Investigation of Motorcycle Design Improvements with Respect to Whole Body Vibration Exposure to the Rider

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Abstract—Various studies on motorcycle rider whole body vibration (WBV) exposure indicate that the vibration magnitude in the vertical direction is the most severe as compared to magnitudes in lateral and transverse directions. For the sake of understanding and doing evaluation of the vibration environment of the motorcycle rider and various motorcycle component bodies, a mathematical model was developed. The main intention was to investigate the physics of motorcycle suspensions, study vibration magnitudes in vertical direction in terms of displacement and acceleration on a cyclist depending on motorcycle and road parameters, also attempt to determine the location of the rider’s seat with least vibration magnitudes. In this case, analytical methods for studying response to harmonic base excitation of the rider and other component bodies of a motorcycle were applied with the aim to analyze their vibrational behaviour. The two wheeled motorcycle with a rider model was introduced. The equations of motion for the model were formulated. The matrix form of the equations was written solved and simulations were done. The vibration behaviour in vertical direction of various component bodies of the motorcycle model was observed.

Keywords—modeling, Motorcycle rider, vibration magnitude

I. INTRODUCTION

In recent days modeling has become a major process in the quest for determining, improving, and optimizing dynamic characteristics of engineering systems. Model creation and analysis of a particular vibration system involves the process of determining the inherent dynamic characteristics of the system in terms of spring characteristics and damping factors which facilitates the formulation of mathematical model for analysis of the dynamic behaviour. Modeling is regarded as part of solution of an engineering problem that aims at producing its mathematical description. This description can be obtained by taking advantage of known laws of physics. The laws cannot be directly applied to real systems; therefore it becomes necessary to introduce many assumptions that simplify the engineering problem to such an extent that physics laws may be applied [6]. Modeling a mechanical system begins with creation of a physical model on which physics laws and mathematical operations are applied to develop a mathematical model [11]. The process of solving the mathematical model is known as analysis and yields the solution to a problem considered.

When riding a motorcycle the type of disturbance mostly affecting comfort of the rider, is random vibration which is caused by unpredicted loads such as road roughness and wind. Human exposure to whole-body vibration results into transmission of vibratory energy to the entire body and leads to localized effects. It affects comfort, normal functioning of the body and health [8], [9]. Recently, [3] carried out a WBV exposure comparison between motorcycles and cars. They concluded that WBV exposure levels of common motorcycle riders are distinctively higher than those of car drivers. Reference [12] also found that motorcycle riders have higher health risks than car drivers because they are sitting on a flat seat without back support causing greater energy absorption of vertical vibrations. A study done to investigate vibration magnitudes to which commercial motorcycle riders of the Dar es Salaam city were subjected, found that magnitudes in Z axis (vertical direction) were the highest as compared to magnitudes in X and Y axes [2]. These results lead to a desire of using mathematical descriptions to investigate the motorcycle construction with respect to vibration magnitudes of various parts including the rider, caused by the nature of terrain.

II. MATHEMATICAL MODEL FORMULATION

A motorcycle with rider model is considered, taking into account pitch motion of the motorcycles’ body. The degrees of freedom considered are; motorcycle vertical body displacement $x_b$, motorcycle body pitch angle $\theta$, front wheel displacement $x_{fw}$ and rear wheel displacement $x_{rw}$. The rider body is represented as $m_r$ the front wheel of the motorcycle is represented by the mass $m_{fw}$ and the spring coefficient $K_{fw}$. Similarly the rear wheel is represented by the mass $m_{rw}$ and the spring coefficient $K_{rw}$. The suspensions of the front and rear wheels are described by the damper’s coefficients $C_{fw}$ and $C_{rw}$ and the spring’s coefficients $K_{fw}$ and $K_{rw}$ respectively. The mass $m_r$ and the inertia $I$
represent the motorcycle body sprung mass. The location of the center of gravity is given by $L_1$ and $L_2$. Typical parameters for lumped seated human coupled with a motorcycle model listed in Table 1, were determined using different methods (experimental methods and reviewing various literature) which ensured that the parameters were actual for various motorcycle components and human. The schematic diagram of lumped seated human coupled with a Motorcycle vibration model is shown in Fig. 1.

The human-body has been considered as a lumped mass with stiffness ($K_b$) and damping ($C_b$) properties obtained from those proposed by [1].

![Figure 1: Schematic diagram of lumped seated human coupled with a Motorcycle vibration model.](image)

**Table 1.** Parameters for lumped seated human coupled with a Motorcycle vibration model.

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front / rear tire stiffness (N/m).</td>
<td>$K_{fr}$</td>
<td>130,000/141,000</td>
</tr>
<tr>
<td>Front / rear wheel and axle masses (Kg).</td>
<td>$m_{fr}$</td>
<td>11.9/14.7</td>
</tr>
<tr>
<td>Linear front and rear suspension damping coefficients (N·s/m).</td>
<td>$C_{fr}$</td>
<td>213/1165</td>
</tr>
<tr>
<td>Front and rear suspension stiffness (N/m).</td>
<td>$K_{fr}$</td>
<td>25000/58570</td>
</tr>
<tr>
<td>Distance between the C.G and front axle (m).</td>
<td>$L_1$</td>
<td>0.70</td>
</tr>
<tr>
<td>Distance between the C.G and rear axle (m).</td>
<td>$L_2$</td>
<td>0.595</td>
</tr>
<tr>
<td>Distance between the C.G and seat (m).</td>
<td>$a$</td>
<td>0.298</td>
</tr>
<tr>
<td>Motorcycle body mass (sprung mass) (Kg).</td>
<td>$m_b$</td>
<td>119</td>
</tr>
<tr>
<td>Rider body mass (Kg).</td>
<td>$m_h$</td>
<td>60.67</td>
</tr>
<tr>
<td>Rider body stiffness properties</td>
<td>$k_b$</td>
<td>390330</td>
</tr>
<tr>
<td>Rider body damping properties</td>
<td>$C_b$</td>
<td>422.41</td>
</tr>
<tr>
<td>Motorcycle body mass moment of inertia (Kg·m²).</td>
<td>$I$</td>
<td>22.013</td>
</tr>
<tr>
<td>Seat mass (Kg).</td>
<td>$m_s$</td>
<td>14.8</td>
</tr>
<tr>
<td>Seat damping coefficients (N·s/m).</td>
<td>$C_{se}$</td>
<td>150</td>
</tr>
<tr>
<td>Seat suspension stiffness (N/m).</td>
<td>$K_{se}$</td>
<td>15,000</td>
</tr>
</tbody>
</table>

**A. Derivation of Equations of Motion**

Schematic diagram of lumped seated human coupled with a Motorcycle vibration model was further simplified into free body diagrams of constituent bodies (Fig. 2).

![Figure 2: Free Body Diagrams of component bodies of the Model](image)

Assuming conditions of dynamic equilibrium to the component bodies constituting the model in Fig. 2, the equations 1 to 10 for various vertical forces acting on constituent bodies of the model were formulated.

\[ F_1 = k_h(x_{se} + x_h) \]  
\[ F_2 = c_h(\dot{x}_{se} - \dot{x}_h) \]  
\[ F_3 = k_2(x_b - x_{se} - a\theta) \]  
\[ F_4 = c_2(\dot{x}_b - \dot{x}_{se} - a\dot{\theta}) \]  
\[ F_5 = k_{sf}(x_{wr} - x_b + L_1\theta) \]  
\[ F_6 = c_{sf}(\dot{x}_{wr} - \dot{x}_b + L_1\dot{\theta}) \]  
\[ F_7 = k_{tr}(x_{wr} - x_b - L_2\Theta) \]  
\[ F_8 = c_{tr}(\dot{x}_{wr} - \dot{x}_b - L_2\dot{\Theta}) \]  
\[ F_9 = k_{tr}(x_{wr} - x_{wr}) \]  
\[ F_{10} = k_{fr}(x_{se} - x_{wr}) \]

Applying Newton’s equations of motion for each body, equations11-16 were formulated:

\[ m_b\ddot{x}_h - F_1 - F_2 = 0 \]  
\[ m_{se}\ddot{x}_{se} + F_1 + F_2 - F_3 - F_4 = 0 \]  
\[ m_b\ddot{x}_b + F_3 + F_4 - F_5 - F_6 - F_7 - F_8 = 0 \]
\[ i \ddot{\theta} - L_2 F_7 - L_2 F_9 - a F_3 - a F_4 + L_1 F_5 + L_1 F_6 = 0 \quad (14) \]
\[ m_{wf} \ddot{x}_{wr} - F_{10} + F_7 + F_8 = 0 \quad (15) \]
\[ m_{wr} \ddot{x}_{wr} - F_9 + F_5 + F_6 = 0 \quad (16) \]

For such a multi degrees of freedom system, the six equations of motions for the five bodies can be combined, rearranged and expressed in matrix form as equation 17.

\[ [M][\ddot{x}] + [C][\dot{x}] + [K][x] = \{F(t)\} \quad (17) \]

Where the force, displacement, velocity and acceleration vectors and also the mass, damping and stiffness matrices would be written as follows:

\[
\begin{bmatrix}
    m_h & 0 & 0 & 0 & 0 \\
    m_{se} & 0 & 0 & 0 & 0 \\
    m_{b} & 0 & 0 & 0 & 0 \\
    0 & 0 & m_{h_0} & 0 & 0 \\
    0 & 0 & 0 & m_{se} & 0 \\
    0 & 0 & 0 & 0 & m_{b} \\
    0 & 0 & 0 & 0 & m_{h_0} \\
    \end{bmatrix}
\]

\[
\begin{bmatrix}
    c_h & -c_h \\
    c_{se} & -c_{se} \\
    c_b & -c_b \\
    0 & c_h \\
    -c_{se} & -c_{se} \\
    c_{b} & -c_{b} \\
    0 & 0 \\
    \end{bmatrix}
\]

\[
\begin{bmatrix}
    k_h & -k_h & 0 & 0 & 0 \\
    -k_h & k_h & k_h & 0 & 0 \\
    -k_h & -k_h & a_{k_h} & 0 & 0 \\
    0 & -k_h & -k_h & a_{k_h} & 0 \\
    0 & 0 & -k_h & -k_h & a_{k_h} \\
    \end{bmatrix}
\]

Where, \( \omega \) is the radian frequency of the road and is equal to \( nV/L \).

Mathematical model of road profile is derived assuming motorcycle with wheelbase \( p \) passing over humps with speed \( V \), will have front ground displacement \( x_{of} \). The rear ground \( x_{of} \) follows the same track as the front with a given time delay \( \tau \) (wheelbase correlation) which is equal to the wheelbase divided by the motorcycle speed \( (\tau = p/V) \). This study assumed that the model travels with the constant speed of 20 km/h (5.5 m/s), \( H_0 \) 0.035m; is the hump height, and \( (L = 1 \text{ m}) \) is the length of the hump. The displacement and acceleration for the model in terms of time domain were obtained by solving derived equations of motion using MATLAB/ SIMULINK software version 7.9.0.529(R2009b). The initial conditions were assumed at equilibrium position. In this assumption, the rider was seated, and therefore, the initial velocity and displacement for each mass were equal to zero.

Substituting the given values in equations 18 and 19, the input excitations were written as:

\[
x_{of} = 0.035 \sin(17.27t) \quad \text{and} \quad x_{or} = 0.035 \sin(17.27t + 4.066),
\]

hence the force vector were written in a form of a single column matrix in Equation 20.

\[
\{F(t)\} = \begin{bmatrix}
0 \\
0 \\
4935 \sin(17.27t + 4.066)
\end{bmatrix}
\]

Input excitations caused by the road profile were introduced to the model through two points representing a modal front and rear wheels. The road profile caused the displacements \( x_{wf} \) and \( x_{wr} \) in vertical direction as motorcycle model was simulated travelling along a path with harmonic profile at constant speed. By doing model simulation, it was possible to observe vertical motion responses of all model components at a given instant.

The MATLAB software was used for solving, simulating and plotting the vibrational displacement and acceleration responses of the rider and other component bodies of a motorcycle for the assumed harmonic base excitations.

### III. Observations from the Model Simulation and Results

#### A. Model front wheel response

Response of a model front wheel to the road profile excitations resulted into varying vertical displacement \( x_{of} \) forming a graphical profile similar to sine wave, attaining an amplitude value of 0.041m after stabilization. This could be described as a vibrational state of a wheel caused by the nature of the terrain as the model motorcycle travels at a given speed. The amplitude displacement value (Fig. 3) was slightly above the hump height \( (H = 0.035) \). This could be described as a result of inertia effects and elastic...
properties on the frontal wheel on striking a hump profile at a speed resulting into motorcycle body pitch motion about a small angle $\theta$.

![Displacement graph](image1)

**Figure 3** Simulation results: Vibration vertical displacement of the model front wheel ($x_{wf}$) in time domain (amplitude after stabilization: 0.041m)

The front wheel of the model attained a very high acceleration ($\dot{x}_{wf}$) in the beginning and stabilized at an amplitude of 12.329m/s² after a first seconds as seen in Fig. 4.

![Acceleration graph](image2)

**Figure 4** Simulation results: Vibration vertical acceleration of the model front wheel ($\ddot{x}_{wf}$) in time domain (amplitude after stabilization: 12.329 m/s²)

B. Model rear wheel response

The response of the model rear wheel did not differ much from those of the frontal wheel in profile, but there were significant differences in both vibration displacement and acceleration magnitudes. The rear wheel attained displacement amplitude of 0.035m almost equal to the hump height $H$ (Fig. 5), and acceleration amplitude of 10.500m/s² (Fig. 6) after stabilization. The magnitude of vibration displacement being equal to the value of the profile hump height, to some extent describes the correctness of the model as during the pitch motion while the frontal wheel raising up, the rear wheel tended to travel the opposite side hence tracing the hump profile. The vibration displacement and acceleration magnitudes were slightly less than those attained by the frontal modal wheel. This difference could have been brought about by the difference in weight ($m_r$ and $m_f$) also stiffness properties ($k_f$ and $k_r$) between the model wheels. Motorcycles are manufactured with such a difference in characteristics between front and rear wheel because of the fact that, when it is travelling at speed, there exists a tendency of decrease in the load on the front wheel and a corresponding increase in load on the rear wheel [5]. The dimensions, stiffness and other properties for the rear wheel are greater than those of the front wheel to make it withstand these changes in load.

C. Model body responses

The motorcycle body was represented by the sprung mass $m_b$ and mass moment of inertia $I$, hence its responses were described by the vertical displacement $x_b$ and pitch angle $\theta$. In simulation results for motorcycle body displacement and acceleration, the amplitudes attained after stabilization were 0.024m and 7.148m/s² respectively. As the vibration reaching the body originates from the front and rear wheels, comparatively there is a high reduction in vibration magnitudes. This difference may have been caused by two physical reasons namely, big mass value $m_b$ as compared to $m_{wf}$ and $m_{wr}$ also the existence of the suspension system separating the tyres from the body (sprung mass). According to Newton’s second law of motion for a given force, the acceleration of a body is inversely proportional to its mass. Some of the vibration energy from the tyres might have been absorbed by the suspension springs and dampers systems and hence resulting in reduced vibration magnitude values.

Pitch motion results produced the highest vibration magnitudes as compared to vibration magnitudes of all other constituent bodies of a model. The highest amplitude attained after stabilization were 0.047m for vibration displacement and 13.943 m/s² for vibration acceleration.

D. Model seat and rider responses

Although they had different magnitudes, the responses of the model seat and rider had similar behaviour. Both the graphs for $x_{se}$ and $x_{re}$ began with 0 followed by a negative increment accompanied by high fluctuations attaining stable amplitude after the end of the first second. Both displacement and acceleration responses behaved in a similar manner. The vibrational acceleration and displacement of model rider and seat in terms of time domain were as indicated in Figures 5- 8.

![Seat and rider response graph](image3)

**Figure 5** Simulation results: Vibration vertical displacement of the model seat ($x_{se}$) in time domain (amplitude after stabilization: 0.042m)
in parameters of the front suspension system were avoided because making those changes on a motorcycle could largely influence the control behaviour of a whole motorcycle [4].

With the aim of observing how was the vibration behaviour of the model as the position of a seat and hence a rider was changed with respect to the center of gravity, the following two steps were done:

i. In the first approach, the seat was assumed to be exactly located on or above the center of gravity hence the parameter \( \alpha \) was made equal to zero \((\alpha = 0)\), the simulation was done and the results observed.

ii. The second approach, the value of the parameter \( \alpha \) was made equal to \( L_1 \), this meant that the seat position moved to the rear side such that \( \alpha = L_1 \), the simulation was done and the results observed.

Having performed the simulations and recorded the results, the parameters were retuned as at the beginning and a similar procedure was repeated with rear suspension system parameter \( K_{sr} \). In this case no dimensional alterations were done, but changes were done by reducing the magnitude of a responsible parameter.

A. Simulation results after parameter \( \alpha \) changes

When the value of \( \alpha \) was changed from 0.298m to 0 and then to 0.595m making the model seat position to change with respect to the center of gravity, the displacement and acceleration attained amplitudes was recorded in Tables 2 and 3.

In the first two seconds, the results were fluctuating and unstable, therefore the recorded values were those attained after stabilization. From the simulation results recorded in Table 2 and 3, the following phenomenon was visible. As the position of the seat and hence the rider moved further from the center of gravity location towards the rear wheel, there was a corresponding reduction in the vibration magnitudes of the model component bodies.

<table>
<thead>
<tr>
<th>Variable before parameter change</th>
<th>Displacement change ((\alpha = 0.298))</th>
<th>Displacement change ((\alpha = 0))</th>
<th>Displacement change ((\alpha = 0.595 = L_1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_h )</td>
<td>0.044</td>
<td>0.045</td>
<td>0.042</td>
</tr>
<tr>
<td>( x_se )</td>
<td>0.042</td>
<td>0.045</td>
<td>0.041</td>
</tr>
<tr>
<td>( x_p )</td>
<td>0.024</td>
<td>0.025</td>
<td>0.023</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.047</td>
<td>0.056</td>
<td>0.037</td>
</tr>
<tr>
<td>( x_{wf} )</td>
<td>0.041</td>
<td>0.044</td>
<td>0.039</td>
</tr>
<tr>
<td>( x_{wr} )</td>
<td>0.035</td>
<td>0.039</td>
<td>0.032</td>
</tr>
</tbody>
</table>

The model simulation results were investigated with the aim of having an over view on how vibration magnitudes behaved with changing parameters of the model components. The investigation began by making alterations in the following parameters of the model: the position of the seat relative to the position of the center of gravity (parameter \( \alpha \)) and rear suspension system stiffness (parameter \( k_{sr} \)). The rear suspension damping coefficient (parameter \( c_{sr} \)) was left unaltered. The decision whether to increase or decrease the magnitude of the damping coefficient \( c_{sr} \), needs a clear knowledge of the values of the critical damping constant, the natural frequency and the forcing frequency. In some cases damping can lead to loss of vibration isolation efficiency [7]. Alterations
is influential on resultant vibration magnitudes as compared to other suspension parameters.

**Table 5 Model simulation results: Vibration acceleration (m/s²) of the six bodies composing a model before and after reducing $k_{sr}$**

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Acceleration before parameter change</th>
<th>Acceleration after $k_{sr}$ parameter change</th>
<th>Acceleration after $c_{sr}$ and $k_{sr}$ parameters change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{x}_h$</td>
<td>13.223</td>
<td>12.821</td>
<td>12.263</td>
</tr>
<tr>
<td>$\ddot{x}_{se}$</td>
<td>12.632</td>
<td>12.248</td>
<td>11.741</td>
</tr>
<tr>
<td>$\ddot{x}_b$</td>
<td>07.148</td>
<td>06.982</td>
<td>06.784</td>
</tr>
<tr>
<td>$\ddot{\theta}$</td>
<td>13.943</td>
<td>13.805</td>
<td>13.670</td>
</tr>
<tr>
<td>$\ddot{x}_{wf}$</td>
<td>12.329</td>
<td>12.241</td>
<td>12.146</td>
</tr>
<tr>
<td>$\ddot{x}_{wr}$</td>
<td>10.500</td>
<td>10.495</td>
<td>10.495</td>
</tr>
</tbody>
</table>

The simulation results in tables 4 and 5 show that out of the five bodies composing a model, the motorcycle body is the part with lowest vibration magnitudes ($\ddot{x}_b$ and $\ddot{x}_{wf}$), but at the same time the magnitudes due to pitch motion ($\ddot{\theta}$ and $\ddot{\theta}$) were the highest. Vibration displacement and acceleration for the seat ($\ddot{x}_{se}$ and $\ddot{x}_{se}$) and those for a rider ($\ddot{x}_h$ and $\ddot{x}_h$) appeared to be greater even more than those at the wheels (input points). This may have been caused by several factors such as the large value of vibration due to pitch motion, their relative position from the front and rear wheels ($a$, $L_2$ and $L_1$) and the applied values of the damping constant. High magnitudes of vibration caused by pitch motion were linked to the assumption that the profile of the path was sinusoidal.

## V. CONCLUSIONS

From the simulation results it may be concluded that

- the location of the rider seat from the centre of gravity of a motorcycle has an influence to the level of vibration magnitude of the rider such that, the farther the rider seat is located from the centre of gravity towards the rear wheel the little the vibration magnitudes of the rider.
- The stiffness of the suspension influences the vibration magnitude of the rider in such a way that the more stiff the suspension the more vibration is transmitted to the rider; hence less stiff suspension is friendlylier with regard to vibration exposure.

## VI. RECOMMENDATION

- In order to achieve the goal of reducing vibration magnitudes of the motorcycle rider caused by the road condition, the following should be put into consideration: Determination of optimum position of the seat relative to the motorcycle body centre of gravity, and rear wheel and applying suspension system which may have variable characteristics (active suspension).
REFERENCES


