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On The Speed of Feynman Quantum Computer

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***Abstract*— Quantum computing research has gained a lot of momentum recently due to several theoretical analyses that indicate that quantum computer would be more efficient at solving certain classes of problems than classical computer. Quantum computer works according to quantum mechanical laws and with it some problem exists, namely, these laws would restrict the time when a computer ends the calculation. We have discussed the Feynman serial model of quantum calculation in which we have found the time in which complete calculations could be defined for a half adder.**

Keywords—Quantum Cumputers; Feynman Quantum Computer

I. INTRODUCTION

This Richard Feynman discussed in the early 1980's [1] that certain quantum mechanical effects cannot be simulated efficiently on a classical computer. This led to idea that perhaps quantum computation could be done more efficiently by quantum computers. In 1994 Peter Shor surprised the world by describing a polynomial time quantum algorithm for factoring integers [2]. His discovery enhanced the activity both among experimentalists trying to build quantum computers and theoreticians trying to find better quantum algorithms.

In 1985 Richard Feynman [3, 4] presented also a model of quantum computation which was quantum mechanically plausible. The Feynman computer is an ideal quantum computer based on the quantum mechanical law which does calculations. Some problem was related to uncertainty of time in which calculation will be completed. Feynman has written that we have quantum computer for making calculation, but time of arrival of the cursor and measurement of the output register (in other words, the time it takes in which to complete the calculation) has undefined time. It is a question of probabilities, and so there is a considerable uncertainty according to his view at what time a calculation will be done.

In this paper we have discussed the Feynman model for an adder (a half adder) where the calculation could be completed at the definite time. This allowed us to estimate a calculation speed in a quantum computer base on the Feynman quantum mechanical model of computer.

II. CALCULATION DETAILS

First, Feynman considered how a quantum computer can be built using the laws of quantum mechanics [3]. It is important to have a Hamiltonian which will describe all the internal computing actions. The quantum computer would be small, for example, it could be composed from atoms or ions. A quantum computation works as follows: we propose quantum register with qubits in a known state, we apply quantum gates on the register, i.e. we apply unitary operations on some qubits and we measure the final state of the register to read its content.

All elements in quantum computers could be built as a combination of primitive elements, namely NOT, CNOT, and CCNOT elements [4]. The Controlled Not element (CNOT) has two entering lines (in a diagram model), *a* and *b* and two exiting lines *a'* and *b'*. The *a'* is always the same as *a*, which is the control line. If the control is activated ($a = 1$) then the output *b'* is the NOT of *b*. Otherwise *b* is unchanged, $b = b'$. The Controlled Controlled Not element (CCNOT) involves three lines. We have two control lines *a, b* which appear unchanged in the output and which change the third lines to NOT *c* only if both lines are activated ($a = 1$ and $b = 1$), otherwise $c' = c$.

We can make an adder by using CNOT and CCNOT elements to produce the sum on the second atom and carry on the third as it is shown in Figure 1. It means the adder represents the simplest computer which makes a calculation.

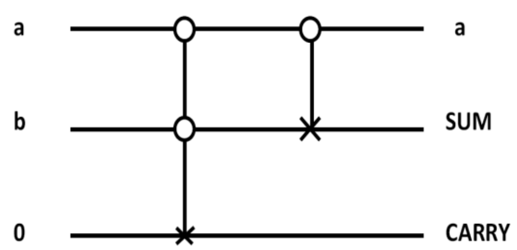

Fig.1. *The diagram representation of an half adder.*

The adder is composed from two logical blocks A_0 (CCNOT) and A_1 (CNOT). There exists a matrix *M* which consists of a sequence of matrices A_0, A_1, where $M = A_1 A_0$. The matrix *M* can easily be seen as the result of two successive primitive operations A_0 and A_1 where [3]

$$A_1 = A_{a,b},\ A_0 = A_{ab,c} \quad (1)$$

and

$$A_{a,b} = 1 + a^+ a(b + b^+ - 1), \quad (2)$$

$$A_{ab,c} = 1 + a^+ ab^+ b(c + c^+ - 1). \quad (3)$$

We use an operator representation of the gates by of creation a and annihilation a^+ operators. The atoms indexed *a, b, c* create a *register.* We add next new set three atoms indexed by 0, 1, 2 which we will call *program counter* (PC). Let us call q_i and q_i^+ the annihilation and creation operators for the PC. If the atom or ion of PC is occupied by as cursor (it could be for example an electron), we write the Hamiltonian of the adder as

$$H = q_1^+ q_0 A_0 + q_2^+ q_1 A_1 + \left(q_1^+ q_0 A_0 + q_2^+ q_1 A_1\right)^+. \quad (4)$$

This is the Feynman version of quantum computer [3]. We put at time *t = 0* the cursor on the atom 0. The term $q_1^+ q_0$ is a term which simple moves the occupied site from the location 0 to the location 1. The matrix then A_0 multiplies the initial state of the three register atoms. If the Hamiltonian begins to operate the second time, this first term will produce nothing because q_0 produces nothing on the number 0 site (the site 0 is now unoccupied). The term which can operate now is the term $q_2^+ q_1 A_1$. The cursor can move from site 1 to site 2 but the matrix A_1 now operates on the register. If we check the cursor at site 2, we remove the cursor and we now know that the register contains the output data, the calculation is completed.

In which time the cursor will be at the end of the program line at site 2? According the quantum mechanical laws there exists an *probability amplitude* to be at site 2 for the time *t > 0*. The probability amplitude $< 2 | \psi(t) >$ that cursor will be at the time *t > 0* on the site 2, if it is at the site 0 at t = 0, can be expressed as follows (it is true when the Hamiltonian of the quantum computer does not depend on the time)

$$< 2 | \psi(t) > = \sum_\alpha b_\alpha < 2 | \psi_\alpha(t) > \quad (5)$$

We can use a standard procedure for the solution of the problem to find the probability amplitude. If we take the Hamiltonian matrix as $H_{00} = H_{11} = H_{22} = 0$, $H_{01} = H_{10} = A = -|A|$, $H_{02} = H_{20} = 0$, $H_{12} = H_{21} = A$ (a tight binding approximation) we get the following expression

$$P_2(t) = \left|< 2 | \psi(t) >\right|^2 = \frac{1}{2} - \frac{1}{2}\cos\left(\frac{t}{\hbar}\sqrt{2}\,|A|\right) - \frac{1}{4}\sin^2\left(\frac{t}{\hbar}\sqrt{2}\,|A|\right) \quad (6)$$

The probability that the cursor is found in state |2> varies harmonically with time. There exists a minimal time in which the cursor is found in the state |2>, and this time can be determined from the equation $P_2(T) = 1$. It follows

$$T = \frac{\pi\hbar}{\sqrt{2}\,|A|}. \quad (7)$$

For the half adder it need not check at the site, say, by scattering electrons, that the site is empty, or that the site 2 has a cursor. We can select the measurement time in which the calculation is completed.

III. Results and Discussion

At the minimum time, in principle, we can take the values of bits from the output registry. For typical value of *A* the order 0.1 electron volts we can evaluate the time *T* of the complete calculation of the half adder. We get the value 1.46 x 10^{-15} second. This represents an improvement over the present values of the time delays in classical transistors by a factor $x\,10^4$.

However, currently, we have to change an idea about the time for which quantum computer will do some calculation. This relates to the physical implementation of the computer. In recent years, have been successfully implemented elements that compose the quantum computers. For example, *CNOT* element as it is written in Blatt et al. [5], was implemented by ions in the linear trap, where ions are controlled by laser pulses. The *CNOT* gate operation [6] was realized with the following sequence of laser pulses:

$$R\left(\frac{\pi}{2},0\right), R^+\left(\frac{\pi}{\sqrt{2}},0\right), R^+\left(\pi,\frac{\pi}{2}\right), R^+\left(\frac{\pi}{\sqrt{2}},0\right), R^+\left(\frac{\pi}{\sqrt{2}},\pi\right), R\left(\frac{\pi}{2},\pi\right). \quad (8)$$

All qubit transitions are described as rotations on corresponding Bloch sphere and they are written as unitary operation *R(θ, Φ), R⁻ (θ, Φ), R⁺ (θ, Φ)* on the carrier, red sideband and blue sideband, respectively. Typical pulse duration for π-pulse ranges from 1 to several 10 μs for the carrier transition and 50-200 μs on the sideband transition. By concatenating pulses on the carrier and sidebands whole quantum algorithm can be implemented [7].

This means that the computation time can be of the order of a millisecond, when the quantum computers uses technology ions trapped in the linear trap. Quantum computer will not do the calculation with high speed. However, if we can insert into the trap of many qubits, we expect, that although the calculation will take a relatively long period of time, for certain types of tasks will calculate the effective compared with classical algorithms [8].

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