Modeling of Anisotropic Rectangular Waveguide Partially Embedded in an Anisotropic Substrate

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***Abstract*—With due account for diffraction, the problem of modeling of an anisotropic rectangular waveguide partially embedded in an anisotropic substrate is being considered in this paper. There have been conducted numerical calculations of the characteristic equations obtained for anisotropic waveguide with a uniform environment and for anisotropic strip waveguide, which were investigated experimentally in the microwave range. A comparison of theoretical and experimental data shows that the single-wave approximation sufficiently accurately describes the basic types of rectangular waveguides.**

Keywords—rectangular waveguide; microwave range; anisotropic waveguide; electromagnetic field; telecommunication devices.

I. INTRODUCTION

Study of the propagation of electromagnetic waves along rectangular anisotropic waveguides has a great scientific and practical interest. This is primarily due to the fact that such waveguides may serve not only for limiting and direction of the electromagnetic signal but also are the basis of the various telecommunication devices. It should be noted that a rigorous theory of rectangular waveguides is far from completion. Existing methods of calculation can be divided into numerical and analytical [1-3]. Computational methods are generally applicable to waveguides with small differential dielectric permittivity [3] or surrounded by a homogeneous medium. Of greatest interest are approximate methods as they are applicable to a wider range of tasks. It should be noted that the approximate methods of analysis [5-9] do not give a complete solution of the boundary problem but they are distinguished by the simplicity of the final results. So, constant spread of modes is determined from characteristic equations for two planar waveguides. At the same time, various methods differ only in the choice of the refractive indices of these waveguides. To get a complete picture of the field distribution in a rectangular waveguide it is necessary to consider the diffraction of waves on the faces of the waveguide.

II. PROBLEM FORMULATION

In this paper, approximately, but with due account of diffraction the problem of modeling of an anisotropic rectangular waveguide partially embedded in an anisotropic substrate is being considered. The problem is solved by the method of partial areas in the single-wave approximation the essence of which is as follows. The field in the central part of the waveguide appears to be the mode of a flat waveguide experiencing reflections from the side faces of the rectangular rod. When interacting with the lateral faces it is assumed that mode is reflected primarily in itself, that is, an inevitable process of transformation in the process of reflection of the incident wave into other types of vibrations is not being taken into consideration. In contrast to [4-9] field, systems of two media separated by a plane boundary are located in the lateral partial areas, either in the form of integrals over plane waves, or in the form of integrals over its own types of waves.

Approximate matching of fields at the boundaries of partial regions leads to the determination of the amplitude expansion and dispersion equation for the propagation constant, which determines the distribution of the field in the entire space.

III. DEVELOPMENT OF MATHEMATICAL MODELS

The main interest is the solution of this problem for an arbitrary orientation of the principal axes of the dielectric tensor with respect to the chosen coordinate system. However, the solution of this problem is very difficult. We therefore confine ourselves to the special case when the anisotropic waveguide has a dielectric permittivity tensor of a diagonal form, in which the two elements are equal to:

$$\varepsilon = \begin{pmatrix} \varepsilon_0 & 0 & 0 \\ 0 & \varepsilon_e & 0 \\ 0 & 0 & \varepsilon_0 \end{pmatrix} \quad (1)$$

We assume that the tensor of dielectric permittivity substrate has a similar form:

$$\varepsilon' = \begin{pmatrix} \varepsilon' & 0 & 0 \\ 0 & \varepsilon' & 0 \\ 0 & 0 & \varepsilon' \end{pmatrix} \qquad (2)$$

We choose a Cartesian coordinate system in such a way that the axis of the waveguide coincides with the coordinate axis Oz, and the region in the transverse plane occupied by the rod is defined by the inequality $|x| \le b, |y| \le a$ (Fig.1). We assume that the waveguide is embedded into the substrate by an amount $(a + y_0)$.

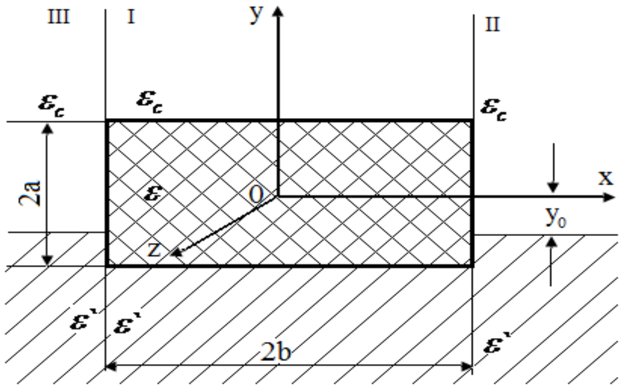

Fig.1. *A rectangular waveguide partially embedded in the substrate*

We also assume that the medium above the waveguide and the substrate is an isotropic dielectric with dielectric permittivity of ε_c. It should be noted that the problem in this formulation cannot be solved by known methods [5-9]. This problem is solved by the method of partial areas in the single-wave approximation.

Let's divide the plane of the cross section into three partial areas. Region *I* is bounded by the planes $x = \pm b$, regions *II* and *III*, are set by the inequalities $x > b$ and $x < -b$ respectively. As before, the solution of Maxwell's equations is sought in the form of harmonic waves [10-12] traveling along the axis of the waveguide.

$$\begin{cases} E = E(x,y)\exp(ihz - i\omega f), \\ H = H(x,y)\exp(ihz - i\omega f). \end{cases} \qquad (3)$$

In this case, the transverse component fields at all points of the system, except for the interfaces of media, are expressed by differentiating through longitudinal E_z and H_z. Let's find the distribution of the field in the areas *I—III*. Because of the symmetry of the system with respect to the plane $x = 0$ there is a possibility of the existence of two independent groups of waves with different parity by x.

As before, we neglect the process of transformation of waves being reflected on the faces $x = \pm b$, that is, the field in a partial region *I* we will approximately assume as one mode of a flat anisotropic waveguide on an anisotropic substrate. Boundary conditions at $y = \mp a$ are satisfied by two types of independent waves with different polarization.

For the first type, which is determined by the condition $E_y = 0$, the expressions for the longitudinal components in the area *I* are in the form

$$\begin{cases} E_z = (x,y) = A_0\phi_0(x,y), \\ H_z(x,y) = A_0\phi_0(x,y). \end{cases} \qquad (4)$$

and the wave number u_0, v_0, q_2 and q_3 are associated with h_0 by the expressions

$$k_0^2\varepsilon_0 = u_0^2 + \vartheta_0^2 + h_0^2, \qquad (5)$$

$$k_0^2\varepsilon_0' = u_0^2 + \vartheta_0^2 - q_3^2, \qquad (6)$$

$$k_0^2\varepsilon_e = u_0^2 + \vartheta_0^2 - q_3^2, \qquad (7)$$

which are usual equations for the propagation constants of uniaxial crystals and the corresponding ordinary waves. Upon that, ϑ_0, q_2 and q_3 satisfy the equation

$$tg(2\vartheta_0 a) = \vartheta_0(q_2 + q_3)/(\vartheta_0^2 + q_2 q_3), \qquad (8)$$

which is similar to the characteristic equation for the ordinary waves of a flat anisotropic waveguide on an anisotropic substrate.

For the second type of waves with polarization $H_y = 0$ in the region we will have

$$\begin{cases} E_z(x,y) = B_e\phi_e(x,y), \\ H_z(x,y) = B_e\phi_e(x,y), \end{cases} \qquad (9)$$

and the wave numbers u_e, ϑ_e, q_2' and q_3' are associated with h_e by the expressions

$$k_0^2\varepsilon_0\varepsilon_e = \varepsilon_e u_e^2 + \varepsilon_0(u_e^2 + h_e^2), \qquad (10)$$

$$k_0^2\varepsilon_0'\varepsilon_e' = \varepsilon_0'(u_e^2 + h_e^2) - q_3'^2\varepsilon_e', \qquad (11)$$

$$k_0^2\varepsilon_e = u_e^2 + h_e^2 - q_2'^2, \qquad (12)$$

corresponding to the extraordinary waves. Upon that, ϑ_e, q_2' and q_3' satisfy the equation

$$tg(2\vartheta_0 a) = \frac{\varepsilon_0\vartheta_e(\varepsilon_0' q_2' + \varepsilon_e q_3)}{\varepsilon_0\varepsilon_0'\vartheta_0^2 + \varepsilon_0^2 q_2' q_3'}, \qquad (13)$$

which is similar to the characteristic equation for the extraordinary waves of a flat anisotropic waveguide on an anisotropic substrate.

Thus, the waves (4) correspond to the ordinary waves $(E_y = 0)$, and (9) – to the extraordinary waves $(H_y = 0)$ of an anisotropic rectangular dielectric waveguide.

In the outer regions *II* $(x > b)$ and *III* $(x < -b)$ let's assume fields in the form of a superposition of waves of a system consisting of two media, one of which has a dielectric permittivity ε_c, and the second is characterized by a dielectric permittivity tensor (2) separated by flat boundary $y = y_0$ (Fig. 1).

For ordinary waves with polarization $E_y = 0$, we have (the upper sign corresponds to the region $x < -b$, while the lower to the region $x > b$):

$$\begin{cases} E_z(x,y) = \int_0^\infty \alpha_\pm(\lambda_1) F_1(\lambda_1, y) \exp[\pm\beta_1(x \pm b)] d\lambda_1 + \\ + \int_0^\infty \gamma \pm(\sigma_1) F_2(\sigma_2, y) \exp[\pm\beta_2(x \pm b)] d\sigma_2, \\ H_z(x,y) = \int_0^\infty \frac{h}{\pm\beta_1 k_0} \alpha_\pm(\lambda_1) E_1(\lambda_1, y) \exp[\pm\beta_1(x \pm b)] d\lambda_1 + \\ + \int_0^\infty \frac{h}{\pm\beta_2 k_0} \gamma \pm(\sigma_2, y) F_2(\sigma_2, y) \exp[\pm\beta_2(x \pm b)] d\sigma_2. \end{cases} \quad (14)$$

Upon that wave numbers $\lambda_1, \lambda_2, \sigma_1, \sigma_2, \beta_1$ and β_2 are associated with each other by expressions

$$\begin{cases} \beta_1 = \sqrt{\lambda_1^2 + h^2 - k_0^2 \varepsilon_0}, \\ \beta_2 = \sqrt{\sigma_2^2 + h^2 - k_0^2 \varepsilon_0'}, \\ \sigma_1^2 = \lambda_1^2 + (\varepsilon_0' - \varepsilon_0) k_0^2, \\ \lambda_2^2 = \sigma_2^2 + (\varepsilon_0' - \varepsilon_0) k_0^2, \end{cases} \quad (15)$$

and the cross-section functions $F_1(\lambda_1, y)$ and $F_2(\sigma_2, y)$ are orthogonal and satisfy the normalization condition:

$$\begin{cases} \int_{-\infty}^{+\infty} F_1(\lambda_1, y) F_2 * (\sigma_2', y) dy = 0, \\ \int_{-\infty}^{+\infty} F_1(\lambda_1, y) F_1 * (\lambda_1', y) dy = \delta(\lambda_2 - \lambda_1'), \\ \int_{-\infty}^{+\infty} F_2(\sigma_2, y) F_2 * (\sigma_2', y) dy = \delta(\sigma_2 - \sigma_2'). \end{cases} \quad (16)$$

For extraordinary waves with polarization $H_y = 0$ expressions for the longitudinal component fields are similar:

$$\begin{cases} H_z(x,y) = \int_0^\infty c_\pm(\lambda_1) \bar{F}_1(\lambda_1, y) \exp[\pm\beta_1(x \pm b)] d\lambda_1 + \\ + \int_0^\infty d \pm(\sigma_2) \bar{F}_2(\sigma_2, y) \exp[\pm\beta_2(x \pm b)] d\sigma_2, \\ E_z(x,y) = \int_0^\infty \frac{h}{\pm\beta_1 k_0} c_\pm(\lambda_1) \bar{E}_1(\lambda_1, y) \exp[\pm\beta_1(x \pm b)] d\lambda_1 + \\ + \int_0^\infty \frac{h}{\pm\beta_2 k_0} d \pm(\sigma_2) \bar{E}_2(\sigma_2, y) \exp[\pm\beta_2(x \pm b)] d\sigma_2. \end{cases} \quad (17)$$

Upon that parameters $\lambda_1, \lambda_2, \sigma_1, \sigma_2, \beta_1$ and β_2 are associated with each other by expressions

$$\begin{aligned} &\beta_1 = \sqrt{\lambda_1^2 + h^2 - k_0^2 \varepsilon_0}, \\ &\beta_2 = \sqrt{\frac{\varepsilon_e'}{\varepsilon_0'} \sigma_2^2 + h^2 - k_0^2 \varepsilon_0'} \\ &\sigma_1^2 = \frac{\varepsilon_e'}{\varepsilon_0'} [\ \lambda_1^2 + (\varepsilon_0' - \varepsilon_0) k_0^2\ ], \\ &\sigma_2^2 = \frac{\varepsilon_e'}{\varepsilon_0'} [\lambda_2^2 + (\varepsilon_0' - \varepsilon_0) k_0^2\ ], \end{aligned} \quad (18)$$

functions are orthogonal and satisfy the normalization condition

$$\begin{cases} \int_{-\infty}^{+\infty} \frac{1}{\varepsilon(y)} \bar{F}_1(\lambda_1, y) \bar{F}_1 * (\lambda_1', y) dy = \delta(\lambda_2 - \lambda_2'), \\ \int_{-\infty}^{+\infty} \frac{1}{\varepsilon(y)} \bar{F}_1(\lambda_1, y) \bar{F}_2 * (\sigma_2', y) dy = 0, \\ \int_{-\infty}^{+\infty} \frac{1}{\varepsilon(y)} \bar{F}_2(\sigma_2, y) \bar{F}_2 * (\sigma_2', y) dy = \delta(\sigma_2 - \sigma_2'). \end{cases} \quad (19)$$

In expressions (17) the lower sign corresponds to region *II*, and the upper to region *III*.

Upon that, if $e_c = e_0' = e_e$ (Medium in regions *II* and *III* is uniform), expressions (14) and (17) are transformed into Fourier integrals in plane waves. Expansion amplitudes $\alpha_\pm(\lambda_1)$ and $y_\pm(\lambda_2)$, $C_\pm(\sigma_1)$ and $d_\pm(\sigma_2)$ define an electromagnetic field in the regions *II* and *III* for the two types of waves of a strip waveguide and, depending on the parity, the solutions by *x* may differ only in sign.

These amplitude and propagation constant h determining the field of a strip waveguide are determined out of matching conditions of the fields at the boundaries $x = \pm b$. Because of the symmetry of the system relative to the plane $x = 0$ it is sufficient to consider only the boundary conditions on one surface $x = b$.

Let's assume that reflection of waves on the faces $x = \pm b$ occurs predominantly without transformation, that is field in the partial region *I* has a form of (4) or (9).

At the same time, as before, the components of the field H_z for waves (4) and E_z, for waves (9) are not taken into account.

Then from matching E_z component for ordinary waves (4) and H_z components for the extraordinary waves (9) on the boundary $x=b$, respectively, and from the conditions of equality H_z components for ordinary waves (4) and E_y components for extraordinary waves, with consideration of (16) and (19) for the two types of waves, respectively, we obtain

$$\left.\begin{matrix}-ctg(ub)\\ tg(ub)\end{matrix}\right\} A_0=\frac{u^2+h^2}{u}\phi_0(b,y)= \tag{20}$$

$$=A_0\int_{-\infty}^{-\infty}\frac{\lambda_1^2-k_0^2\varepsilon_0}{\beta}F_1(\lambda_1,y)d\lambda_1\int_{-\infty}^{-\infty}\phi_e(b,y')F_1*(\lambda_1,y')d'y'-$$

$$-A_e\int_{-\infty}^{-\infty}\frac{k_0^2\varepsilon'-\sigma_2^2}{\beta_2}\bar{F}_2=(\sigma_2y')d\sigma_2\int_{-\infty}^{+\infty}\phi_e(b,y')F_2*(\sigma_2,y')d'y'$$

$$\left.\begin{matrix}-tg(ub)\\ ctg(ub)\end{matrix}\right\} B_e\frac{u^2+h^2}{u\tilde{B}(y)}\phi_0(b,y)= \tag{21}$$

$$=B_e\int_{-\infty}^{-\infty}\frac{k_0^2\varepsilon_e-\lambda_1^2}{\varepsilon(y)\beta_1}\bar{F}_1(\lambda_1,y)d\lambda_1\int_{-\infty}^{-\infty}\frac{1}{\varepsilon(y')}\phi_e(b,y')F_1'*(\lambda_1,y')d'y'+$$

$$+B_e\frac{\varepsilon_e'}{\varepsilon_0'}\int_0^{\infty}\frac{k_0^2\varepsilon'-\sigma_2^2}{\varepsilon(y)\beta_2}\bar{F}_2(\sigma_2y)d\sigma_2\int_{-\infty}^{+\infty}\frac{1}{\varepsilon(y')}\phi_e(b,y')F_2'*(\sigma_2,y')d'y',$$

where $\tilde{\varepsilon}(y)=\varepsilon_e$ as $|y|<a$, $\tilde{\varepsilon}(y)=\varepsilon_0$ as $y>a$, $\tilde{\varepsilon}(y)=\varepsilon$ as $y>-a$.

In accordance with the single-wave approximation, let's multiply expression (20) by $\phi_0'(b,y)$ and expression (21) by $\varphi_e'(b,y)$ and integrate by *y* from $-\infty$ to $+\infty$. As a result, we obtain the characteristic equations for determining the propagation constant k for two types of waves of a strip waveguide

$$\left.\begin{matrix}-ctg(ub)\\ tg(ub)\end{matrix}\right\}\frac{u^2+h^2}{u}\int_{-\infty}^{+\infty}|\phi_e(b,y')|^2\,dy=$$

$$=\int_{-\infty}^{+\infty}\frac{\lambda_1^2-k_0^2\varepsilon_e}{\beta_1}*\left|\int_{-\infty}^{+\infty}\phi_e(b,y)F_1'(\lambda_1,y)d\lambda_1\right|^2 d\lambda_1+ \tag{22}$$

$$+\int_0^{\infty}\frac{\sigma_2^2-k_0^2\varepsilon_0'}{\beta_2}\left|\int_{-\infty}^{+\infty}\phi_0(b,y)F_2'(\sigma_2,y)dy\right|^2 d\sigma_2$$

(ordinary wave),

$$\left.\begin{matrix}-tg(ub)\\ ctg(ub)\end{matrix}\right\}\frac{u^2+h^2}{u}\int_{-\infty}^{+\infty}\left|\frac{1}{\sqrt{\varepsilon'(y)}}\phi_e(b,y)\right|^2 dy=$$

$$=\int_{-\infty}^{+\infty}\frac{k_0^2\varepsilon_e-\lambda_1^2}{\beta_1}*\left|\int_{-\infty}^{+\infty}\phi_e(b,y)\bar{F}_1'(\lambda_1,y)d\lambda_1\right|^2 d\lambda_1+ \tag{23}$$

$$+\frac{\varepsilon_e'}{\varepsilon_0'}\int_0^{\infty}\frac{k_0^2\varepsilon_0'-\sigma_2^2}{\beta_2}\left|\int_{-\infty}^{+\infty}\phi_e(b,y)\bar{F}_2'(\sigma_2,y)dy\right|^2 d\sigma_2$$

(extraordinary wave).

Wherein integrals in expressions (22) and (23) for y are easily taken in explicitly at any $|y_0|<a$. The equation for ordinary waves (22) id solved together with (5) - (8) and (15), and for extraordinary waves (23) together with (10) - (13) and (18). Besides the obvious transition to an isotropic medium, the resulting solution of the problem contains a number of other special cases. For example, comparing the dielectric permittivities of a substrate and the environment, we obtain the result of the previous paragraph for the waveguide in a homogeneous medium. If we equate permittivities of a substrate and the environment only in the lateral regions *II* and *III* ($|x|>b$), we obtain the result for a rectangular waveguide on a stand.

IV. COMPARISON OF THE THEORY WITH THE EXPERIMENT

In this paper there have been made numerical calculations of the characteristic equations obtained for an anisotropic waveguide with a uniform environment and for an anisotropic strip waveguide, which were investigated experimentally in the microwave range.

Comparison of theoretical and experimental results allowed to establish the limits of applicability of the adopted method of calculation.

To ensure the reliability of the results, experimental studies were conducted in the millimeter (6-16 mm) wavelength range for which the measurement procedure is well developed and allows one to precisely control the shape and dimensions of the waveguide. Samples of the test material were placed in an open Fabry-Perot cavity formed by the mesh reflector and copper mirror. Through the mesh reflector desired type of oscillations were excited in the resonator using a dielectric waveguide, which was simultaneously used for indicating resonance. Equipment (klystron oscillator, heterodyne cymometer) used in the process provides a measure of the refractive index of the waveguide with an accuracy of 1%. Upon that, the measured values do not differ from the theoretical only in the third decimal place, i.e. they are the same within experimental accuracy.

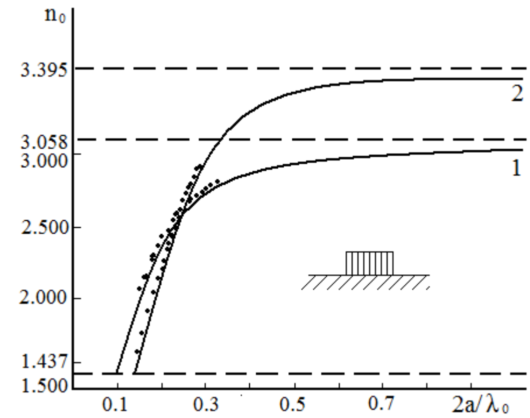

Fig. 2. *Theoretical and experimental data for two main types of waves of a ruby waveguide on a substrate made of PTFE-4*

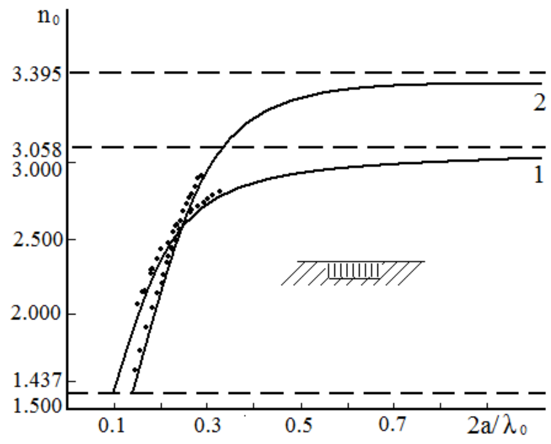

Fig. 3. *Basic types of waves of a ruby waveguide embedded in a substrate made of PTFE-4*

V. RESULTS AND CONCLUSIONS

Thus, a comparison of theoretical and experimental data shows that a single-wave approximation quite accurately describes the main types of rectangular waveguides. Upon that the accuracy of the method increases with

increasing ratio of the sides of the waveguides.

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