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Reducing wave reflection in the split Hopkinson bar device by modifying the bar end

Abdulrahman Al-Mohemmed, Ramzi Othman, Khalid Almitani
Mechanical Engineering Department, Faculty of Engineering
King Abdulaziz University, P.O. Box 80248
Jeddah 21589, Saudi Arabia
Rothman1@kau.edu.sa

***Abstract*—The split Hopkinson bar set-up is a mechanical device widely used to test materials at high strain rates. Because of the multiple wave reflections in bars the test duration is limited to some hundreds of micro-seconds. In order to increase the test duration and consequently extend the use of this machine to intermediate strain rate range, we propose here a new design of the output bar end in order to reduce wave reflections within this bar. Namely, the flat end is replaced by a stepped end. This highly reduces the amplitude of the reflected wave due to the progressive change in the mechanical impedance. More precisely, the reflected wave amplitude is reduced by 37% using one-step end and by more than 50% using two-step end. This result is highly promising as it proves the possibility of reducing the reflected wave amplitude in Hopkinson bars using non-flat ends.**

Keywords—Hopkinson bar; reflected wave; mechanical impedance; intermediate strain rate.

I. INTRODUCTION (*Heading 1*)

The SHB (split Hopkinson bar) has become a standard experimental technique for performing tests under dynamic loading conditions. However, the use of this method is rather limited to high strain rates because of multiple wave reflections within the bars which yields short test duration.

Waves reflect at the bars' ends because of the mechanical impedance mismatch between steel and air. The output bar end is assumed free of stress. Therefore, compressive wave are reflected back into the bar as tensile waves and vice versa. In the literature, authors were focused on the analysis of these multiple reflections. Consequently, they proposed wave separation techniques to increase the test duration of the split Hopkinson bar [1-14]. However, these solutions are mostly based on signal processing background which needs important mathematical skills.

Studying airborne ultrasonic transducers, Saffar et al. [15-17] proposed the use of progressive change in the mechanical impedance in order to reduce wave reflections and increase power transmission into the air. In this paper, we aim at reducing wave reflections in split Hopkinson bar machine by modifying bars ends, namely, using stepped ends. This will lead to progressive change of the mechanical impedance.

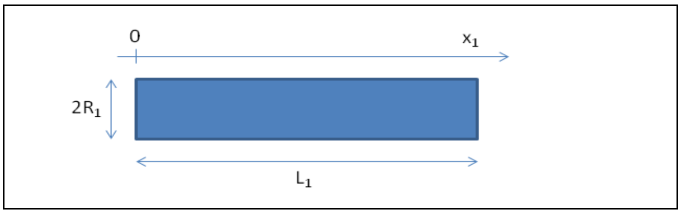

Fig.1. *Schematic of the main rod.*

II. METHOD

A. Problem statement

We consider an elastic rod of length L_1 and radius R_1 (Fig. 1). In a cross-section x_1, the displacement velocity and force Fourier transforms read [18-19]:

$$\tilde{U}_1(x_1,\omega) = F_1(\omega)\, e^{-i\xi_1(\omega)x_1} + D_1(\omega)\, e^{i\xi_1(\omega)x_1}, \quad (1)$$

$$\tilde{V}_1(x_1,\omega) = i\omega\left(F_1(\omega)\, e^{-i\xi_1(\omega)x_1} + D_1(\omega)\, e^{i\xi_1(\omega)x_1}\right), \quad (2)$$

and

$$\tilde{N}_1(x_1,\omega) = i\xi_1(\omega)A_1E_1 \times \left(-F_1(\omega)\, e^{-i\xi_1(\omega)x_1} + D_1(\omega)\, e^{i\xi_1(\omega)x_1}\right), \quad (3)$$

respectively, where $F_1(\omega)$ and $D_1(\omega)$ are the incident and reflected waves, $\xi_1(\omega)$ is the wave dispersion in the rod, E_1 its Young's modulus and A_1 its cross-sectional area. If the right bar end is free,

$$\tilde{N}_1(L_1,\omega) = 0, \quad (4)$$

Thus,

$$\frac{|D_1(\omega)|}{|F_1(\omega)|} = 1, \quad (5)$$

i.e., the amplitude of the reflected wave $D_1(\omega)$ is equal to the amplitude of the incident wave $F_1(\omega)$.

In this paper, we aim at modifying the end of the rod in order to reduce the reflected-to-incident waves ratio. The above rod or bar is called hereafter the main rod. It is considered 1 m in length and 10 mm in radius. Two solutions are investigated here: one-step ended and two-step ended rods.

B. Use of one-step end

In this section, we are first interested in the reduction of the reflected wave by using one step at the end of the main rod. A schematic of the one-step ended bar is given is Fig. 2.The Equations (1)-(3) hold for the main or first rod. Likewise, the velocity and force in a cross-section x_2 of the second bar read:

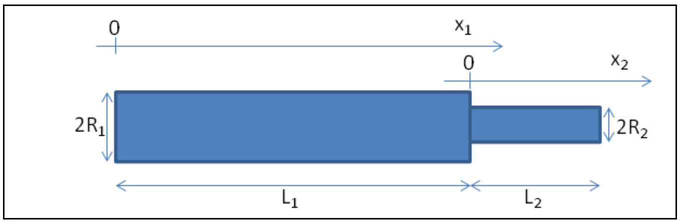

Fig.2. *Schematic of the one-step ended rod.*

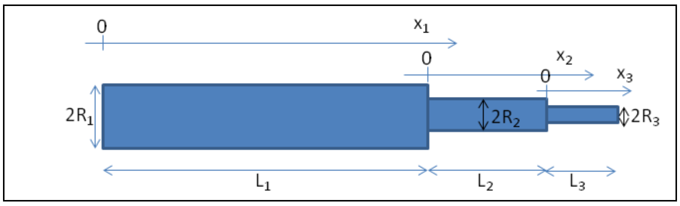

Fig.2. *Schematic of the two-step ended rod.*

$$\tilde{V}_2(x_2,\omega) = i\omega \times \left(F_2(\omega)\, e^{-i\xi_2(\omega)x_2} + D_2(\omega)\, e^{i\xi_2(\omega)x_2}\right), \quad (6)$$

and

$$\tilde{N}_2(x_2,\omega) = i\xi_2(\omega)A_2E_2 \times \left(-F_2(\omega)\, e^{-i\xi_2(\omega)x_2} + D_2(\omega)\, e^{i\xi_2(\omega)x_2}\right), \quad (7)$$

where $\xi_2(\omega)$ is the wave dispersion of the second rod, E_2 its Young's modulus and A_2 its cross-sectional area.

In order to express the reflected-to-incident waves ratio, i.e., $|D_1(\omega)|/|F_1(\omega)|$, the boundary conditions are considered. First, the right end of the second bar is free. Thus,

$$\tilde{N}_2(L_2,\omega) = 0. \quad (8)$$

Moreover, we assume the continuity of force and velocity at the interface between the two bars. Consequently,

$$\tilde{V}_1(L_1,\omega) = \tilde{V}_2(0,\omega), \quad (9)$$

and

$$\tilde{N}_1(L_1,\omega) = \tilde{N}_2(0,\omega), \quad (10)$$

Eliminating $F_2(\omega)$ and $D_2(\omega)$ yields:

$$\frac{D_1(\omega)}{F_1(\omega)} = e^{-2i\xi_1(\omega)L_1}\,\frac{Z_1\cos(\xi_2(\omega)L_2) - i\,Z_2\sin(\xi_2(\omega)L_2)}{Z_1\cos(\xi_2(\omega)L_2) + i\,Z_2\sin(\xi_2(\omega)L_2)}, \quad (11)$$

where $Z_1 = \xi_1(\omega)A_1E_1/\omega$ and $Z_2 = \xi_2(\omega)A_2E_2/\omega$ are the mechanical impedances of the main and second bars. The modulus of this ratio then reads:

$$\left|\frac{D_1(\omega)}{F_1(\omega)}\right| = \left|\frac{Z_1\cos(\xi(\omega)L_2) - i\,Z_2\sin(\xi(\omega)L_2)}{Z_1\cos(\xi(\omega)L_2) + i\,Z_2\sin(\xi(\omega)L_2)}\right|, \quad (12)$$

The aim of this work is to minimize this ratio. Hence this ratio is evaluated for L_2 between 0 and 2m and for R_2 between 0 and 10 mm. The best design corresponds to the peer (L_2, R_2) that gives the lowest ratio.

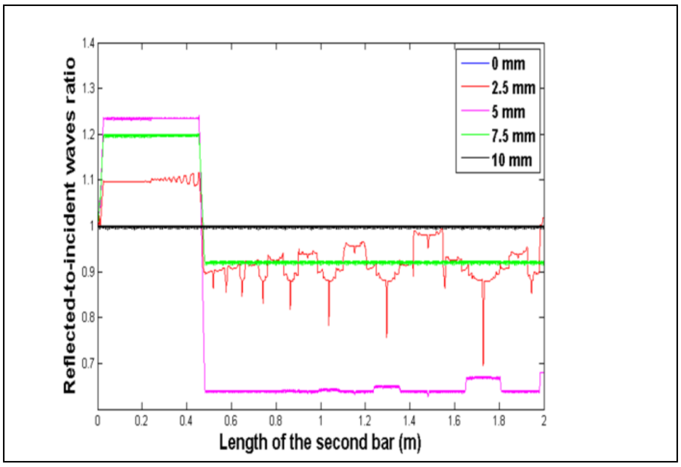

Fig.4. *Reduction of the reflected wave in terms of the length of the second bar*

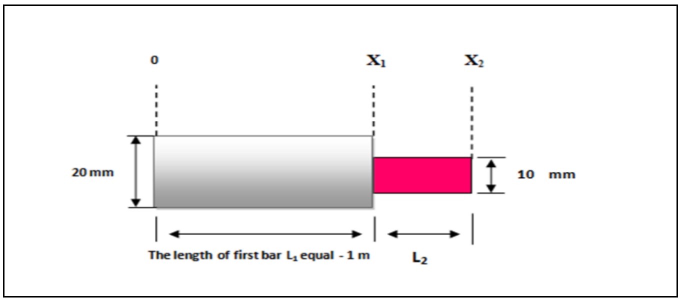

Fig.5. *Optimum design with one-step end*

C. Use of two-step end

Instead of using only one-step end, it is also possible to use two-step end (Fig. 3). In this case, the Equations (1) to (3) can also be used for the main rod

(1^{st} bar). The Equations (6) and (7) can be used to predict velocity and force in any cross-section of the 2^{nd} bar. Moreover, similar equations can be used for the 3^{rd} bar. More precisely, the velocity and force in a cross-section x_3 of the third bar read:

$$\tilde{V}_3(x_3,\omega) = i\omega \times \left(F_3(\omega)\, e^{-i\xi_3(\omega)x_3} + D_3(\omega)\, e^{i\xi_3(\omega)x_3}\right), \quad (13)$$

and

$$\tilde{N}_3(x_3,\omega) = i\xi_3(\omega)A_3E_3 \times \left(-F_3(\omega)\, e^{-i\xi_3(\omega)x_3} + D_3(\omega)\, e^{i\xi_3(\omega)x_3}\right), \quad (14)$$

where $\xi_3(\omega)$ is the wave dispersion of the third rod, E_3 its Young's modulus and A_3 its cross-sectional area.

Considering that the right end of 3^{rd} bar is free yields:

$$\tilde{N}_3(L_3,\omega) = 0. \quad (15)$$

Moreover, we assume the continuity of force and velocity at the interface between the main and second bar and at the interface between the second and third bar. Therefore,

$$\tilde{V}_1(L_1,\omega) = \tilde{V}_2(0,\omega), \quad (16)$$

$$\tilde{N}_1(L_1,\omega) = \tilde{N}_2(0,\omega), \quad (17)$$

$$\tilde{V}_2(L_2,\omega) = \tilde{V}_3(0,\omega), \quad (18)$$

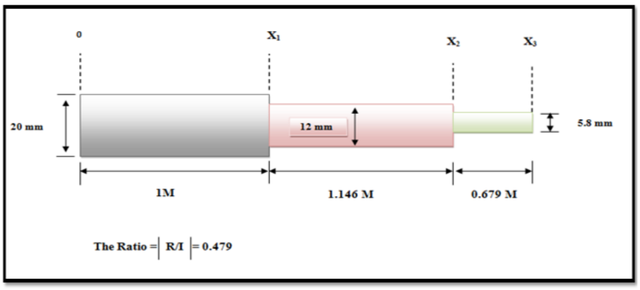

Fig.5. *Optimum design with a two-step end bar*

and

$$\tilde{N}_2(L_2,\omega)=\tilde{N}_3(0,\omega). \qquad (19)$$

Eliminating $F_2(\omega)$, $D_2(\omega)$, $F_3(\omega)$ and $D_3(\omega)$ yields:

$$\frac{D_3(\omega)}{F_3(\omega)}=e^{-2i\xi_1(\omega)L_1}\times \frac{Z_1\left(1+\varphi_2(\omega)e^{-2i\xi_2(\omega)L_2}\right)-Z_2\left(1-\varphi_2(\omega)e^{2i\xi_2(\omega)L_2}\right)}{Z_1\left(1+\varphi_2(\omega)e^{-2i\xi_2(\omega)L_2}\right)+Z_2\left(1-\varphi_2(\omega)e^{2i\xi_2(\omega)L_2}\right)}. \qquad (20)$$

where

$$\varphi_2(\omega)=e^{-2i\xi_2(\omega)L_2}\frac{Z_2\cos(\xi_3(\omega)L_3)-i\,Z_3\sin(\xi_3(\omega)L_3)}{Z_2\cos(\xi_3(\omega)L_3)+i\,Z_3\sin(\xi_3(\omega)L_3)}. \qquad (21)$$

and $Z_3=\xi_3(\omega)A_3E_3/\omega$ is the mechanical impedance of the third bar. In order to get an optimal design that minimizes wave reflection in the main bar, the best set of (L_2,R_2,L_3,R_3) is determined using an optimization procedure in order to have the lowest reflected-to-incidentt waves ratio:

$$\left|\frac{D_3(\omega)}{F_3(\omega)}\right|=\left|\frac{Z_1\left(1+\varphi_2(\omega)e^{-2i\xi_2(\omega)L_2}\right)-Z_2\left(1-\varphi_2(\omega)e^{2i\xi_2(\omega)L_2}\right)}{Z_1\left(1+\varphi_2(\omega)e^{-2i\xi_2(\omega)L_2}\right)+Z_2\left(1-\varphi_2(\omega)e^{2i\xi_2(\omega)L_2}\right)}\right|. \qquad (22)$$

III. Results

A. Use of one-step end

Considering (12), the reflected-to-incident waves ratio is calculated. Fig. 4 shows this ratio in terms of the length of the added step for several values of this step radius. Hence, increasing the second bar (the added step) length yields to an increase of the ratio. This means that the reflected wave is rather for short second bars (length lower than 0.5). This what we would like to avoid. Fortunately, the reflected-to-incident waves ratio drops for step lengths longer than 0.5. Moreover, this ratio drops lower than 1 which means that the reflected wave is reduced. The best reduction is obtained for a step radius of 5 mm which is half the radius of the main bar. In this case, the ratio can be as low as 0.63. Thus the optimum design, using one-step bar, is to have the radius of the second bar equal to 5 mm and its length higher than 0.5 m (Fig. 5). Hence, the reflected wave can be reduced by 37%.

B. Use of two-step end

In order to improve the reduction of the reflected wave a two-step end is investigated in this section. Considering Eqs. (20) and (21), the reflected-to-incident waves ratio depends on the second and third bar lengths and also their radii. An optimization procedure was used to obtain the best set of these geometrical parameters in order to get the lowest ratio. The optimum solution is schematized in Fig. 6. It gives a waves ratio of 0.479 which means that the reflected wave is reduced by more than 50%.

IV. Conclusion

In this paper, a new design of the Hopkinson bars is proposed. More precisely, the flat end of the output bar is replaced here by either one-step or two-step end. Using the one-dimensional wave propagation in bars, the reflected-to-incident waves ratio is expressed in terms of the lengths and radii of the added steps. Subsequently, a parametric study and an optimization procedure give the best geometrical parameters that minimize the waves ratio and consequently the reflected wave amplitude. Using one-step end gives a reduction of the reflected wave amplitude by 37% whereas the two-step end can achieve more than 50% reduction of the reflected wave amplitude. These results are highly promising. They show that it is possible to reduce the amplitude of the reflected wave by using non-fat Hopkinson bar end. This work should be followed by further investigations in order to achieve higher reduction of the reflected wave.

References

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