Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 2 Issue 1, January - 2015

# Vibration Analysis of Non-homogeneous Rectangular Orthotropic Plates

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***Abstract*—This paper presents a differential quadrature solution for analysis of transverse vibrations of non-homogeneous rectangular orthotropic plates of linearly varying thickness resting on Winkler foundation. Following Lévy approach i.e. two parallel edges are simply supported, the governing equation of motion has been solved for three different combinations of clamped, simply supported and free boundary conditions at the other two edges. Numerical results for first three natural frequencies for various values of parameters are presented in tables and graphs. The accuracy and convergence results are examined and verified.**

***Keywords* — *DQM, orthotropy, variable thickness, non-homogeneity.***

## I. INTRODUCTION

The theory and numerical method of vibration of non-homogeneous rectangular orthotropic plates of linearly varying thickness plates on foundation have been widely concerned. Leissa provides an excellent bibliography on plate vibration up to 1987 in [1-5]. Subsequently the study of free vibrations of homogeneous isotropic rectangular and square plates of linearly varying thickness has been reported [6, 7]. Considerable amount of work dealing with natural frequencies of homogeneous rectangular orthotropic plates of uniform and non-uniform thickness have appeared [8-15]. Various numerical methods have been employed to study the vibrational behavior of uniform/variable thickness plates and are reported [8-14]. Of these, Rayleigh-Ritz method with characteristics orthogonal polynomials has been employed to obtain the natural frequencies of transverse vibrations of rectangular plates of non-uniform thickness [8]. In [9] adopted a semi-analytical approach in the differential quadrature method to investigate free vibration of isotropic and orthotropic rectangular plates with linearly varying thickness. Rossi [10] employed the finite element method in the study of vibrations of thin orthotropic rectangular plate. Bambill et al. [11] used the Rayleigh-Ritz method and finite element method to analyze the transverse element method to analyze the transverse vibration of an orthotropic rectangular plate with linearly varying thickness. Ashour [12] studied the flexural vibrations of orthotropic plates with variable thickness in one direction by employing the finite strip transition matrix technique. In the reference [13] Chebysev collocation method has been used in the study of transverse vibrations of non-uniform rectangular orthotropic plates. Recently Lal and Dhanpati [14] have presented Quintic spline solution for transverse vibration of non-homogeneous orthotropic rectangular plates.

In the present work, the analysis of vibrational behavior of non-homogeneous orthotropic rectangular plates with linearly varying thickness along one direction resting on Winkler foundation on the basis of the classical plate theory have been investigated. The two opposite sides are simply supported while the other two may be clamped or simply supported or free. The partial differential equation governing the motion of plate has been reduced into fourth order ordinary differential equation with variable coefficients. The resulting equation is then solved by differential quadrature method (DQM) to study the effect of various parameters for a Huber type orthotropic plate material “ORTHO1” [15] for the first three mode of vibration.

## II. MATHEMATICAL FORMULATION

Consider an orthotropic nonhomogeneous rectangular plate of varying thickness $h(x, y)$ occupying the domain $0 \le x \le a, 0 \le y \le b$ in $xy-$plane, where $a$ and $b$ are the length and the breadth of the plate respectively and resting on a Winkler foundation with foundation modulus $k_f$. The middle surface being $z = 0$ and origin is at the one of the corners of the plate. The $x$ and $y$ axes are taken along the principal directions of orthotropy and the axis of $z$ is perpendicular to the $xy$ plane. The differential equation governing the transverse vibration of such plates is given by [14]

$$
\begin{aligned}
& D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} \\
& + 2\frac{\partial H}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 w}{\partial y \partial x^2} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 w}{\partial x^3} \\
& + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 w}{\partial y^3} + \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 w}{\partial y^2} \\
& + \frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 w}{\partial y \partial x} \\
& + \rho h \frac{\partial^2 w}{\partial t^2} + k_f w = 0, \qquad (1)
\end{aligned}
$$

where $w(x, y, t)$ is the transverse deflection, $t$ the time, $\rho$ the mass density and $E_x, E_y, \nu_x, \nu_y$ and $G_{xy}$ are material constants in proper directions defined by an orthotropic stress-strain law.

The two opposite edges $y$ = 0 and $b$ are assumed to be simply supported. For a harmonic solution, the displacement $w$ is expressed as

$$
w(x, y, t) = \bar{w}(x)\sin(p\pi y / b)e^{i\omega t} , \qquad (2)
$$

where $p$ is a positive integer and $\omega$ is the frequency in radians.

Let the thickness of the plate, Young's moduli $E_x, E_y$ and density $\rho$ be the functions of space variable $x$ only and shear modulus is $G_{xy} = \sqrt{E_x E_y} / 2(1 + \sqrt{\nu_x \nu_y})$.

By introducing the non-dimensional variables $X$ = $x/a$, $Y$ = $y/b$, $\bar{h} = h / a$, $W = \bar{w} / a$ and using (2), (1) reduces to

$$
\begin{aligned}
& \bar{h}^3 E_x W^{iv} + [2(\bar{h}^3 E_x' + 3\bar{h}^2 \bar{h}' E_x)] W''' \\
& + [(6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x + 6\bar{h}^2 \bar{h}' E_x' + \bar{h}^3 E_x'' \\
& - 2\lambda^2 \bar{h}^3 (E^* + 2G_{xy})(1 - \nu_x \nu_y)] W'' \\
& - [2\lambda^2 \{3\bar{h}^2 \bar{h}'(\nu_y E_x + 2(1 - \nu_x \nu_y) G_{xy}) \\
& + \bar{h}^3 (\nu_y E_x' + 2(1 - \nu_x \nu_y) G_{xy}')\}] W' \\
& + [\lambda^4 \bar{h}^3 E_y - \lambda^2 \nu_y \{\bar{h}^3 E_x'' + 6\bar{h}^2 \bar{h}' E_x' + (6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x\} \\
& - 12\,(1 - \nu_x \nu_y)(\rho \bar{h} a^2 \omega^2 - a k_f)] W = 0 , \qquad (3)
\end{aligned}
$$

where $\lambda^2 = p^2 a^2 \pi^2 / b^2$ and primes denote differentiation with respect to $X$.

For linear variation in thickness [10, 11] i.e. $\bar{h} = h_0$ (1+ α $X$) and following [14] for non-homogeneity of the plate material in $X$ direction as follows:

$$
E_X = E_1 e^{\mu X}, E_y = E_2 e^{\mu X}, \rho = \rho_0 e^{\beta X} \qquad (4)
$$

where $h_0$, $\rho_0$ are the thickness and density of the plate at $X = 0$, $\alpha$ the taper parameter, $\mu$ the non-homogeneity parameter, $\beta$ the density parameter and $E_1, E_2$ the Young's modulii in proper directions at $X$=0.

Equation (3) now reduces to

$$
A_0 W^{iv} + A_1 W''' + A_2 W'' + A_3 W' + A_4 W = 0 \qquad (5)
$$

where,

$$
\begin{aligned}
& A_0 = 1, A_1 = 2(\mu + \frac{3\alpha}{(1 + \alpha X)}), \\
& A_2 = \frac{6\alpha^2}{(1 + \alpha X)^2} + \frac{6\mu\alpha}{(1 + \alpha X)} + \mu^2 \\
& - 2\lambda^2 (\nu_y + \frac{\sqrt{E_2 / E_1}}{1 + \sqrt{\nu_x \nu_y}}(1 - \nu_x \nu_y)), \\
& A_3 = -2\lambda^2 (\frac{3\alpha}{(1 + \alpha X)} + \mu) \\
& (\nu_y + \frac{\sqrt{E_2 / E_1}}{1 + \sqrt{\nu_x \nu_y}}(1 - \nu_x \nu_y)), \\
& A_4 = \lambda^4 E_2 / E_1 - \lambda^2 \nu_y \{\mu^2 + \frac{6\mu\alpha}{(1 + \alpha X)} \\
& + \frac{6\alpha^2}{(1 + \alpha X)^2}\} - \frac{\Omega^2}{(1 + \alpha X)^2} e^{(\beta - \mu)X} + \\
& \frac{12K}{h_o^3 (1 + \alpha X)^3} e^{-\mu x}
\end{aligned}
$$

where $K$= a $k_f$ (1- $\nu_x$ $\nu_y$)/ $E_1$ ,

$\Omega^2 = 12 \rho_0 (1 - \nu_x \nu_y) a^2 \omega^2 / E_1 h_0^2$.

The solution of (5) together with the boundary conditions at the edges $X$ = 0 and $X$ = 1 gives rise to a two-point boundary value problem with variable coefficients whose closed form solution is not possible. An approximate solution is obtained by employing differential quadrature method.

## III. METHOD OF SOLUTION: DIFFERENTIAL QUADRATURE METHOD

Let $X_1$, $X_2$, $X_m$ be the $m$ grid points in the applicability range [0, 1] of the plate. According to the

DQM, the $n^{th}$ order derivative of $W(X)$ w.r.t. $X$ can be expressed discretely at the point $X_i$ as

$$\frac{d^n W(X_i)}{dX^n} = \sum_{j=1}^{m} c_{ij}^{(n)} W(X_j) \text{ , n = 1,2,3,4}$$

and $i$ =1,2,…, $m$ (6)

where $c_{ij}^{(n)}$ are the weighting coefficients associated with the nth order derivative of W($X$) w. r. to $X$ at discrete point $X_i$. Following Shu [16, pages 31, 35] are given by

$$c_{ij}^{(1)} = \frac{M^{(1)}(X_i)}{(X_i - X_j)M^{(1)}(X_j)}, \; i, \; j\text{=1,2,…,}m;\; i\neq j \tag{7}$$

$$M^{(1)}(X_i) = \prod_{\substack{j=1 \\ j\neq i}}^{m} (X_i - X_j) \tag{8}$$

$$c_{ij}^{(n)} = n\left( c_{ii}^{(n-1)} c_{ij}^{(1)} - \frac{c_{ij}^{(n-1)}}{x_i - x_j} \right)$$

$$for \;\; i, j = 1,2,....,m, \; j \neq i \;\; and \;\; n = 2,3,4 \tag{9}$$

$$c_{ii}^{(n)} = -\sum_{\substack{j=1 \\ j\neq i}}^{m} c_{ij}^{(n)} \quad for \;\; i = 1,2,....m, \tag{10}$$

$$and \;\; n = 1,2,3,4$$

Discretizing equation (5) at grid points $X_i$,

$i$ = 3, 4,…….,$m$-2, it reduces to,

$$A_{0,i} W^{iv}(X_i) + A_{1,i} W'''(X_i) + A_{2,i} W''(X_i) + A_{3,i} W'(X_i) + A_{4,i}\, W(X_i) = 0. \tag{11}$$

Substituting for $W(X)$ and its derivatives at the $i^{th}$ grid point in (11) and using relations (6) to (10), (11) becomes

$$\sum_{j=1}^{m} (A_{0,i} c_{ij}^{(4)} + A_{1,i} c_{ij}^{(3)} + A_{2,i} c_{ij}^{(2)} + A_{3,i} c_{ij}^{(1)}) W(X_j) + A_{4,i} W(X_i) = 0. \tag{12}$$

For $i$ = 3, 4 …., ($m$-2), one obtains a set of ($m$-4) equations in terms of unknowns $W_j (\equiv W(X_j)),\; j = 1,2,\cdots, m,$ which can be written in the matrix form as

[$B$][$W$*]=[0] (13)

where $B$ and $W$* are matrices of order ($m$-4) x $m$ and ($m$ x 1) respectively.

Here, the ($m$-2) internal grid points chosen for collocation are the zeros of shifted Chebyshev polynomial of order ($m$-2) with orthogonality range

[0, 1] given by $X_{k+1} = \frac{1}{2}[1 + \cos(\frac{2k-1}{m-2}\frac{\pi}{2})]$ ,

$k$ =1, 2,…, $m$-2 (14)

## IV. BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS

The three different combinations of boundary conditions namely, C-C, C-S, C-F have been considered here, where C, S, F stand for clamped, simply supported and free edge, respectively and

first symbol denotes the condition at the edge $X$=0 and second symbol at the edge $X$=1.By satisfying the relations for clamped, simply supported and free edge conditions, respectively, a set of four homogeneous equations in terms of unknown $W_j$ are obtained.

$$W = \frac{dW}{dX} = 0;$$

$$W = \frac{d^2W}{dX^2} - (E^*/E_x^*)\lambda^2 W = 0; \quad and$$

$$\frac{d^2W}{dX^2} - (E^*/E_x^*)\lambda^2 W = \frac{d}{dX}\left( E_x \bar{h}^3 \left\{ \frac{d^2W}{dX^2} - \lambda^2 \nu_y W \right\} \right)$$

$$-4\lambda^2(1 - \nu_x \nu_y) G_{xy} \bar{h}^3 \frac{dW}{dX} = 0,$$

These equations together with field equation (13) give a complete set of $m$ homogeneous equations in $m$ unknowns. For C-C plate this set of equations can be written as

$$\begin{bmatrix} B \\ B^{CC} \end{bmatrix} [W^*] = [0] \tag{15}$$

where $B^{CC}$ is a matrix of order 4×$m$. For a non-trivial solution of (15), the frequency determinant must vanish and hence,

$$\begin{vmatrix} B \\ B^{CC} \end{vmatrix} = 0. \tag{16}$$

Similarly for C-S and C-F plates, the frequency determinants can be written as

$$\begin{vmatrix} B \\ B^{CS} \end{vmatrix} = 0 \;\&\; \begin{vmatrix} B \\ B^{CF} \end{vmatrix} = 0 \text{ respectively} \tag{17, 18}$$

## V. NUMERICAL RESULTS AND DISCUSSION

The frequency equations (16-18) have been solved numerically to compute the values of the frequency parameter $\Omega$ for various values of plate parameters and the effect of foundation parameter, non-homogeneity parameter, density parameter, taper parameter and aspect ratio on frequency parameter $\Omega$ has been analysed for C-C, C-S and C-F plates vibrating in first three modes of vibration. The elastic constants for the plate material are taken as $E_1 = 1\times 10^{10}\, MPa$, $E_2 = 5\times 10^{9}\, MPa$, $\nu_x = 0.2$, $\nu_y = 0.1$, given by [15] ('ORTHO1'). This is obtained by taking p = 1.0 and thickness $h_0$ = 0.1at the edge $X = 0$. To choose the appropriate number of collocation points m, convergence studies have been carried out for various sets of parameters for all the three plates. Convergence graphs are shown in the Figs. 1(a-c) for $a/b$ = 1.0, $K$ = 0.02, $\mu$ = 0.5, $\alpha$ = -0.5 and $\beta$ = 0.5 for C-C, C-S and C-F plates respectively. For these data the maximum deviation were observed. In all the computations, m = 20 has been fixed since a further increase in m does not improve the results even in the fourth place of decimal.

Table 1 presents a comparison of the results which shows the computational accuracy of DQM for homogeneous ($\mu = \beta = 0$), isotropic ($E_2 / E_1$= 1) plate of uniform thickness ($\alpha$=0.0) for $\upsilon_x = \upsilon_y = 0.3$ and p=1.0 with approximate results obtained by quintic spline technique, Chebyshev collocation method, Frobinius method and finite element method and exact value reported in [1].

The results are presented in tables (2-4) and Figs. (2 & 3). It is found that the frequency parameter Ω for a C-S plate is greater than that for C-F plate while smaller than that for a C-C plate for the same set of values of plate parameters.

The tables (2-4) presents the values of frequency parameter for $E_2 / E_1$ = 0.5, $\upsilon_x$ = 0.2 and different values of non-homogeneity parameter $\mu$ = -0.5, 0.0, 0.5, density parameter $\beta$ = -0.5, 0.0, 1.0 foundation parameter $K$= 0.0, 0.02, taper parameter $\alpha$ = -0.5, 0.0, 0.5 and aspect ratio $a/b$= 0.5, 1.0 for C-C, C-S and C-F plates vibrating in first three modes of vibration. It is observed that the frequency parameter Ω decreases with increasing values of density parameter $\beta$ keeping other parameters fixed. The rate of decrease in frequency parameter Ω with $\beta$ increases with the increase in the values of non-homogeneity parameter$\mu$, foundation parameter $K$ and taper parameter $\alpha$. This rate of decrease for a C-S plate is higher than that for C-F plate but smaller than that for C-C plate. The rate of decrease in Ω with $\beta$ increases with increase in number of modes for all the plates. Further, it is found that frequency parameter Ω increases with increasing value of boundary conditions. The rate of increase in Ω with $\mu$ decreases in the order C-C, C-S and C-F non-homogeneity parameter $\mu$ for all the three plates respectively. This rate decreases with the increase in the values of foundation parameter $K$ or density parameter $\beta$ or both while increases with the increasing value of taper parameter $\alpha$ for all the three plates for first three modes. This rate of increase gets pronounced in higher modes. The frequency parameter Ω increases with the increasing values of the aspect ratio $a/b$ for C-C, C-S and C-F plates vibrating in first three modes of vibration. The rate of increase of frequency parameter Ω with $a/b$ in case of C-S plate is smaller than that for a C-F plate but higher than that for a C-C plate, irrespective of other plate parameters. However, when the plate is vibrating in the first mode due to the effect of elastic foundation this rate of increase does not follow order of the boundary conditions i.e. this rate of increase in C-F plate is smaller than that for C-S plate. This rate increases with the increase in the values of taper parameter $\alpha$, non-homogeneity parameter$\mu$, and foundation parameter $K$ while decreases with the increasing value of density parameter $\beta$ for all the three plates. This rate of increase increases with the increase in number of modes.

Fig. 2(a) depicts the behavior of frequency parameter Ω with taper parameter $\alpha$ for $a/b$= 1,

$\beta$ = -0.5, $K$ = 0.0, 0.02, and $\mu$ = -0.5, 0.5 for the first mode of vibration. The frequency parameter Ω is found to increase continuously with the increasing values of taper parameter $\alpha$ in the absence of elastic foundation ($K$=0.0) for all the three plates. However, in the presence of an elastic foundation, the frequency parameter Ω is found to increase with increasing values of $\alpha$ for C-C and C-S plates, but in case of C-F plate for $\mu$ = -0.5, the frequency parameter Ω decreases with the increasing values of taper parameter $\alpha$, while for $\mu$ = 0.5, it first decreases and then increases with a local minima in the vicinity of $\alpha$= 0.2. In particular, for a C-S plate for $K$=0.02 and $\mu$ = -0.5, the frequency parameter Ω first decreases and then increases, with a local minima in the vicinity of $\alpha$ = -0.4. In case of second mode of vibration, Fig. 2 (b), the frequency parameter Ω increases with increasing values of taper parameter α for all the boundary conditions. The rate of increase in frequency parameter Ω is found to increase with the increasing values of non-homogeneity parameter $\mu$ but it decreases with the increase in the value of foundation parameter $K$. As far as the behavior of the plate vibrating in the third mode is concerned,

Fig. 2 (c), it is same as for the second mode. The rate of increase of frequency parameter Ω with taper parameter $\alpha$ is higher in third mode as compared to the first two modes.

Figs. 3(a-c) show the plots of the frequency parameter Ω versus foundation parameter $K$ for taper parameter $\alpha$ = 0.5, non-homogeneity parameter $\mu$ = 0.5, density parameter $\beta$ = -0.5, 0.5 and aspect ratio $a/b$ =1.0. It is observed that the frequency parameter Ω increases with the increasing values of $K$ for all the three boundary conditions. The rate of increase in frequency parameter Ω with foundation parameter $K$ is higher in case of C-F plate as compared to C-S and C-C plates for the same set of values of other plate parameters. The rate of increase goes on decreasing with the increase in the order of modes.

Figs. 4(a-c) present the plots for normalized transverse displacements for a square plate i.e. $a/b$ = 1.0 and $K$ = 0.02, $\beta$ = -0.5, $\mu$ = -0.5, 0.5, $\alpha$ = -0.5, 0.5 for the first three mode of vibration for clamped, simply supported and free plate, respectively. The nodal lines are found to shift towards the edge $X$ = 1 as $\alpha$ increases from – 0.5 to 0.5 i.e. as the plate becomes thicker at outer edge. A similar pattern of nodal lines is seen for different values of $\beta$ and $K$.

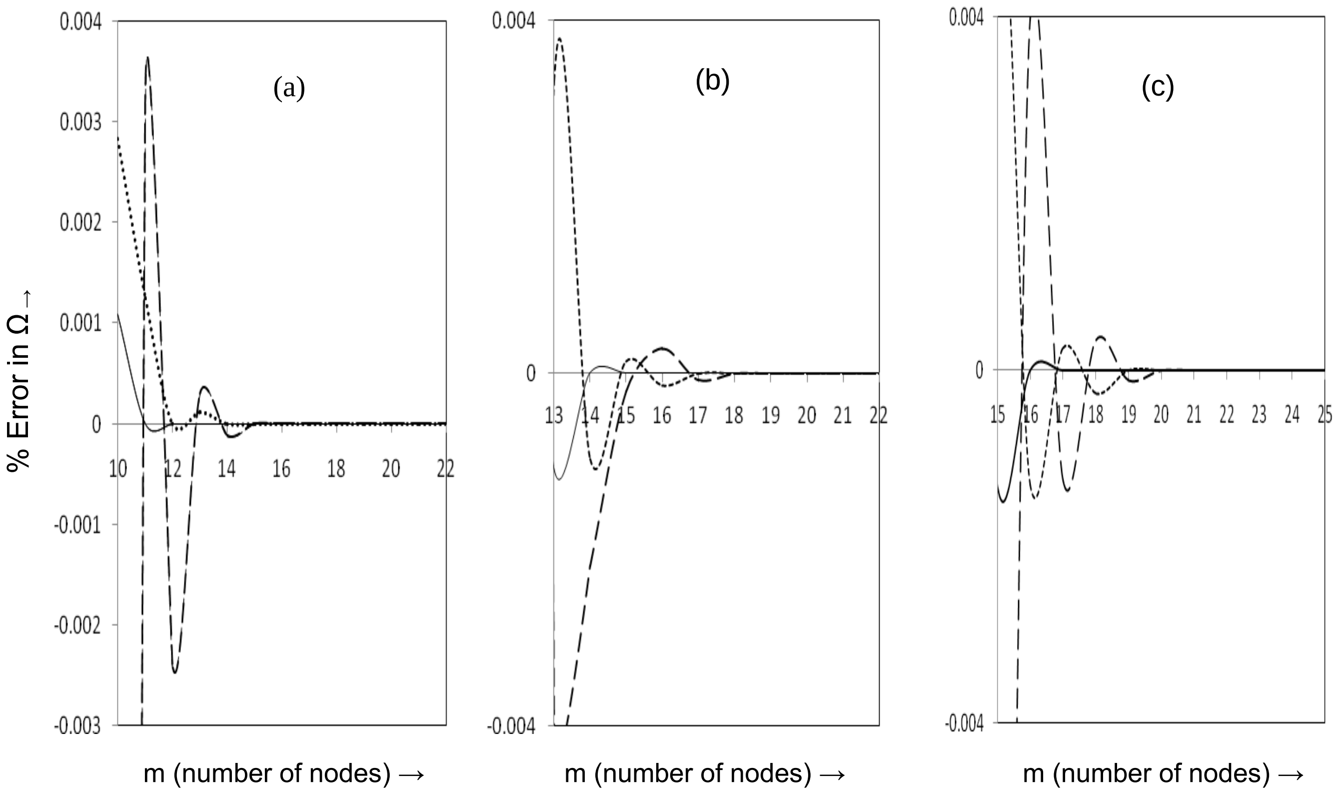


Fig.1. Percentage error in frequency parameter Ω; for (a) C-C, (b) C-S and (c) C-F plate, for *a/b*= 1.0, *K*= 0.02, *μ*= 0.5, *β*= 1.0, *α* = 1.0, ———, first mode,……..., second mode, – – – – – , third mode.
% error = $[(\Omega_m - \Omega_{20})/ \Omega_{20}] \times 100$.

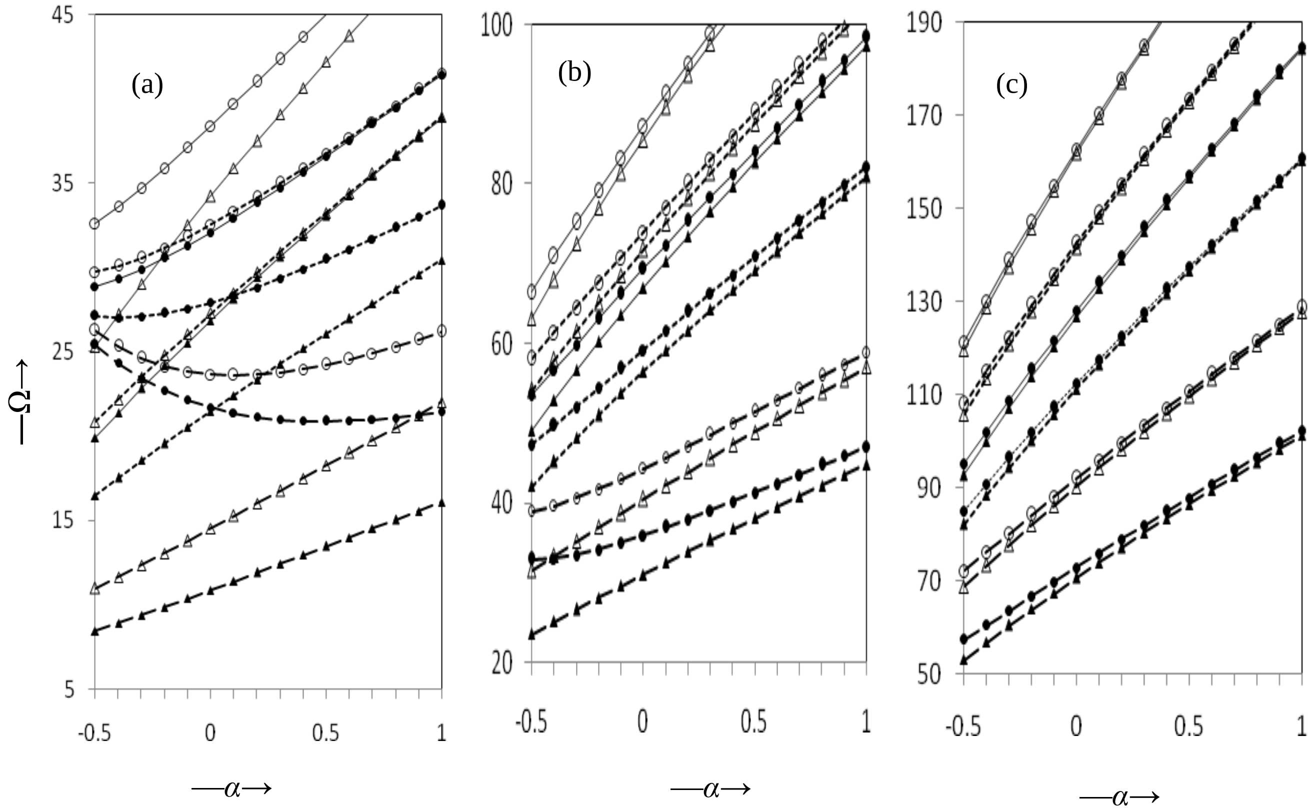


Fig.2. Natural frequencies of C-C, C-S and C-F plates: (a) first mode (b) second mode and (c) third mode, for *a/b*= 1.0,*β* = - 0.5.——— , C-C; ……, C-S; – – – , C-F; ▲,*μ* = - 0.5, *K*= 0.0; Δ,*μ* = 0.5, *K*= 0.0; ●, *μ* = -0.5, *K*= 0.02; ○, *μ* = 0.5, *K*= 0.02.

## VI. Conclusion

The effect of non-homogeneity, which is presumed to arise due to variation in Young's moduli and density on natural frequencies of rectangular orthotropic plates of linearly varying thickness resting on Winkler foundation has been studied on the basis of classical plate theory. It is observed that frequency parameter $\Omega$ increases with the increase in non-homogeneity parameter $\mu$, aspect ratio $a/b$, foundation parameter *K*, and other plate parameters being fixed. Further Ω is found to decrease with the increasing value of density parameter *β* keeping all other plate parameters fixed for all the three boundary conditions. However, the behavior with taper parameter *α* is not monotonous. The results will help design engineers to have desired natural frequency by a proper choice of plate parameters.

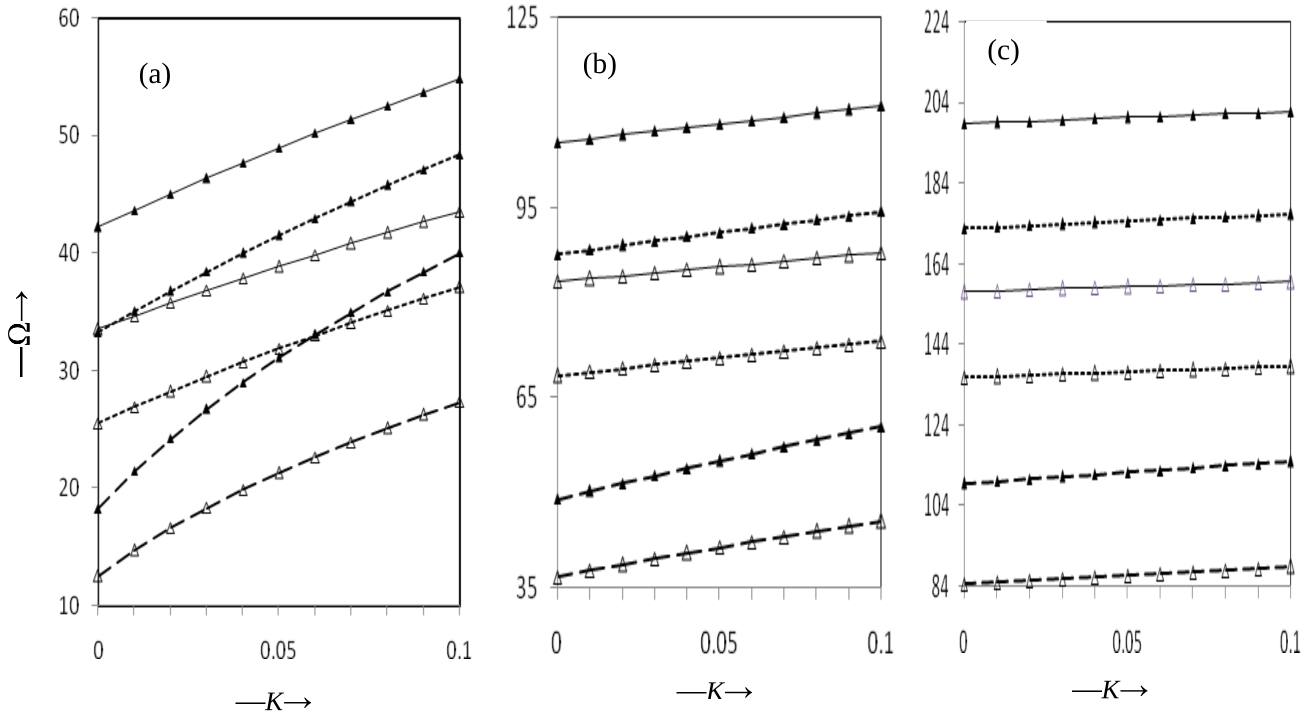


Fig. 3. Natural frequencies of C-C, C-S and C-F plates: (a) first mode (b) second mode and (c) third mode, for *α* = 0.5, *μ* = 0.5, *a/b* = 1.0. ——, C-C; ……, C-S; – – –, C-F; ▲, *β*= - 0.5; ∆, *β*= 0.5.0.

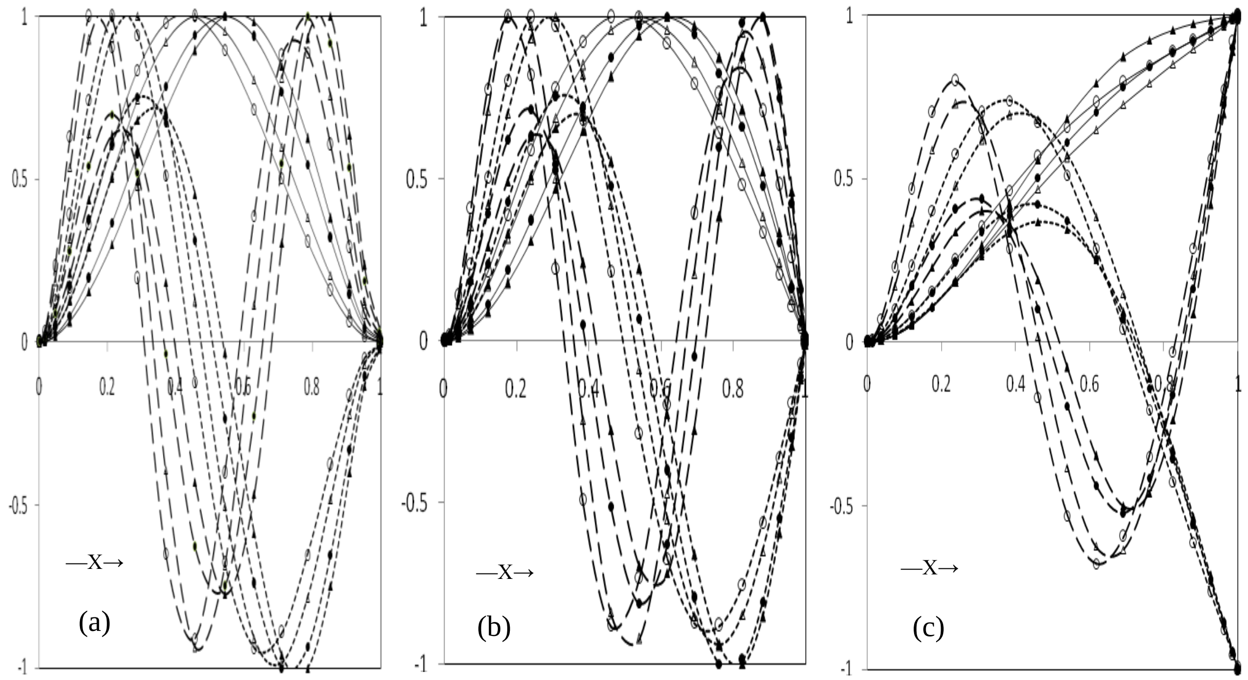


Fig. 4: Normal displacements: (a) C-C plate, (b) C-S plate, (c) C-F plate, for *a/b* = 1.0, *β* = -0.5, *K* = 0.02, ———, first mode; ……..... , second mode; – – – – ,third mode;
▲, *μ* = -0.5, *α* = -0.5; ∆, *μ* = -0.5, *α* = 0.5; ●, *μ* =0.5, *α* = -0.5; ○, *μ* = 0.5, *α* = 0.5.

**Table 1:** Comparison of frequency parameter Ω for isotropic ($E_2/E_1$=1), homogeneous ($\mu = \beta$ = 0), and uniform ($\alpha$ = 0) C-C, C-S and C-F plates for υ= 0.3.

| Boundary Conditions | | K=0.0 | | | | K=0.01 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **a/b** | 0.5 | | 1.0 | | 0.5 | | 1.0 | |
| | **Ref. /Mode** | I | II | I | II | I | II | I | II |
| C-C | Liessa [1] | — | — | 28.946 | 69.320 | — | — | — | — |
| | Lal et al.[13] | 23.816 | 63.635 | 28.951 | 69.327 | 26.214 | 64.472 | 30.954 | 70.187 |
| | Jain &Soni [17] | 23.816 | 63.535 | 28.951 | 69.327 | — | — | — | — |
| | Lal and Dhanpati [14] | 23.820 | 63.603 | 28.950 | 69.380 | 26.219 | 64.539 | 30.953 | 70.239 |
| | **Present** | **23.815** | **63.5345** | **28.950** | **69.3270** | **26.2142** | **64.472** | **30.954** | **70.1872** |
| C-S | Liessa [1] | — | — | 23.646 | 58.641 | — | — | — | — |
| | Lal et al.[13] | 17.332 | 52.098 | 23.646 | 58.646 | 20.503 | 53.237 | 26.060 | 59.661 |
| | Jain &Soni [17] | 17.332 | 52.097 | 23.646 | 58.646 | — | — | — | — |
| | Lal and Dhanpati [14] | 17.335 | 52.150 | 23.647 | 58.688 | 20.506 | 53.288 | 26.061 | 59.702 |
| | **Present** | **17.3318** | **52.0979** | **23.6363** | **58.6464** | **20.5034** | **53.2372** | **26.0605** | **59.6607** |
| C-F | Liessa [1] | — | — | 12.680 | — | — | — | — | — |
| | Lal et al.[13] | 5.704 | 24.944 | 12.687 | 33.065 | 12.351 | 27.243 | 16.762 | 34.839 |
| | Lal and Dhanpati [14] | 5.703 | 24.949 | 12.684 | 33.064 | 12.350 | 27.248 | 16.760 | 34.831 |
| | **Present** | **5.7039** | **24.9438** | **12.6874** | **33.0651** | **12.3505** | **27.3432** | **16.7621** | **348325** |

**Table 2:** Values of frequency parameter Ω for C-C plate, and $E_2/E_1$=0.5, $\nu_x$=0.2.

| $\beta$ | $\mu$ / α | *a/b* = 0.5 | | | | | | *a/b* =1 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *K*= 0.0 | | | *K* = 0.02 | | | *K*= 0.0 | | | *K* = 0.02 | | |
| | | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 |
| **Mode I** | | | | | | | | | | | | | |
| | -0.5 | 17.3309 | 19.5172 | 24.7181 | 27.1434 | 28.4703 | 32.0482 | 19.9152 | 22.4537 | 28.4674 | 28.8919 | 30.5771 | 35.0229 |
| -0.5 | 0.0 | 23.4124 | 26.4407 | 33.6694 | 29.2746 | 31.7139 | 37.8936 | 26.8656 | 30.3388 | 38.5883 | 32.1037 | 35.0284 | 42.3197 |
| | 0.5 | 28.7852 | 32.5612 | 41.6017 | 32.7875 | 36.1431 | 44.4538 | 33.0632 | 37.3736 | 47.6262 | 36.6002 | 40.5319 | 50.136 |
| | -0.5 | 15.1153 | 17.0844 | 21.7947 | 23.7312 | 24.9684 | 28.2893 | 17.356 | 19.6446 | 25.0995 | 25.2361 | 26.7982 | 30.911 |
| 0.0 | 0.0 | 20.6239 | 23.3762 | 29.9834 | 25.7943 | 28.0437 | 33.7492 | 23.663 | 26.8257 | 34.3836 | 28.2832 | 30.9777 | 37.7125 |
| | 0.5 | 25.506 | 28.9566 | 37.2652 | 29.0501 | 32.14 | 39.8184 | 29.3043 | 33.2526 | 42.7018 | 32.4368 | 36.0605 | 44.9504 |
| | -0.5 | 11.3747 | 12.9494 | 16.7595 | 17.8768 | 18.9398 | 21.7632 | 13.0328 | 14.8643 | 19.2852 | 18.968 | 20.2917 | 23.7602 |
| 1.0 | 0.0 | 15.831 | 18.0739 | 23.5215 | 19.7805 | 21.6661 | 26.4632 | 18.1473 | 20.7313 | 26.9859 | 21.671 | 23.9232 | 29.5857 |
| | 0.5 | 19.8083 | 22.6518 | 29.5803 | 22.5395 | 25.1236 | 31.5926 | 22.7538 | 26.0198 | 33.9372 | 25.1644 | 28.1978 | 35.7092 |
| **Mode II** | | | | | | | | | | | | | |
| | -0.5 | 46.2639 | 52.4264 | 66.998 | 50.9064 | 56.5038 | 70.1383 | 50.9064 | 55.8272 | 71.3772 | 53.6331 | 59.6725 | 74.3333 |
| -0.5 | 0.0 | 63.0322 | 71.2619 | 90.6526 | 65.4474 | 73.3937 | 92.3171 | 65.4474 | 75.8394 | 96.4661 | 69.3526 | 77.8461 | 98.0321 |
| | 0.5 | 77.748 | 87.7788 | 111.3597 | 79.3217 | 89.1741 | 112.4603 | 79.3217 | 93.4399 | 118.491 | 84.2518 | 94.7518 | 119.5259 |
| | -0.5 | 40.4219 | 45.9476 | 59.0832 | 44.4182 | 49.4697 | 61.8145 | 44.4182 | 48.9297 | 62.945 | 46.8057 | 52.251 | 65.5158 |
| 0.0 | 0.0 | 55.5401 | 62.9853 | 80.6179 | 57.6603 | 64.8626 | 82.0929 | 57.6603 | 67.0307 | 85.7853 | 61.1021 | 68.7977 | 87.1729 |
| | 0.5 | 68.8485 | 77.9711 | 99.525 | 70.2453 | 79.2132 | 100.5108 | 70.2453 | 82.9972 | 105.8938 | 74.6098 | 84.1652 | 106.8208 |
| | -0.5 | 30.5756 | 34.9684 | 45.5228 | 33.579 | 37.6325 | 47.6156 | 33.579 | 37.2411 | 48.5006 | 35.3891 | 39.7534 | 50.4703 |
| 1.0 | 0.0 | 42.7258 | 48.752 | 63.173 | 44.3809 | 50.2264 | 64.3457 | 44.3809 | 51.8846 | 67.2204 | 47.0295 | 53.2726 | 68.3238 |
| | 0.5 | 53.4914 | 60.9541 | 78.7685 | 54.605 | 61.9505 | 79.5688 | 54.605 | 64.8824 | 83.8027 | 57.9954 | 65.8196 | 84.5556 |
| **Mode III** | | | | | | | | | | | | | |
| | -0.5 | 89.5352 | 101.7175 | 130.4754 | 92.0379 | 103.897 | 132.1348 | 92.0379 | 105.352 | 135.1572 | 95.1448 | 107.4571 | 136.7597 |
| -0.5 | 0.0 | 122.3857 | 138.4249 | 176.0137 | 123.6488 | 139.5365 | 176.8792 | 123.6488 | 143.3354 | 182.2635 | 127.9443 | 144.4092 | 183.0996 |
| | 0.5 | 151.1668 | 170.5338 | 215.7212 | 151.9832 | 171.2571 | 216.2922 | 151.9832 | 176.6086 | 223.3991 | 157.3412 | 177.3072 | 223.9505 |
| | -0.5 | 78.2837 | 89.1905 | 115.0683 | 80.427 | 91.0625 | 116.5021 | 80.427 | 92.382 | 119.1976 | 83.151 | 94.1902 | 120.5822 |
| 0.0 | 0.0 | 107.8505 | 122.3346 | 156.4457 | 108.9575 | 123.3116 | 157.2108 | 108.9575 | 126.6728 | 161.9918 | 112.7443 | 127.6166 | 162.7309 |
| | 0.5 | 133.831 | 151.4103 | 192.6247 | 134.5563 | 152.0547 | 193.1363 | 134.5563 | 156.7965 | 199.4628 | 139.2962 | 157.4188 | 199.9568 |
| | -0.5 | 59.3352 | 67.9879 | 88.7267 | 60.9454 | 69.4027 | 89.8234 | 60.9454 | 70.4323 | 91.9179 | 63.0234 | 71.7991 | 92.9769 |
| 1.0 | 0.0 | 83.0386 | 94.7304 | 122.5393 | 83.9096 | 95.5036 | 123.1517 | 83.9096 | 98.0942 | 126.8777 | 86.8321 | 98.8411 | 127.4693 |
| | 0.5 | 103.9964 | 118.3343 | 152.2789 | 104.5823 | 118.8579 | 152.6992 | 104.5823 | 122.5406 | 157.666 | 108.267 | 123.0462 | 158.0719 |

**Table 3:** Values of frequency parameter Ω for C-S plate, and $E_2/E_1$=0.5, $\nu_x$=0.2.

| $\beta$ | $\mu$ / α | *a/b* = 0.5 | | | | | | *a/b* =1.0 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *K*= 0.0 | | | *K* = 0.02 | | | *K*= 0.0 | | | *K* = 0.02 | | |
| | | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 |
| **Mode I** | | | | | | | | | | | | | |
| | -0.5 | 13.6268 | 15.2319 | 18.9755 | 25.4862 | 26.2645 | 28.382 | 16.4709 | 18.5367 | 23.4154 | 27.1344 | 28.3122 | 31.4955 |
| -0.5 | 0.0 | 17.2496 | 19.2682 | 23.9503 | 24.8254 | 26.2316 | 29.7773 | 21.4938 | 24.2308 | 30.7459 | 27.9292 | 30.0491 | 35.4494 |
| | 0.5 | 20.4138 | 22.7904 | 28.3062 | 25.787 | 27.7009 | 32.382 | 26.0871 | 29.4576 | 37.5406 | 30.4725 | 33.3979 | 40.6971 |
| | -0.5 | 11.7294 | 13.1545 | 16.4948 | 22.0268 | 22.7546 | 24.7197 | 14.1728 | 16.0093 | 20.372 | 23.4337 | 24.5215 | 27.4494 |
| 0.0 | 0.0 | 14.9556 | 16.7577 | 20.9572 | 21.533 | 22.8215 | 26.0616 | 18.6538 | 21.1049 | 26.9734 | 24.2479 | 26.1805 | 31.1057 |
| | 0.5 | 17.7759 | 19.9046 | 24.867 | 22.4517 | 24.1907 | 28.4457 | 22.759 | 25.7914 | 33.1062 | 26.5817 | 29.2385 | 35.8876 |
| | -0.5 | 8.5911 | 9.6967 | 12.3141 | 16.1629 | 16.7969 | 18.4696 | 10.3658 | 11.7918 | 15.2196 | 17.1671 | 18.0842 | 20.5222 |
| 1.0 | 0.0 | 11.1092 | 12.523 | 15.8487 | 15.97 | 17.0331 | 19.6929 | 13.8697 | 15.8013 | 20.4799 | 18.004 | 19.5794 | 23.6004 |
| | 0.5 | 13.3151 | 14.9964 | 18.9504 | 16.7919 | 18.2033 | 21.6602 | 17.0925 | 19.5041 | 25.3894 | 19.9362 | 22.0866 | 27.5028 |
| **Mode II** | | | | | | | | | | | | | |
| | -0.5 | 39.1374 | 44.2335 | 56.1974 | 44.7119 | 49.1615 | 60.0388 | 42.3099 | 47.8931 | 61.0523 | . 475116 | 52.4791 | 64.6098 |
| -0.5 | 0.0 | 51.9901 | 58.5442 | 73.8128 | 54.9271 | 61.1518 | 75.873 | 56.5469 | 63.804 | 80.8236 | 59.2605 | 66.2073 | 82.7125 |
| | 0.5 | 63.3044 | 71.1352 | 89.2931 | 65.2327 | 72.8546 | 90.6657 | 69.1836 | 77.9288 | 98.3895 | 70.9528 | 79.502 | 99.6375 |
| | -0.5 | 34.0191 | 38.564 | 49.2905 | 38.7664 | 42.7753 | 52.5951 | 36.7745 | 41.7494 | 53.533 | 41.2051 | 45.6684 | 56.5926 |
| 0.0 | 0.0 | 45.5707 | 51.4702 | 65.2857 | 48.132 | 53.7512 | 67.0986 | 49.5453 | 56.0655 | 71.4275 | 51.9109 | 58.1665 | 73.0882 |
| | 0.5 | 55.7655 | 62.8541 | 79.376 | 57.469 | 64.3777 | 80.5998 | 60.9041 | 68.7992 | 87.3514 | 62.4661 | 70.1922 | 88.4629 |
| | -0.5 | 25.4799 | 29.0563 | 37.5876 | 29.0017 | 32.2003 | 40.0861 | 27.5408 | 31.4504 | 40.8038 | 30.8277 | 34.3758 | 43.116 |
| 1.0 | 0.0 | 34.7122 | 39.4439 | 50.6429 | 36.6989 | 41.2243 | 52.0759 | 37.7145 | 42.9271 | 55.3247 | 39.5484 | 44.5661 | 56.6362 |
| | 0.5 | 42.9067 | 48.6581 | 62.2071 | 44.2597 | 49.8763 | 63.1987 | 46.8065 | 53.1817 | 68.2971 | 48.0467 | 54.2951 | 69.1969 |
| **Mode III** | | | | | | | | | | | | | |
| | -0.5 | 78.8503 | 89.4641 | 114.4294 | 81.7353 | 91.9831 | 116.3568 | 82.2011 | 93.3139 | 119.4958 | 84.9721 | 95.7321 | 121.3443 |
| -0.5 | 0.0 | 106.3604 | 120.0538 | 151.9469 | 107.8201 | 121.3416 | 152.955 | 111.098 | 125.4917 | 159.105 | 112.4969 | 126.725 | 160.0691 |
| | 0.5 | 130.5309 | 146.8937 | 184.7772 | 131.4769 | 147.7341 | 185.4445 | 136.5639 | 153.8172 | 193.9029 | 137.4685 | 154.6201 | 194.5391 |
| | -0.5 | 68.7561 | 78.2316 | 100.6326 | 71.2113 | 80.3813 | 102.2867 | 71.6761 | 81.5916 | 105.067 | 74.0343 | 83.6551 | 106.6531 |
| 0.0 | 0.0 | 93.4877 | 105.8241 | 134.7008 | 94.7626 | 106.9521 | 135.5887 | 97.6289 | 110.5828 | 140.9762 | 98.8504 | 111.6627 | 141.8248 |
| | 0.5 | 115.2736 | 130.0957 | 164.5832 | 116.1122 | 130.8428 | 165.1798 | 120.5551 | 136.1617 | 172.5873 | 121.3567 | 136.8752 | 173.1558 |
| | -0.5 | 51.8541 | 59.3328 | 77.1941 | 53.6859 | 60.9459 | 78.4498 | 54.0584 | 61.8781 | 80.5741 | 55.8178 | 63.4264 | 81.7777 |
| 1.0 | 0.0 | 71.6443 | 81.5622 | 105.0138 | 72.645 | 82.4528 | 105.7231 | 74.7929 | 85.1893 | 109.8164 | 75.7514 | 86.0417 | 110.494 |
| | 0.5 | 89.176 | 101.2244 | 129.544 | 89.8529 | 101.8311 | 130.0343 | 93.206 | 105.8619 | 135.6786 | 93.8529 | 106.4411 | 136.1455 |

**Table 4:** Values of frequency parameter Ω for C-F plate, and $E_2/E_1$=0.5, $\nu_x$=0.2.

| *B* | μ / α | *a/b* = 0.5 | | | | | | *a/b* = 1 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *K* = 0.0 | | | *K* =0.02 | | | *K* = 0.0 | | | *K* =0.02 | | |
| | | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 | -0.5 | 0.0 | 1.0 |
| **Mode I** | | | | | | | | | | | | | |
| | -0.5 | 5.3919 | 5.8943 | 7.0633 | 24.4483 | 24.6092 | 24.9236 | 8.4598 | 9.6288 | 12.5369 | 25.4182 | 25.7938 | 26.8625 |
| -0.5 | 0.0 | 5.8396 | 6.4679 | 8.0511 | 19.7418 | 19.9129 | 20.4308 | 10.8824 | 12.56 | 16.8466 | 21.7072 | 22.553 | 25.1075 |
| | 0.5 | 6.5232 | 7.3219 | 9.3921 | 17.2672 | 17.5784 | 18.5241 | 13.4733 | 15.6712 | 21.3255 | 20.8866 | 22.3584 | 26.6118 |
| | -0.5 | 4.4151 | 4.8358 | 5.8167 | 20.4668 | 20.5507 | 20.7604 | 6.9348 | 7.9146 | 10.365 | 21.1826 | 21.4819 | 22.3972 |
| 0.0 | 0.0 | 4.7872 | 5.3111 | 6.6339 | 16.2147 | 16.377 | 16.8525 | 8.9587 | 10.3697 | 13.9981 | 17.8958 | 18.6422 | 20.8794 |
| | 0.5 | 5.3548 | 6.0201 | 7.7495 | 14.168 | 14.4477 | 15.2801 | 11.1299 | 12.9857 | 17.795 | 17.2478 | 18.5215 | 22.2014 |
| | -0.5 | 2.932 | 3.2222 | 3.9013 | 13.7329 | 13.8067 | 13.9983 | 4.6095 | 5.2852 | 6.9897 | 14.1882 | 14.433 | 15.1634 |
| 1.0 | 0.0 | 3.1854 | 3.5441 | 4.4529 | 10.7364 | 10.883 | 11.2783 | 5.997 | 6.9748 | 9.516 | 11.9335 | 12.4983 | 14.1611 |
| | 0.5 | 3.5712 | 4.026 | 5.2135 | 9.4026 | 9.6218 | 10.2488 | 7.4924 | 8.7864 | 12.1795 | 11.5666 | 12.4916 | 15.1605 |
| **Mode II** | | | | | | | | | | | | | |
| | -0.5 | 20.4511 | 22.8938 | 28.5271 | 30.9878 | 32.4706 | 36.3893 | 24.1824 | 27.3337 | 34.8319 | 33.4359 | 35.6482 | 41.4911 |
| -0.5 | 0.0 | 25.2801 | 28.2023 | 34.9767 | 31.0851 | 33.479 | 39.3146 | 31.3768 | 35.5017 | 45.4577 | 36.2187 | 39.8271 | 48.8836 |
| | 0.5 | 29.687 | 33.0912 | 41.0622 | 33.6359 | 36.6723 | 43.9947 | 38.1424 | 43.2345 | 55.6431 | 41.2926 | 46.0357 | 57.844 |
| | -0.5 | 17.4696 | 19.608 | 24.5619 | 26.0028 | 27.4148 | 31.0414 | 20.661 | 23.4124 | 29.9792 | 28.2081 | 30.2277 | 35.4781 |
| 0.0 | 0.0 | 21.7456 | 24.3208 | 30.3097 | 26.6996 | 28.8358 | 34.0394 | 26.9836 | 30.6019 | 39.3523 | 31.1145 | 34.2998 | 42.2919 |
| | 0.5 | 25.6368 | 28.6453 | 35.7046 | 29.0562 | 31.754 | 38.2623 | 32.9226 | 37.3994 | 48.3269 | 35.6502 | 39.8307 | 50.246 |
| | -0.5 | 12.6426 | 14.2655 | 18.0606 | 34.7737 | 19.8116 | 22.7255 | 14.9566 | 17.0378 | 22.041 | 20.2995 | 21.8931 | 26.0031 |
| 1.0 | 0.0 | 15.9556 | 17.9369 | 22.5769 | 19.6583 | 21.3304 | 25.4114 | 19.7988 | 22.5659 | 29.2954 | 22.8874 | 25.3481 | 31.5349 |
| | 0.5 | 18.9581 | 21.287 | 26.7798 | 21.5616 | 23.6684 | 28.7629 | 24.3442 | 27.7876 | 36.2364 | 26.4277 | 29.6589 | 37.7361 |
| **Mode III** | | | | | | | | | | | | | |
| | -0.5 | 49.4734 | 55.92 | 70.9768 | 54.1699 | 60.0601 | 74.1976 | 53.3063 | 60.4273 | 77.2296 | 57.7002 | 64.2906 | 80.2193 |
| -0.5 | 0.0 | 64.7572 | 72.799 | 91.4123 | 67.1622 | 74.9348 | 93.1038 | 70.7096 | 79.8452 | 101.3423 | 72.9259 | 81.8052 | 102.8797 |
| | 0.5 | 78.364 | 87.8425 | 109.6856 | 79.9347 | 89.2451 | 110.8099 | 86.3731 | 97.3583 | 123.2107 | 87.8021 | 98.6273 | 124.2143 |
| | -0.5 | 42.8419 | 48.5582 | 61.9767 | 46.7749 | 52.0349 | 64.6955 | 46.1322 | 52.4288 | 67.3466 | 49.8106 | 55.6705 | 69.8657 |
| 0.0 | 0.0 | 56.5474 | 63.7478 | 80.4925 | 58.6311 | 65.6032 | 81.9697 | 61.6484 | 69.7843 | 88.991 | 63.5652 | 71.4832 | 90.3294 |
| | 0.5 | 68.7629 | 77.297 | 97.0511 | 70.1467 | 78.5363 | 98.0502 | 75.6193 | 85.4386 | 108.6071 | 76.8758 | 86.5572 | 109.496 |
| | -0.5 | 31.892 | 36.3486 | 46.9181 | 62.1875 | 38.9105 | 48.9444 | 34.3062 | 39.1927 | 50.8692 | 36.9989 | 41.5782 | 52.742 |
| 1.0 | 0.0 | 42.8107 | 48.5375 | 61.9879 | 44.4305 | 49.9888 | 63.1578 | 46.5473 | 52.9587 | 68.2026 | 48.0344 | 54.2846 | 69.2592 |
| | 0.5 | 52.5743 | 59.4408 | 75.4866 | 53.6824 | 60.4398 | 76.3027 | 57.59 | 65.3911 | 83.9084 | 58.5946 | 66.2913 | 84.6328 |

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