Tz)}(t)$$

$$= F_{\alpha(z,Tz)}(t) * 1 * 1 * F_{\alpha(z,Tz)}(t) * 1 * F_{\alpha(z,Tz)}(t)$$

$$\geq F_{\alpha(z,Tz)}(t).$$

Thus, by Lemma 2, we have z = Tz. Since z = STz, we have z = Sz.

Therefore, z = Az = Bz = Pz = Qz = Sz = Tz, that is, z is the common fixed point of A,B, P, Q, S and T.

Case II. P is continuous, and (P,AB) and (Q, ST) are compatible of type (P-1).

Since P is continuous, $PPx_{2n} \to Pz$ and $P(AB)x_{2n} \to Pz$. Since (P,AB) is compatible of type (P-1), $AB(AB)x_{2n} \to Pz$.

Step 8. By taking $x = ABx_{2n}$, $y = x_{2n+1}$ with β = 1 in (1.2), we have

$$F_{\alpha(P(AB)x_{2n},Qx_{2n+1})}(kt) \geq F_{\alpha(AB(AB)x_{2n},STx_{2n+1})}(t) * F_{\alpha(P(AB)x_{2n},AB(AB)x_{2n})}(t) * F_{\alpha(Qx_{2n+1},STx_{2n+1})}(t) * F_{\alpha(P(AB)x_{2n},STx_{2n+1})}(t) * F_{\alpha(Qx_{2n+1},AB(AB)x_{2n})}(t).$$

This implies that, as $n \to \infty$

$$F_{\alpha(z,Pz)}(kt) \geq F_{\alpha(z,Pz)}(t) * F_{\alpha(Pz,Pz)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(z,Pz)}(t) * F_{\alpha(z,Pz)}(t)$$

$$= F_{\alpha(z,Pz)}(t) * 1 * 1 * F_{\alpha(z,Pz)}(t) * F_{\alpha(z,Pz)}(t)$$

$$\geq F_{\alpha(z,Pz)}(t).$$

Thus, by Lemma 2, it follows that z = Pz. Now using Step 5-7, we have

z = Qz = STz = Sz = Tz.

Step 9. Since Q(X)⊂AB(X), there exists w ∈X such that z = Qz =ABw. By taking x = w, $y = x_{2n+1}$ with β = 1 in (1.2), we have

$$F_{\alpha(Pw,Qx_{2n+1})}(kt) \geq F_{\alpha(ABw,STx_{2n+1})}(t) * F_{\alpha(Pw,ABw)}(t) * F_{\alpha(Qx_{2n+1},STx_{2n+1})}(t) * F_{\alpha(Pw,STx_{2n+1})}(t) * F_{\alpha(Qx_{2n+1},ABw)}(t)$$

which implies that, as $n \to \infty$

$$F_{\alpha(z,Pw)}(kt) \geq F_{\alpha(z,z)}(t) * F_{\alpha(z,Pw)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(z,Pw)}(t) * F_{\alpha(z,z)}(t)$$

$$= 1 * F_{\alpha(z,Pw)}(t) * 1 * F_{\alpha(z,Pw)}(t) * 1$$

$$\geq F_{\alpha(z,Pw)}(t).$$

Thus, by Lemma 2, we have z = Pw. Since z = Qz = ABw, Pw = ABw.

Since (P,AB) is compatible of type (P-1), we have Pz = ABz. Also z = Bz follows from Step 4. Thus, z = Az = Bz = Pz. Hence, z is the common fixed point of the six maps in this case also.

Step 10. For uniqueness, let v (v ≠ z) be another common fixed point of A,B, P, Q, S and T.

Taking x = z, y = v with β = 1 in (1.2), we have

$$F_{\alpha(Pz,Qv)}(kt) \geq F_{\alpha(ABz,STv)}(t) * F_{\alpha(Pz,ABz)}(t) * F_{\alpha(Qv,STv)}(t) * F_{\alpha(Pz,STv)}(\beta t) * F_{\alpha(Qv,ABz)}((2-\beta)t)$$

$$\Rightarrow F_{\alpha(z,v)}(kt) \geq F_{\alpha(z,v)}(t) * F_{\alpha(z,z)}(t) * F_{\alpha(v,v)}(t) * F_{\alpha(z,v)}(t) * F_{\alpha(v,z)}(t)$$

Thus,

$$= F_{\alpha(z,v)}(t) * 1 * 1 * F_{\alpha(z,v)}(t) * F_{\alpha(z,v)}(t)$$

$$\geq F_{\alpha(z,v)}(t).$$

by Lemma 2, we have z = v.

This completes the proof of the theorem.

If we take A = B = S = T = IX (the identity map on X) in Theorem 1, we have the following:

Corollary . Let P and Q be self maps on a complete Fuzzy Menger space (X, F_α, *)

with continuous t-norm * and t*t ≥ t for all t ∈ [0, 1]. If there exists a constant

k ∈ (0, 1) such that

$$F_{\alpha(Px,Qy)}(kt) \geq F_{\alpha(x,y)}(t) * F_{\alpha(x,Px)}(t) * F_{\alpha(y,Qy)}(t) * F_{\alpha(y,Px)}(\beta t) * F_{\alpha(x,Qy)}((2-\beta)t)$$

for all x, y ∈ X, β ∈ (0, 2) and t > 0, then P and Q have a unique common fixed point.

References

[1] Y. J. Cho, H. K. Pathak, S. M. Kang, and J. S. Jung, Common fixed points of compatible maps of type (β) on fuzzy metric spaces, Fuzzy Sets and Systems **93** (1998), 99-111.

[2] G. Jungck, Compatible mappings and common fixed points, Internat. J. Math. Sci. **9**(1986), 771-779.

[3] S.Kutukcu, S. Sharma and H.Tokgoz “A Fixed Point Theorem in Fuzzy metric Spaces”Int. Journal of Math. Analysis, Vol. 1, 2007, no. 18, 861– 872.

[4]S.Kutukcu, A. Tuna, and A. T. Yakut, Generalized contraction mapping principle in intuitionistic Menger spaces and an application to differential equations, Appl. Math.Mech. **28** (2007), no. 6, 779-809.

[5] K. Menger, Statistical metric, Proc. Nat. Acad. **28** (1942), 535-537.

[6] S. N. Mishra, Common fixed points of compatible mappings in PM-spaces, Math. Japon.**36** (1991), 283-289.

[7] B. Schweizer and A. Sklar, Probabilistic Metric Spaces, North-Holland, Amsterdam,1983.

[8] V. M. Sehgal and A. T. Bharucha-Reid, Fixed point of contraction mapping on PM spaces, Math. Systems Theory **6** (1972), 97-100.

[9] S. Sessa, On weak commutativity condition of mappings in fixed point considerations, Publ. Inst. Math. Beograd **32** (1982), 149-153.

[10] S. Sharma, Common fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems **127** (2002), 345-352.

[11] R.Shrivastav, V.Patel and V.B.Dhagat, "Fixed point theorem in fuzzy menger spaces satisfying occasionally weakly compatible mappings"Int. J. of Math. Sci. & Engg. Appls. , Vol.6 No.VI, 2012, 243-250.

[12] B. Singh and S. Jain, A fixed point theorem in Menger space through weak compatibility,J. Math. Anal. Appl. **301** (2005), 439-448.