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# Constrained Flow-Shop Scheduling Problem with m Machines

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***Abstract*— *This paper provides a heuristic procedure for solving the constrained flow-shop scheduling problems of n-jobs on m-machines. We obtained a schedule with a set of n-jobs to minimize the mean weighted flow-time. This method is very easy to understand and to apply. It will also help managers in the scheduling related issues by aiding them in the decision making process and providing an optimal or near optimal schedule in a simple and effective manner. A numerical example is given to illustrate the procedure.***

***Keywords*— *Flow shop, Transportation time, Weight of jobs, Break-down interval***

## I. Introduction

Prior to the year (1980) almost entire work in deterministic scheduling theory conforms to the studies of scheduling problems in which all jobs are assumed to be of equal importance for processing them on the machines in a flow-shop. But this assumption regarding all jobs are of eqi-importance in the flow-shop seems to be relatively restrictive from the practical view point. Thus, there are practical situations in sequencing in which all jobs are not equally important, due to different inventory costs associated with the jobs. For example, in the processing of a sequence S of jobs, a job $\alpha$ in S may have higher inventory cost over a job $\beta$ in S, and this way the job $\alpha$ becomes more important than the job $\beta$ for processing on the machines in the sequence S. Hence we can associate 'weight' in the sense of relative importance for performance with a job in a flow-shop problem. Miyazaki et. al. (1980) studied scheduling flow-shop problems. In each problem the idea of 'weight' of a job is introduced. The computational algorithms for obtaining optimal or near optimal solutions are described in their study, which conforms to minimization of the weighted mean flow-time for the problems.

Johnson (1954) and Bellman (1956) studied the problem of scheduling of n jobs on two machines arranged in tandem where time required to transport jobs from first machine to the second was assumed to be negligible. Practically the machines in the flow-shop, processing the jobs may be planted at distant places, so that a job after completion on the first machine may involve certain minimum time for processing it at the subsequent machine. The minimum time which elapses between completion of a job at the first machine and starting it on the second machine in the 2-machine flow-shop problem is called 'transportation time'. A job in a schedule S may have a transportation time as a sum of loading time; moving time and unloading time for the job in the processing procedure of the job from one machine to another machine (see Maggu and Das (1980)). P. L. Maggu et al., (1984) introduced the idea of 'weight and transportation time' of a job and obtain optimal or near optimal solution.

As the problem size increases, NP- hardness of flow-shop problems necessitates the development of heuristics to get near optimal solutions. Garey et al., (1976) discussed the complexity of flow-shop. Campbell et al., (1970) proposed a heuristic algorithm to minimize the makespan. In their work, they split the given *m* machine problem into a series of equivalent two machine flow-shop problem and solving it using Johnson's method. Another heuristic was proposed by Nawaz et al., (1983) for minimizing makespan time. Apart from the makespan objective, other significant objectives like total flow time, tardiness and idle time of machines have been proposed by different authors. Ho and Chang (1976) have attempted not only minimizing makespan but also minimizing total flow time and machine idle time. Chou and Lee (1999) attempted to solve two machine flow-shop bi-criteria scheduling problem with release dates for the jobs, in which the objective function is to minimize the weighted sum of total flow time and makespan. Rajendran and Ziegler (1997) have considered the objective of minimizing the total weighted flow times of jobs. Maggu and Das (1980) used the concept of transportation time in going from one stage to the other. They studied a system in which an infinite number of transport agents were available and no transport agent was required to return to stage one from stage two. It was assumed that Machine A starts processing the next item immediately after finishing the preceding one. Khodadadi (2008) developed a new heuristic for three machine flow-shop scheduling problem with transportation time of job. Chandramouli (2005) propose a heuristic approach for n-job, 3-machine flow-shop scheduling problem involving transportation time, break-down time and weights of jobs. Laha D. and Chakraborty U. K. (2009) propose a constructive heuristic for minimizing makespan in no-wait flow-shop scheduling. Khodadadi A. (2011) solved constrained flow shop scheduling problem with three machines involving transportation time only.

In this paper, we have developed a method for flow-shop scheduling problems involving transportation time, weight of jobs and break down time (constrained flow-shop scheduling problems) with m-machines to obtain an optimal or near optimal solution.

The weighted mean flow-time is defined by

$\bar{F}_w = \dfrac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i}$, where $f_i$ is the flow time of $i^{th}$ job.

The proposed method provides an important tool for decision makers when they design a schedule for constrained flow-shop scheduling problems with m-machines.

## II. Machine Flow-shop Problem

The flow-shop sequencing problem is a production planning problem: *n* jobs (items, tasks,…) have to be processed in the same sequence on m machines; the processing time of job *i* on machine *j* is given by $p_{ij}$. These times are fixed, non-negative and some of them may be zero if some job is not processed on a machine (Taillard, 1990). The transportation time of job $i$ on machine $j$ to machine $j+1$ is given by $t_i(j, j+1)$ $(i = 1, 2, ..., n;\ j = 1, 2, ..., m)$. The problem consists of minimizing the time between the beginning of the execution of the first job on the first machine and the completion of the execution of the last job on the last machine, this time is called makespan. In other words, the objective is to find the sequence of jobs minimizing the maximum flow time (makespan) (Baker, 1974). The general flow-shop scheduling problem is NP-hard. Let n jobs be processed through m machines $M_1, M_2, ..., M_m$ in the order $M_1 M_2 ... M_m$, $t_i(j, j+1)$ be the transportation time of job i from machine j to j+1, job i be assigned with a weight $w_i$ according to its relative importance for performance in the given sequence. The performance measure studied is weighted mean flow time. Let the break down interval (*a*, *b*) is already known (deterministic) i.e. the break down interval length (b-a) is known. Our aim is to find out the optimal or near optimal sequence of jobs so as to minimize the total elapsed time and weighted flow time. Formulation of the problem in tabular form is given below:

Table-1

| Job $(i)$ | M1 $(M_{i1})$ | $t_i(1,2)$ | M2 $(M_{i2})$ | $t_i(2,3)$ | … | Mm $(M_{im})$ | $w_i$ |
|---|---|---|---|---|---|---|---|
| 1 | $p_{11}$ | $t_1$ | $p_{12}$ | $t_1$ | … | $p_{1m}$ | $w_1$ |
| 2 | $p_{21}$ | $t_2$ | $p_{22}$ | $t_2$ | … | $p_{2m}$ | $w_2$ |
| . | . | . | . | . | … | . | . |
| . | . | . | . | . | … | . | . |
| . | . | . | . | . | … | . | . |
| n | $p_{n1}$ | $t_n$ | $p_{n2}$ | $t_n$ | | $p_{nm}$ | $w_n$ |

Let us assume that the above problem satisfies any one or both of the following structural conditions involving the processing time and transportation time of jobs hold.

$Minimum\{M_{i1}\} \geq Maximum\{t(j-1, j) + M_{ij}\}$

$for\ all\ \ j = 2,3,...,m-1$

or / and

$Minimum\{M_{im}\} \geq Maximum\{t(j-1, j) + M_{ij}\}$

$for\ all\ \ j = 2,3,...,m-1$

Then we have developed a new method as follows:

## III. Algorithm

**Step 1**: Reduce the given problem into two machines flow-shop scheduling problem by introducing two fictitious machines, *G* and H having the processing times as

$G_i = M_{i1} + t(j-1, j) + M_{ij}\ \ for\ all\ \ j = 2,3,...,m-1$
$H_i = M_{ij} + t(j, j+1) + M_{im}\ \ for\ all\ \ j = 2,3,...,m-1$

**Step 2.** Compute *Minimum* $\{G_i, H_i\}$

(i) If *Minimum* $\{G_i, H_i\} = G_i$, then define $G_i' = G_i + w_i$ and $H_i' = H_i$

(ii) If *Minimum* $\{G_i, H_i\} = H_i$, then define $G_i' = G_i$ and $H_i' = H_i + w_i$

**Step 3.** Formulate a new reduced two machines scheduling problem as follows:

Table-2

| Job *(i)* | $G_i''$ | $H_i''$ |
|---|---|---|
| 1 | $\frac{G_1'}{w_1}$ | $\frac{H_1'}{w_1}$ |
| 2 | $\frac{G_2'}{w_2}$ | $\frac{H_2'}{w_2}$ |
| . | . | . |
| . | . | . |
| . | . | . |
| n | $\frac{G_n'}{w_n}$ | $\frac{H_n'}{w_n}$ |

**Step 4.** Determine the optimal sequence to the new reduced two machines scheduling problem obtained in the Step 3 by using Johnson's algorithm.

**Step 5.** Identify the effect of break-down interval (*a*, *b*) on different jobs.

**Step 6.** Modify the given problem using the new machine processing times $M_{i1}', M_{i2}', ..., M_{im}'$ which are obtained as

(i) If the break-down interval (*a*, *b*) has no effect on job *i*, at the time of processing the machines $M_1, M_2, ..., M_m$, then $M_{ij}' = M_{ij}$ for all *j*.

(ii) If the break-down interval (*a*, *b*) has affected on job *i*, at the time of processing the machines $M_1, M_2, ..., M_m$ then $M_{ij}' = M_{ij} + (b-a)$ for all *j*.

**Step 7.** Using the modified scheduling problem and the optimal sequence obtained in step-4, determine the total elapsed time and weighted mean-flow time.

## IV. NUMERICAL EXAMPLE

Consider the following constrained flow-shop scheduling problem of five jobs on three machines with processing times, transportation times and the weights of jobs: given that the break-down interval (a, b) = (19, 27).

| Job $(i)$ | $M_1$ $(M_{i1})$ | $t_i(1,2)$ | $M_2$ $(M_{i2})$ | $t_i(2,3)$ | $M_3$ $(M_{i3})$ | $w_i$ |
|---|---|---|---|---|---|---|
| 1 | 9 | 1 | 7 | 2 | 5 | 3 |
| 2 | 11 | 3 | 6 | 5 | 9 | 1 |
| 3 | 13 | 2 | 3 | 4 | 5 | 4 |
| 4 | 10 | 5 | 2 | 1 | 6 | 5 |
| 5 | 9 | 4 | 5 | 3 | 7 | 2 |

Solution:

Since, $Minimum\ \{M_{i1}\} \geq Maximum\ \{t(1,2)+M_{i2}\}$ the first structural condition is satisfied. The above problem can be converted into two machines problem *G* and H having the processing times as:

| Job $(i)$ | $G_i$ | $H_i$ | $w_i$ |
|---|---|---|---|
| 1 | 17 | 14 | 3 |
| 2 | 20 | 20 | 1 |
| 3 | 18 | 12 | 4 |
| 4 | 17 | 9 | 5 |
| 5 | 18 | 15 | 2 |

From the above table *Minimum* $\{G_i, H_i\} = H_i$ , then $G_i' = G_i$ and $H_i' = H_i + w_i$

So the new reduced two machines scheduling problem is as follows:

| Job $(i)$ | $G_i''$ | $H_i''$ |
|---|---|---|
| 1 | 17/3 | 17/3 |
| 2 | 20/1 | 21/1 |
| 3 | 18/4 | 16/4 |
| 4 | 17/5 | 14/5 |
| 5 | 18/2 | 17/2 |

Now, using the step-4 of the proposed method, we obtain that (1, 2, 5, 3, 4) is an optimal sequence for the given problem.

Now, the total elapsed time for the optimal sequence (1, 2, 5, 3, 4) is calculated as follows:

| Job $(i)$ | $M_1$ $(M_{i1})$ | $t_i(1,2)$ | $M_2$ $(M_{i2})$ | $t_i(2,3)$ | $M_3$ $(M_{i3})$ | $w_i$ |
|---|---|---|---|---|---|---|
| 1 | 0-9 | 1 | 10-17 | 2 | <u>19-24</u> | 3 |
| 2 | <u>9-20</u> | 3 | <u>23-29</u> | 5 | 34-43 | 1 |
| 5 | <u>20-29</u> | 4 | 33-38 | 3 | 41-48 | 2 |
| 3 | 29-42 | 2 | 44-47 | 4 | 51-56 | 4 |
| 4 | 42-52 | 5 | 57-59 | 1 | 60-66 | 5 |

Therefore, the total elapsed time is 66 hrs.

Now Job1, Job2 and Job5 have been affected by the break down interval (19, 27) on the optimal sequence (1, 2, 5, 3, 4). Using step-6 of the proposed method, we modify the processing times for affected jobs and we obtain the following new scheduling problem in the tabular form as follows:

| Job $(i)$ | $M_1$ $(M_{i1}')$ | $t_i(1,2)$ | $M_2$ $(M_{i2}')$ | $t_i(2,3)$ | $M_3$ $(M_{i3}')$ | $w_i$ |
|---|---|---|---|---|---|---|
| 1 | 9 | 1 | 7 | 2 | 13 | 3 |
| 2 | 19 | 3 | 14 | 5 | 9 | 1 |
| 3 | 13 | 2 | 3 | 4 | 5 | 4 |
| 4 | 10 | 5 | 2 | 1 | 6 | 5 |
| 5 | 17 | 4 | 5 | 3 | 7 | 2 |

Now, using this modified scheduling problem and the optimal sequence obtained in step-4, we determine the total elapsed time and weighted mean-flow time as below:

| Job $(i)$ | $M_1$ $(M_{i1})$ | $t_i(1,2)$ | $M_2$ $(M_{i2})$ | $t_i(2,3)$ | $M_3$ $(M_{i3})$ | $w_i$ |
|---|---|---|---|---|---|---|
| 1 | 0-9 | 1 | 10-17 | 2 | 19-32 | 3 |
| 2 | 9-28 | 3 | 31-45 | 5 | 49-58 | 1 |
| 5 | 28-45 | 4 | 49-54 | 3 | 58-65 | 2 |
| 3 | 45-58 | 2 | 60-63 | 4 | 67-72 | 4 |
| 4 | 58-68 | 5 | 73-75 | 1 | 76-82 | 5 |

The mean weighted flow time

$$\bar{F}_w = \frac{\sum_{i=1}^{n} w_i f_i}{\sum_{i=1}^{n} w_i}$$

$$= \frac{32\times 3+(58-9)\times 1+(65-28)\times 2+(72-45)\times 4+(82-58)\times 5}{3+1+2+4+5}$$

$$= \frac{447}{15} = 29.8$$

Hence, the total elapsed time is 82 units and the mean weighted flow time is 29.8 units.

## V. Conclusion

We have developed a heuristic procedure for solving the constrained flow-shop scheduling problems of n-jobs on m-machines. We obtained a schedule with a set of n-jobs to minimize the mean weighted flow-time. This method is very easy to understand and to apply. It will also help managers in the scheduling related issues by aiding them in the decision making process and providing an optimal or near optimal schedule in a simple and effective manner. Determining a best schedule for given sets of jobs can help decision makers effectively to control job flows and to provide a solution for job sequencing.

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