Studies Of The Transient Thermal Behaviour Of Semiconductor Device By Using Fourier Series

Transient thermal behaviour

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***Abstract*—In this paper, model based on the Fourier series has been used in studies of the transient thermal behavior of semiconductor devices. An explicit expression for the temperature space-time dependence has been derived.**

Keywords—thermal; transient; analytical; semiconductor;

I. Introduction

It is well known that the thermal system optimization plays very important role in electrical optimization. The thermal analysis of microelectronic devices has been the subject of increasing interest for last several years due to a number of applications. Undesirable effects such as thermal runway, thermal coupling between neighboring devices, substrate thinning, multilayer substrates, surface metallization, etc., unfavorably affecton the performance of semiconductor devices and circuits [1,2]. Most of the published papers related to these problems assuming steady-state conditions [3,4], while relatively a few of them deal with the transient thermal modeling of microelectronic devices [5].

In this paper, a simple analytical expression for the transient thermal impedance based on the Fourier series approaches is derived. The proposed relation provides a good physical description of the phenomenon and is in very good agreement with the expression derived by coupling the electrical model of a component with the description of its thermal properties using an electric analog model [6].

II. Descripton of a Model

In this paper we consider an improved version of the simple analytical on one dimensional (1-D) model which geometry is depicted in Figure 1. The heat source (shaded region) is located at the distance *D*, at which the adiabatic condition is assumed. At the bottom of the device (*x* = *L*) the heat convection condition is proposed. In general, the propagation of heat in a system can take place in three different ways, convection, heat radiation or heat conduction. Since electronic components usually have only heat conduction, we start with one dimension heat transfer equation:

$$\frac{\partial^2\Theta}{\partial x^2}-\frac{1}{a^2}\frac{\partial\Theta}{\partial t}=-\frac{P_g(x,t)}{\lambda}, \tag{1}$$

with boundary conditions:

$$\frac{\partial\Theta}{\partial x}=0, \text{ for } x=-D, \tag{2}$$

$$\frac{\partial\Theta}{\partial x}=\frac{\alpha}{\lambda}\Theta, \text{ for } x=L, \tag{3}$$

and an initial condition:

$$\Theta(x,0)=0, \tag{4}$$

where $\alpha^2=\lambda/c\rho$ is the thermal diffusivity, λ is the thermal conductivity of substrate, *c* is the specific heat, ρ is the density of the semiconductor material and *x* describes the coordinates in the direction of heat propagation.

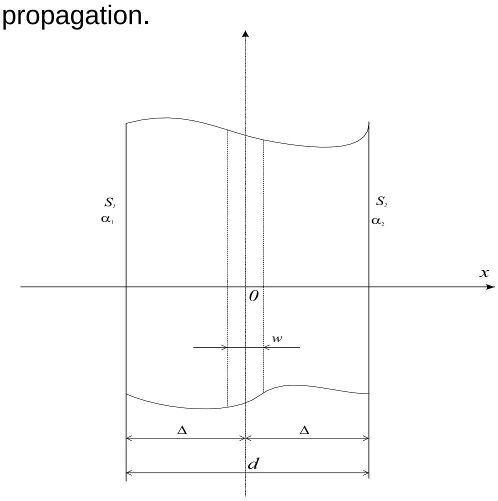

Fig. 1. *Schematic view of a structure with p-n junction in the middle*.

$P_g(x,t)$ is the volume density of dissipated power and we shall suppose in following that the thickness of dissipating region is negligible. Under the assumption that dependence of the function $P_g(x,t)$ on the spatial coordinate and on the time can be resolved separately, or the function $P_g(x,t)$ can be expressed as:

$$P_g(x,t) = P_{gS}\delta(x)f(t) \tag{5}$$

where the function $f(t)$ describes time dependence of the power dissipation. So, the problem we are solving is defined by eq. (1), boundary and initial conditions (2)-(4) and the assumption (5).

III. Fourier Series

It is well known that Fourier series is a method of expressing an arbitrary periodic function as a sum of cosine terms as illustrated in Figure 2. In this paper, we shall used this method in order to obtain an analytical expression for the thermal impedance. In its original interpretation homogenous form of the equation (5) is supposed. To deal with the heat source (the right side) of the equation (1) we assume that the time dependence of it is given by the Heaviside unit step function: $f(t) = \eta(t)$. In that case, the stationary solution of the boundary value problem (1)-(4) is given by the simple relation:

$$\Theta(x,\infty) = P_{gS}\left\{\frac{1}{\alpha} + \frac{1}{\lambda}\left[L - x\eta(x)\right]\right\}. \tag{6}$$

With this, it is possible to write down solution of time dependent case as a sum of two terms:

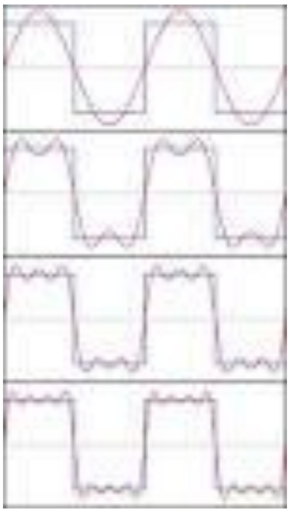

Fig. 2. *Illustrations of sums of the Fourier series.*

$$\Theta(x,t) = \Theta_h(x,t) + \Theta(x,\infty) \tag{7}$$

where $\Theta_h(x,t)$ is the solution of homogenous problem with additional constraint

$$\Theta_h(x,0) = -\Theta(x,\infty), \tag{8}$$

originating from the initial condition $\Theta(x,0) = 0$. Homogenous solution $\Theta_h(x,t)$ can be expressed as well known infinite series:

$$\Theta_h(x,t) = \sum_{n=0}^{\infty} \exp\left(-\gamma_n^2 \alpha^2 t\right)\left[A_n \cos(\gamma_n x) + B_n \sin(\gamma_n x)\right], \tag{9}$$

where constants A_n and B_n and eigen values γ_n have to be determined from boundary and initial conditions. From the boundary conditions the equation defining γ_n can be obtained:

$$tg(\gamma_n D) = \frac{\frac{\alpha}{\lambda}\cos(\gamma_n L) - \gamma_n \sin(\gamma_n L)}{\frac{\alpha}{\lambda}\sin(\gamma_n L) + \gamma_n \cos(\gamma_n L)}. \tag{10}$$

The expression defining thermal impedance is :

$$Z_{th}(t) = \frac{\Theta(0,t)}{SP_{gS}} = \frac{1}{S}\left\{\left(\frac{1}{\alpha} + \frac{L}{\lambda}\right) - \frac{1}{\lambda}\sum_{n=0}^{\infty}\frac{1}{\gamma_n^2} ctg^2(\gamma_n D)\exp\left(-\alpha^2\gamma_n^2 t\right)\right\}. \tag{11}$$

IV. Results

Although the relation (18) has a simple form, it could be greatly simplified when $D \to 0$. Equation (18) then becomes $\gamma_n tg(\gamma_n L) = \alpha / \lambda$ and its smallest zero is simply $\gamma_0^2 \approx \alpha/(\lambda L)$. The same relation leads to approximate value of the coefficient $ctg^2(\gamma_0 D) \approx \gamma_0^2 (L + \lambda/\alpha)$ also and retaining the first term in (11) only, we get the final straightforward expression for thermal impedance:

$$Z_{th}(t) = \frac{1}{S}\left(\frac{1}{\alpha} + \frac{L}{\lambda}\right)\left[1 - \exp\left(-\frac{t}{\tau_r}\right)\right],\ \tau_r = L\frac{c\rho}{\alpha}. \tag{12}$$

So that, we have shown that it is possible to describe the dynamic of the thermal response with one parameter only-rise time τ_r . In the case when *D* cannot be neglected it would be difficult to derive similar relation, but one must remember that relation *D<<L* always holds. The exponential time dependence of the Z_{th} is shown in Figure 3.

We have shown that it is possible to describe transient behavior of the thermal impedance with the single parameter- rise time . Our analytical expression derived by using Fourier series approach has the same form as that obtained by using the phenomenological model, which confirms the accuracy of our procedure.

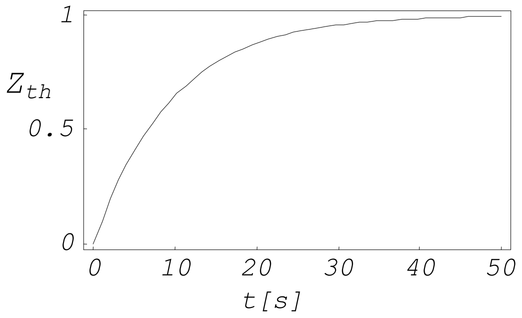

Fig. 3. *The time dependence of the thermal impedance.*

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