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Free Oscillations of the Curvilinear Sections of Pipelines with the Fluid Flow

Safarov Ismail Ibrahimovich
Doctor of physical and mathematical sciences, Professor.,
safarov54@mail.ru;
(+998 93) 625-08-15

Akhmedov Maqsud Sharipovich
Researcher,
maqsud.axmedov.1985@mail.ru
(+99890) 612-01-02

Boltaev Zafar Ihterovich
Scientific - Researcher
(+99891) 401-01-51
Bukhara Technological- Institute of Engineering
Republic of Uzbekistan,
15 K. Murtazoyev Street.

***Abstract*—Are considered natural oscillations of the toroidal shell with stationary flow of an ideal fluid. The problem is reduced linear system of homogeneous algebraic equations for with complex coefficients. Frequency equations are solved by Muller. Numerical results obtained for a steel casing. It has been established that the greater the curvature of the pipe, the more rigid it becomes, and the thicker the pipe wall, the more rigid it is.**

Keywords—Toroidal shell, ideal fluid, algebraic equations whose curve with frequency equation, the method of Mueller.

Introduction

Studies of the natural oscillations of curvilinear sections of pipelines with a constant flow of fluid within the core of the theory were developed in the second half of the last century. One of the first works in this field is Article V.S.Ushakova [1], which was obtained by the equation of motion of a circular section of the pipeline and to investigate its own vibrations at a constant flow rate and internal pressure. The flow rate was considered low, which allowed dropping some small terms of the equation and making the decision to study oscillations of a circular rod. Further research in this area began to develop quite rapidly in [5,6,7,8,9,10,11,12,13,14,15,16,17,18].

Natural oscillations of a fluid flow to the curved sections of thin-walled large-

Diameter pipeline, which constitute one of the most complicated geometry types of membranes (toroidal shells), which are the most vulnerable section of the pipeline in operation, it was not possible to investigate (due to difficulties in determining the value of the hydrodynamic fluid pressure).

Statement of the problem and solution methods.

A curved portion viewed in a pipeline of large diameter thin-walled tube through which flows an ideal non-compressible fluid with a constant velocity $U = const$ and a constant hydrostatic pressure $p_0 = conts$. n addition, the pressure acting on the wall of the pipe arising from the pressure of the hydrodynamic fluid movement. The aim is to study frequencies and modes of flexural vibrations in the plane of curvature of the section of the pipeline as a thin toroidal shell taking into account the dynamic effect of the flowing fluid, internal pressure and deformation of the middle surface of the shell at considerable movements. Damping is considered small and is neglected. Viewed conduit portion is represented as a portion with a radius of of the toroidal shells R a longitudinal axis extending through the centers of gravity of its cross-sections. Cross-sections - circular with a radius of midline section r, shell thickness h. The ratio $\frac{h}{r}$ assumed to be small, so you can use the ratio of the theory of shells based on Kirchhoff-Love. The end section of the shell is pivotally secured believe. Inside the shell with a velocity $U = const$ ideal incompressible fluid flows with density $p_0 = conts$.

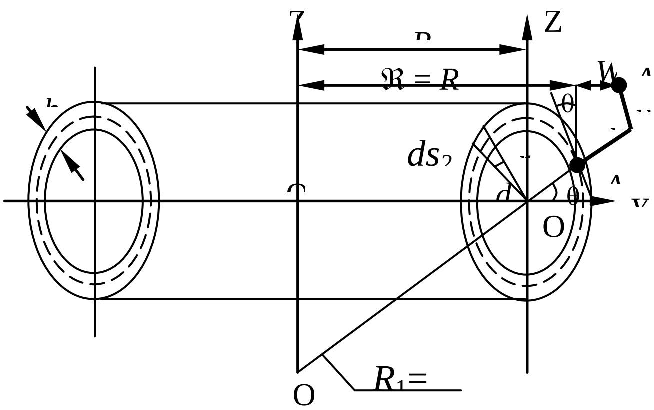

Fig. 1. The curved section of the pipeline in the toroidal coordinates

The geometry of the curved portion of the pipeline shown in Figure 1 a of the toroidal shell having a median toroidal curved surface coordinates β, θ, где β means the central angle of the torus, and θ - angle in cross section shell $(0 \le \theta \le 2\pi)$. If the longitudinal axis of the shell is half of a circle of radius R, as shown in Figure 1. the angle β varies within $0 \le \beta \le \pi$. When considering the middle surface in curvilinear coordinates β, θ differentials segments arcs coordinate lines ds_1 and ds_2 (see Figure 1) differentials associated with the coordinates themselves through the Lame parameters A_1 and A_2:

$ds_1 = (R + r\cos\theta)d\beta, \quad ds_2 = rd\theta,$ Therefore

$$A_1 = R + r\cos\theta, \quad A_2 = r \qquad (1)$$

Curvature of normal sections of the middle surface in the unstrained state at $R_1 = \frac{\Re}{\cos\theta}$ and $R_2 = r$ on Figure 1 is given by:

$$\frac{1}{R_1} = \frac{\cos\theta}{R + r\cos\theta}, \ \frac{1}{R_2} = \frac{1}{r}. \qquad (2)$$

Components moving point A median surface (Figure 1) to A^*, related to the radius r (i.e. dimensionless) and directed along the coordinate β, θ, y and the external normal to median surface are denoted by u, v, W_y, w. The rotation angle of the tangent to the center line of the cross-sectional contour, denoted by ϑ. When considering the deformation of the toroidal shell that occurs when bending vibrations in the plane of curvature assumptions used polubezmomentnoy V.Z.Vlasova shell theory [12] and strictly justified in A.L. Goldenveyzera [13] for a sufficiently long oblochek (whose length is much larger than the radius of the cross section). In toroidal coordinates these assumptions formulated in the following way:

- Elongation in the circumferential direction ε_2 is small compared with the relative movement of w and derivative $\frac{\partial v}{\partial\theta}$, i.e. from $\varepsilon_2 = w + \frac{\partial v}{\partial\theta}$ have $w + \frac{\partial v}{\partial\theta} = 0$

- The relative shift of median surface γ is small compared with the coordinate lines of the angles of rotation, i.e. from $\gamma = \frac{r}{R}\frac{\partial v}{\partial\beta} + \frac{\partial u}{\partial\theta}$ have $\frac{r}{R}\frac{\partial v}{\partial\beta} + \frac{\partial u}{\partial\theta} = 0$

- Rotation angle of the tangent to the mean cross-sectional contour defined by the expression $\vartheta = \frac{\partial w}{\partial\theta} - v$

- arising in the shell of force and deformation are related by:

$$M_1 = \nu D\chi_2; \qquad T_1 = Eh\varepsilon_1;$$
$$H_1 = H_2 = H = (1-\nu)D\tau;$$
$$M_2 = D\chi_2; \qquad \varepsilon_2 + \nu\varepsilon_1 = 0; \qquad (3)$$
$$S_1 = S_2 = S = \frac{Eh}{2(1+\nu)}\gamma;$$

- In all the equations of equilibrium shell element except fourth value can be lowered lateral forces Q_1 and twisting moments H.

This (3) denotes:

M_1 and M_2 - the bending moments, T_1 - longitudinal force, S - shearing force, E - modulus of elasticity the shell, ν - the Poisson coefficient, χ_2 - change of curvature, D - a cylindrical shell stiffness

$$D = \frac{Eh^3}{12(1-\nu^2)}.$$

All parameters specified by the index 1 correspond to coordinate lines β and index 2 lines θ. The equation of motion of bending vibrations of the toroidal shell (Figure 1) are introduced on the basis of general relations geometrically nonlinear theory of shells middle bend described in the monograph H.M. Mushtari and K.Z. Galimov [7]. This theory considers such a bend membranes, in which the maximum deflection (in this case - the radial displacement of the points, of the middle surface w) is the same order of magnitude of wall thickness, or even exceed it, but is small compared with other linear dimensions of the shell.

According to this theory, the force equilibrium equations for the moments element toroidal shell being in the deformed state, have the form (indices 1 and 2 refer to the toroidal coordinates β and θ respectively):

$$\frac{\partial}{\partial\beta}(A_2T_1) + \frac{\partial}{\partial\theta}(A_1S_2) + S_1\frac{\partial A_1}{\partial\theta} - T_2\frac{\partial A_2}{\partial\beta} +$$
$$A_1A_2\left(\frac{Q_1}{R_1^*} + \tau Q_2 + X_1\right) = 0,$$
$$\frac{\partial}{\partial\beta}(A_2Q_1) + \frac{\partial}{\partial\theta}(A_1Q_2) -$$
$$A_1A_2\left(\frac{T_1}{R_1^*} + \frac{T_2}{R_2^*} + S_1\tau + S_2\tau - X_3\right) = 0,$$
$$\frac{\partial}{\partial\beta}(A_2M_1) + \frac{\partial}{\partial\theta}(A_1H_2) - H_1\frac{\partial A_1}{\partial\theta} -$$
$$M_2\frac{\partial A_2}{\partial\beta} - A_1A_2Q_1 = 0,$$
$$\frac{\partial}{\partial\theta}(A_1M_2) + \frac{\partial}{\partial\beta}(A_2H_1) + H_2\frac{\partial A_2}{\partial\beta} -$$
$$M_1\frac{\partial A_1}{\partial\theta} - A_1A_2Q_2 = 0, \qquad (4)$$

where X_1, X_2, X_3- vector components of the external force. The first three equations (4) are the equations of equilibrium of forces, the last two - the equations of equilibrium points. Differential equations of equilibrium shell element (4) are non-linear, as contain pieces efforts and deformations. In addition, they obtained for shell being in the deformed state. Therefore, in these equations includes radii of curvature R_1^* and R_2^* deformed middle surface. Their connection to the curvature of the initial state (2) is expressed in accordance with [4] the following relations:

$$\frac{1}{R_1^*}=\frac{1}{R}\left(\cos\theta-\frac{r}{R}\frac{\partial^2 w}{\partial\beta^2}\right),\quad \frac{1}{R_2^*}=\frac{1}{r}\left(1-\frac{\partial\vartheta}{\partial\theta}\right).\quad (5)$$

Change the curvature of the cross section midline shell χ_2 and torsion τ expressed in terms of the angle of rotation ϑ the following relations:

$$\chi_2=-\frac{1}{r}\frac{\partial\vartheta}{\partial\theta},\ \tau=-\frac{1}{R}\frac{\partial\vartheta}{\partial\beta} \quad (6)$$

In accordance with the assumptions (3) - (5) shell theory V.Z. Vlasova [1] in the first three equilibrium equations (4), a transverse force ignore Q_1, a in the last two - torque H. As a result, after substitution in (4) parameters Lame (1) was obtained in accordance with the principle of d'Alembert system of equations of motion of the shell in the efforts:

$$\begin{aligned}
&\frac{r}{R}\frac{\partial T_1}{\partial\beta}+\frac{\partial S}{\partial\theta}+r\tau Q_2+rX_1^*=0,\\
&\frac{r}{R}\frac{\partial S}{\partial\beta}+\frac{\partial T_2}{\partial\theta}+\frac{r}{R}T_1\sin\theta+\frac{r}{R_2^*}Q_2+rX_2^*=0,\\
&\frac{r}{R_1^*}T_1+\frac{r}{R_2^*}T_2+2r\tau S-\frac{\partial Q_2}{\partial\theta}-rX_3^*=0,\\
&\frac{r}{R}\frac{\partial M_1}{\partial\beta}+\frac{\partial H}{\partial\theta}-rQ_1=0,\\
&\frac{\partial M_2}{\partial\theta}-rQ_2=0, \qquad (7)
\end{aligned}$$

Where X_1, X_2, X_3 - components of the inertial forces to the coordinates β,θ and normal to the median surface respectively. Out of the equation (7), all forces and moments, except T_1 and M_2, come to one equation of motion in the efforts:

$$\begin{aligned}
&\frac{r^2}{R^2}\frac{\partial^2 T_1}{\partial\beta^2}+\frac{r}{R}\frac{\partial}{\partial\beta}\left(\tau\frac{\partial M_2}{\partial\theta}\right)+\frac{\partial^2}{\partial\theta^2}\left(\frac{R_2^*}{R_1^*}T_1\right)-\\
&\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\left(R_2^*\frac{\partial^2 M_2}{\partial\theta_2}\right)- \qquad (8)\\
&-\frac{\partial}{\partial\theta}\left(\frac{r}{R}T_1\sin\theta\right)-\frac{\partial}{\partial\theta}\left(\frac{1}{R_2^*}\frac{\partial M_2}{\partial\theta}\right)+\\
&\frac{r^2}{R}\frac{\partial X_1^*}{\partial\beta}-r\frac{\partial X_2^*}{\partial\theta}-\frac{\partial^2}{\partial\theta^2}\left(R_2^*X_3^*\right)=0
\end{aligned}$$

To solve dynamic problems of this section of the pipeline is necessary to obtain the equation of motion in the toroidal shell displacements.

Therefore, we transform equation (8), expressing effort T_1 and M_2 and deformations ε_1 and τ in displacements, using the relations between the forces, strains and displacements on the membrane theory of shells (3), as well as expressions for the principal curvatures of the shell in the deformed state (5) and for the change of curvature χ_2 and torsion τ relations (8):

$$\begin{aligned}
&T_1=Eh\varepsilon_1=Eh\frac{r}{R}\left(\frac{\partial u}{\partial\beta}+W_y\right),\\
&M_2=D\chi_2=\frac{Eh^3}{12\left(1-\nu^2\right)}\left(-\frac{1}{r}\frac{\partial\vartheta}{\partial\theta}\right), \qquad (9)\\
&W_y=w\cos\theta-\nu\sin\theta,
\end{aligned}$$

where u,v,w - attributed to the radius r dimensionless displacement components, W_y - projection on the axis of movement of point *A* of the middle surface of the shell in position A^* as a result of the deformation of the contour (see Figure 1), ϑ - rotation angle of the tangent to the middle shell section line due to the deformation of the cross section.

Substituting relations (6), (7) and (9) into equation (12), neglecting the small non-linear terms, we obtain the governing equation of motion of the toroidal shell, expressed in terms of displacements

$$\begin{aligned}
&\frac{r^2}{R^2}\frac{\partial^3 u}{\partial\beta\partial\theta^2}\cos\theta+\frac{r^3}{R^3}\frac{\partial^3 u}{\partial\beta^3}-\frac{r^2}{R^2}\frac{\partial^3 u}{\partial\beta^3}-\\
&\frac{r^2}{R^2}\frac{\partial}{\partial\theta}\left(\frac{\partial u}{\partial\beta}\sin\theta\right)+\frac{r^3}{R^3}\frac{\partial^2 W_y}{\partial\beta^2}+\\
&+\frac{r^2}{R^2}\frac{\partial}{\partial\theta}\left[\frac{\partial}{\partial\theta}\left(W_y\cos\theta\right)-W_y\sin\theta\right]+ \qquad (10)\\
&\frac{h^2}{r^2 12\left(1-\nu^2\right)}\frac{\partial^3}{\partial\theta^3}\left(\frac{\partial^2\vartheta}{\partial\theta^2}+\vartheta\right)=\\
&=-\frac{r^2}{EhR}\frac{\partial X_1^*}{\partial\beta}+\frac{r}{Eh}\frac{\partial X_2^*}{\partial\theta}+\frac{r}{Eh}\frac{\partial X_2^*}{\partial\theta}+\\
&\frac{r}{Eh}\frac{\partial X_2^*}{\partial\theta}+\frac{r}{Eh}\frac{\partial}{\partial\theta}\left(\frac{\partial^2\vartheta}{\partial\theta^2}X_3^*+\frac{\partial X_3^*}{\partial\theta}\right),
\end{aligned}$$

Where X_i^* - components of the forces of inertia:

Tangential components of the coordinates β and θ

$$X_1^*=-rhp\frac{\partial^2 u}{\partial t^2},\ X_2^*=-rhp\frac{\partial^2 v}{\partial t^2}$$

Normal component (normal to the middle surface of the shell)

$$X_3^* = -rhp\frac{\partial^2 w}{\partial t^2} + p;$$

p - Internal pressure, including hydrodynamic, fluid motion occurs, - density of the shell material.

The equation of motion, of the toroidal shell (10) is a differential inhomogeneous partial differential equation with four unknowns, u, v, w, ϑ. Acceded to by three sex ratio of the membrane theory of shells

$$\frac{\partial v}{\partial \theta} + w = 0, \quad \frac{r}{R}\frac{\partial v}{\partial \beta} + \frac{\partial u}{\partial \theta} = 0, \quad \vartheta = \frac{\partial w}{\partial \theta} - v \quad (11)$$

Obtain a complete system of equations with four unknowns. At steady stream of liquid solution of equation (10), (11) to determine the frequency and form of the natural oscillations of the curved section of the pipeline.

Determination of the hydrodynamic pressure induced flow of liquid. One of the main factors determining the solution of dynamic problems for pipes with flowing liquid is the hydrodynamic pressure of the fluid in the pipe wall. In addressing these challenges within the core of the theory [5, 6, 7, 8] for straight and curved sections of the pipeline hydrodynamic pressure on the non-deformable wall of the pipe without difficulty determined by the well-known component of the velocity of the liquid. Dynamic problems for thin-walled large diameter pipes are solved on the basis of the theory of shells with deformed middle surface, and here, for the determination of the hydrodynamic pressure methods are used hydro- and aerodynamics.

In this paper we present a solution to the problem of determining the hydrodynamic pressure of the fluid on the wall of the curved section of the pipeline, obtained in [9] in the toroidal coordinates based on the theory of potential flow of an incompressible fluid.

The curved portion of the pipeline is seen as a toroidal shell with a radius line cross-section r, within which proceeds speed $U = const$ ideal incompressible fluid with a density $\rho_0 = const$. Region bounded by the toroidal completely filled with liquid, covered in toroidal coordinates α, β, θ where $0 \le \alpha \le r$ - radial coordinate in the plane of the cross section of the torus (see Figure 1), $0 \le \beta \le \beta_0$ и $-\pi \le \theta \le \pi$. Lame coefficients it $\alpha = const$ have the form [10]:

$$H_\alpha = H_\beta = \frac{c}{ch\alpha - \cos\beta}, \ H_\theta = \frac{csh\alpha}{ch\alpha - \cos\beta}, \ (12)$$

Where c - scale factor. The velocity field of an ideal incompressible fluid in the membrane fluctuations is rotational potential field with potential $\varphi = \varphi(\alpha, \beta, \theta, t)$. The system of basic equations of the potential flow of an ideal incompressible fluid includes [11]:

- The equation of continuity (Laplace) $\nabla^2 \varphi = 0$, (13)

- The equation of motion (Euler) $\frac{\partial \varphi}{\partial t} + Q(p) = 0,$ (14)

- equation of state $\rho_0 = const,$ (15)

Where $Q(p)$ - united in the fluid flow pressure function, determined at $\rho_0 = const$ equality

$$Q(p) = \frac{1}{\rho_0}(p - p_0), \quad (16)$$

Where p and p_0 - hydrodynamic and hydrostatic pressure, respectively.

(13) - (15) establishes a connection between the hydrodynamic pressure p and potential perturbed velocity φ:

$$p = p_0 - \rho_0\left(\frac{\partial \varphi}{\partial t} + \frac{U}{R}\frac{\partial \varphi}{\partial \beta}\right) \quad (17)$$

Considering the fluid flow velocity vector $\overline{U}$ in toroidal coordinates, we write expressions for its components on α, β, θ:

$$U_\alpha = \frac{1}{H_\alpha}\frac{\partial \varphi}{\partial \alpha}, \ U_\beta = \frac{1}{H_\beta}\frac{\partial \varphi}{\partial \beta}, \ U_\theta = \frac{1}{H_\theta}\frac{\partial \varphi}{\partial \theta} \quad (18)$$

For the components of the velocity vector U_α, directed along the normal to the surface of the deformed sheath, the condition of the surface of the smooth flow of the liquid stream [12]:

$$U_\alpha \Big|_{\alpha=r} = \frac{1}{H_a}\frac{\partial \varphi}{\partial \alpha}\Big|_{\alpha=r} = -r\left(\frac{\partial w}{\partial t} + \frac{U}{H_\beta}\frac{\partial w}{\partial \beta}\right)\Big|_{a=r}, \quad (19)$$

Where w - referred to the radius r dimensionless component of movement points of the middle surface of the shell.

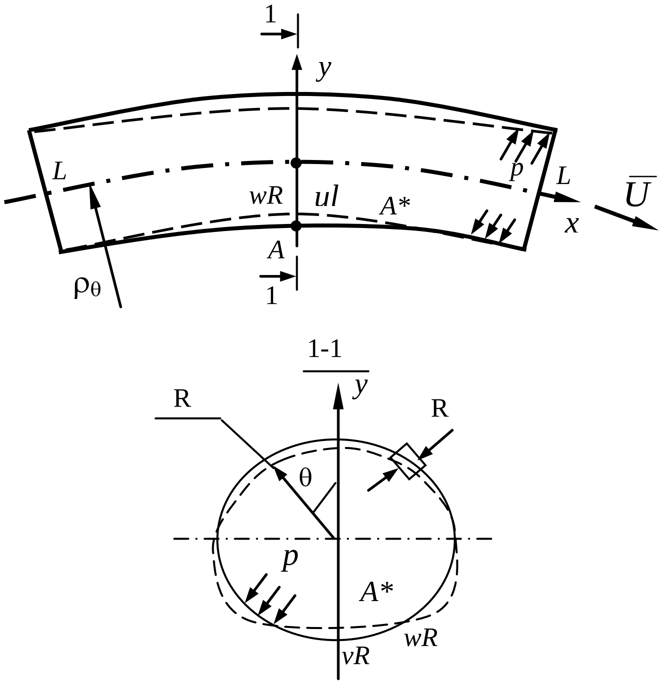

Fig. 2. The pipe with flowing liquid.

Thus, the problem of determining the hydrodynamic pressure of the fluid in the pipe wall is reduced to finding potential φ, satisfies the Laplace equation (13) and the conditions (18), (19) $\alpha = r$. The Laplace equation (17) into the toroidal coordinates system α, β and θ has the form

$$\frac{\partial}{\partial\alpha}\left(\frac{\sin\alpha}{ch\alpha - \cos\beta\partial\alpha}\right) + \frac{\partial}{\partial\beta}\left(\frac{\sin\alpha}{ch\alpha - \cos\beta}\frac{\partial\varphi}{\partial\beta}\right) + \frac{1}{(ch\alpha - \cos\beta)sh\alpha}\frac{\partial^2\varphi}{\partial\theta^2} = 0 \quad (20)$$

As a result of separation of variables after substitution $\varphi = (2ch\alpha - 2\cos\beta)^{1/2}\psi$ and presentation of unknown function $\psi(\alpha,\beta,\theta,t)$ in the form of:

$$\psi = A(\alpha)B(\beta)C(\theta)\Phi(t)$$

We obtain from (20) well-known equation of the torus:

$$A''_\alpha + \frac{ch\alpha}{sh\alpha}A'_\alpha - \left[\left(n - \frac{1}{2}\right)^2 + \frac{\mu^2}{sh^2\alpha}\right]A = 0 \quad (21)$$

Where $\mu = const, n = const$.

The general solution of the torus (21) is determined by a linear combination of the functions independent of the torus $P_{n-\frac{1}{2}}(ch\alpha)$ and $Q_{n-\frac{1}{2}}(ch\alpha)$, represents one of the kinds Legendre functions of the 1st and 2nd kind:

$$A(\alpha) = A_1 P_{n-\frac{1}{2}}(ch\alpha) + A_2 Q_{n-\frac{1}{2}}(ch\alpha) \quad (22)$$

Taking into account that the task is considered an area bounded by the surface of a torus coordinate α, the changing within the limits $0 \le \alpha \le r$, and that in $\alpha \to 0$ Legendre function of the 2nd kind $Q_{n-\frac{1}{2}}(chr) \to \infty$, in the solution (22) to be put $A_2 = 0$. Therefore, the solution of the torus (21) will be expressed only through the Legendre function of the 1st kind:

$$A(\alpha) = A_1 P_{n-\frac{1}{2}}(ch\alpha) \quad (23)$$

and the solution of Laplace's equation (20) with (21), (22) and (23) will have the form:

$$\varphi(\alpha,\beta,\theta,t) = (2ch\alpha - 2\cos\beta)^{1/2} P_{n-\frac{1}{2}}(ch\alpha)A_1\zeta(\beta,\theta,t) \quad (24)$$

Artwork $A_1\zeta(\beta,\theta,t)$ we find from (23) by taking the partial derivative $\left(\frac{\partial\varphi}{\partial\alpha}\right)$. Substituting then the value of this product in (24), we obtain an expression for the velocity potential:

$$\varphi = -\frac{rH_\alpha\left(\frac{\partial w}{\partial t} + \frac{U}{H_\beta}\frac{\partial w}{\partial\beta}\right)B^{1/2}P_{n-\frac{1}{2}}(chr)}{B^{-1/2}shrP_{n-\frac{1}{2}}(chr) - B^{1/2}P'_{n-\frac{1}{2}}(chr)} \quad (25)$$

Where $B = 2(chr - \cos\beta)$. The hydrodynamic pressure of the flowing fluid on the wall of the shell is found from (17) and neglecting small 2nd order arising in the calculation of a private function φ by β:

$$p = p_0 - p_0 rH_\alpha\Phi_n\left[\frac{\partial^2 w}{\partial t^2} + \left(\frac{1}{R} + \frac{1}{H_\beta}\right)\frac{\partial^2 w}{\partial\beta\partial t}U + \frac{U^2}{RH_\beta}\frac{\partial^2 w}{\partial\beta^2}\right], \quad (26)$$

Where indicated

$$\Phi_n = -\left(\frac{shr}{B} + \frac{P'_{n-\frac{1}{2}}(chr)}{P_{n-\frac{1}{2}}(chr)}\right)^{-1} \quad (27)$$

In the formula (26) for the hydrodynamic pressure on the bracketed expression analogy cylindrical [13,14] to be considered given acceleration (based on the velocity U) элемента оболочки, and the value of $p_0\Phi_n$, depending on the density p_0, regarded as a connected body of liquid

$$p = p_0 + p_{ж},$$

$$p_{ж} = -p_0 r^2 \Phi_n^* \left(\frac{\partial^2 w}{\partial t^2} + \frac{U^2}{Rr} \frac{\partial^2 w}{\partial \beta^2} \right),$$

$$\Phi_n^* = -\left(\frac{1}{2} + \frac{P'_{n-\frac{1}{2}}(chr)}{P_{n-\frac{1}{2}}(chr)} \right)^{-1}, \qquad (28)$$

where $P_{n-\frac{1}{2}}(chr)$ and $P'_{n-\frac{1}{2}}(chr)$ - Legendre function of the first kind and its first derivative. To find the parameter Φ_n^* formula (28) is not necessary to calculate the Legendre functions and their derivatives, as this ratio formula contains a derivative of the function the Legendre function itself. From the directory for special functions [16]:

$$\left(\frac{P'_{n-\frac{1}{2}}(chr)}{P_{n-\frac{1}{2}}(chr)} \right) = -\left(n + \frac{1}{2} \right) \frac{1}{chr} = -\left(n + \frac{1}{2} \right) \frac{r}{R} \qquad (29)$$

Where $n = 1,2,3...$ - wave numbers, define the shape of the oscillations. From formulas (28) and (29) we see that the value of Φ_n^* determining the hydrodynamic pressure of fluid flow on the wall of the shell. Hydrodynamic pressure increases as the curvature $\frac{r}{R}$, but within the accepted assumption that $\frac{r}{R} \le \frac{1}{10}$. To solve the system of equations (10) and (11) represent the bending vibrations occurring during the normal component of the toroidal shell move $w(\beta, \theta, t)$ as satisfying the boundary conditions at the edges of the shell:

$$w \Big|_{\substack{\beta=0 \\ \beta=\pi}} = 0, \qquad \frac{\partial^2 w}{\partial \beta^2} \Big|_{\substack{\beta=0 \\ \beta=\pi}} = 0 \ . \qquad (30)$$

As well as satisfying the conditions for cycling circumferential coordinate θ:

$$w(\beta, \theta, t) = f(t) a_m \cos m\theta \sin n\beta \qquad (31)$$

Where $f(t)$- time function t, $a_m = const$, m, n-wave numbers, defining the envelope waveform in the circumferential and longitudinal directions, respectively.

From the relations (31) between the components move at a value for (30) we obtain expressions for the other components of the displacement and rotation angle:

$$u = -\frac{r}{R} \frac{n}{m^2} f(t) a_m \cos m\theta \cos n\beta,$$
$$v = -\frac{1}{m} f(t) a_m \sin m\theta \sin n\beta, \qquad (32)$$
$$\vartheta = -\frac{m^2 - 1}{m} f(t) a_m \sin m\theta \sin n\beta,$$
$$W_\alpha = \frac{1}{2}\left(a_{m+1} \frac{m+2}{m+1} + a_{m-1} \frac{m-2}{m-1} \right) \cos m\theta \sin n\beta.$$

Substituting expression (31), (32) for moving the component and the rotation angle in the equation of motion of the shell (10) and calculating the partial derivatives with β and θ, obtain the governing equation for the unknown amplitude values a_m and comprising a function of time $f(t)$ and the second time derivative $f''(t)$:

$$f''(t) \left(\begin{array}{l} \frac{r^4}{ER^2} p b_m \frac{n^2}{m^3} \sin m\theta + \frac{r^2}{E} p b_m \frac{1}{m} \sin m\theta + \\ \frac{r^2}{Eh} p_0 \Phi_n^* b_m \sin m\theta \end{array} \right) =$$
$$f(t) \left\{ \begin{array}{l} -\frac{r^4}{R^4} \frac{n^4}{m^3} a_m \sin m\theta + \\ \frac{r^3}{2R^3} \frac{n^2}{m} \end{array} \right. a_m (\sin(m-1)\theta + \sin(m+1)\theta) +$$
$$+ \frac{r^3}{2R^3} \frac{n^2}{m} \left(a_{m+1} \frac{m+2}{m+1} + b_{m-1} \frac{m-2}{m-1} \right) \sin m\theta - \qquad (33)$$
$$\frac{r^2}{R^2} \left(a_{m+1} \frac{m+2}{m+1} + b_{m-1} \frac{m-2}{m-1} \right) \times$$
$$\times \left(\frac{m-2}{4} \sin(m-1)\theta + \frac{m+2}{4} \sin(m+1) \right) \theta -$$
$$-\left(h_p{}^2 \left[m(m^2 - 1) \times (m^2 - 1) + \frac{r^3 12(1 - \nu^2)}{Eh^3} p_0 \right] \sin m\theta + \right.$$
$$\left. \frac{r^3}{Eh} p_0 b_m \Phi_n^* \frac{U^2}{Rr} n^2 m \right) \sin m\theta \}$$

To simplify the form of equation (33) we introduce the dimensionless parameter shell thickness h_ν :

$$h_\nu = \frac{h}{r c_\nu}, \quad c_\nu = \sqrt{12(1 - \nu^2)}, \qquad \text{where} \qquad \nu \text{ -}$$

Poisson's ratio.

Divide each term of the equation (33) to h_ν^2. As a result, We obtain:

$$f''(t) \Omega_{m1} - f(t) \Omega_{m2} = 0 \qquad (34)$$

Where

$$\Omega_{m1} = \left\{ -\frac{r^4}{R^4 h_\nu^2} \cdot \frac{n^4}{m^3} a_m \sin m\theta + \frac{r^3}{2R^3 h_\nu^2} \frac{n^2}{m} \cdot \right.$$
$$\cdot \begin{bmatrix} a_m(\sin(m-1)\theta + \sin(m+1)\theta) + \\ \left(a_{m+1}\frac{m+2}{m+1} + a_{m-1}\frac{m-2}{m-1}\right)\sin m\theta \end{bmatrix} -$$
$$-\frac{r^2}{2R^2 h_\nu^2}\left(a_{m+1}\frac{m+2}{m+1} + a_{m-1}\frac{m-2}{m-1}\right) \times$$
$$((m-2)\sin(m-1)\theta + (m+2)\sin(m+1)\theta) - m(m^2-1) \cdot$$
$$\cdot \left(m^2 - 1 + \frac{r}{Ehh_\nu^2} p_0\right) \times b_m \sin m\theta +$$
$$\frac{r}{R}\frac{r}{Ehh_\nu^2} p_0 \Phi_n^* U^2 m n^2 b_m \sin m\theta;$$
$$\Omega_{2m} = \left[\frac{r}{Ehh_\nu^2} p\left(\frac{r^2}{R^2}\frac{n^2}{m^3} + m + \frac{1}{m}\right) + r^2 \frac{r}{Ehh_\nu^2} p_0 \Phi_n^* m\right] a_m \sin m\theta$$

To simplify (34), we introduce the following notation:

$$\rho^* = \frac{r}{Ehh_\nu^2} p, \quad \rho_0^* = \frac{r}{Ehh_\nu^2} p_0,$$
$$p_0^* = \frac{r}{Ehh_\nu^2} p_0$$

In addition, equation (34) using the curvature parameter toroidal shell μ , adopted in shell theory [7, 13], which characterizes not only the geometry of the shell, but also its material t. k. includes a Poisson coefficient:

$$\mu = \frac{r^2}{Rh} c_\nu, \qquad c_\nu^2 = 12(1-\nu^2).$$

Entering a marked transformation in equation (34) and assuming that the proper bending vibrations occur toroidal shell harmonically with the circular frequency ω , i.e.

$$f(t) = d_m \sin \omega t, \quad f'(t) = d_m \omega \cos \omega t,$$
$$f''(t) = -\omega^2 d_m \sin \omega t$$

Obtain:

$$-\frac{1}{4}\mu^2\left(a_{m+1}\frac{m+2}{m+1} + b_{m-1}\frac{m-2}{m-1}\right)((m+2)\sin(m+1)\theta) -$$
$$m(m^2-1)(m^2 - 1 + p_0^*) a_m \sin m\theta +$$
$$+ \mu h_\nu p_0^* \Phi_n^* u^2 m n^2 a_m \sin m\theta \} +$$
$$\sin \omega t \left[\frac{rhp^*}{m}(m^2-1) + r^2 p_0^* \Phi_n^* m\right] \omega^2 d_m \sin m\theta - -d_m \sin \omega t \times$$
$$\times \left\{ -\mu^4 h_\nu^2 \frac{n^4}{m^3} a_m \sin m\theta + \frac{1}{2}\mu^3 h_\nu \frac{n^2}{m} \cdot \begin{bmatrix} a_m(\sin(m-1)\theta + \sin(m+1)\theta) + \\ \left(a_{m+1}\frac{m+2}{m+1} + b_{m-1}\frac{m-2}{m-1}\right)\sin m\theta \end{bmatrix} \right. = 0$$

(35)

Here we have that in the penultimate term of equation (34) the value of discarded $\frac{r^2}{R^2}$, small compared with the wave number $m = 1,2,3....$

Dynamic equations of motion of the toroidal shell with a stationary fluid flow (35) obtained on the basis of a geometrically nonlinear variant theory of shells and the theory of potential flow of an ideal incompressible fluid is a homogeneous equation. All the members of which have coefficients of the trigonometric functions $\sin m\theta$, $m = 1,2,3...$, describing the deformation of the cross-sections of the shell when bending vibrations. At $m = 1$ fluctuations in the shell occur during deformation cross-sectional contours are displaced in the process of oscillation as hard to aim. Therefore, the waveforms do not affect the internal pressure p_0, as a member of the equation (35), containing the pressure vanishes at $m = 1$. All other waveforms ($m = 2,3,4...$), is connected with the deformation contour of the cross section (see Figure 1) and pressure. Just natural frequencies and mode shapes depend on the physical - mechanical properties of the material and liquid membranes. Natural frequencies of the curved portion of the pipeline for all forms of a shell oscillations is determined by equating the coefficients of like trigonometric functions $\sin m\theta$ at $m = 1,2,3...$ The resulting system of homogeneous linear algebraic equations can be written in compact form:

$$\lfloor c_{ij} \rfloor \{d\} = 0 \quad (36)$$

Where $m = 1,2,3,...;$ $m - 1 > 0;$ $m - 2 > 0$, and the coefficients $c_{i,j}$ determined by the expressions:

$$c_{m,m} = A_{mn} - B_{mn} - C_{mn}\omega^2, \quad c_{m,m\pm1} =$$
$$-\frac{m^2(3m\pm2)}{2(m\pm1)}\mu^3 h_\nu n^2, \; c_{m,m\pm2} = \frac{(m\pm3)(m\mp1)}{4(m\pm2)}\mu^2 m^3,$$
$$A_{mn} = \mu^4 h_\nu^2 n^4 + m^4(m^2-1)(m^2 - 1 + p_0^*) +$$
$$0{,}5\mu^2 m^2(m^2+1),$$
$$B_{mn} = p_0^* \Phi_n^* \mu h_\nu U^2 m^4 n^2,$$
$$C_{mn} = p_0^* r^2 \Phi_n^* m^4 + p^* rhm^2(m^2+1)$$

Posed the problem of determining natural frequencies of the curved portion of the pipeline with the flowing liquid is reduced to an eigenvalue problem of the coefficient matrix of the system of homogeneous linear algebraic equations (36).

The numerical results.

Investigation of frequencies of natural bending vibrations curvilinear sections of pipelines (steel) with a stationary fluid flow. In line with the speed of the water flowing from and 0 to $50\frac{M}{c}$. The results obtained allowed us to estimate the effect of flow rate on the frequency of the first four waveforms ($m = 1,2,3,4$ at $n = 1,2,3$). The calculations were performed for the curved pipe with the relative values $\frac{h}{r} = \frac{1}{30}, \frac{1}{40}, \frac{1}{60}$ and different curvatures $\frac{r}{R}=\frac{1}{10},\frac{1}{20}$, that match the curvature $\mu=5{,}8$; 11,6 and 23,1. These parameters, in turn, correspond to the following values of the coefficients of curvature bends and twists pipelines: $\lambda=0{,}57; 0{,}28$ and 0, 14. Modulus of elasticity of steel, which made the pipe, it is assumed $E=2\cdot10^5\,МПа$, Poisson coefficient $\nu=0{,}3$.

Results of calculations are shown in Tables 1-2 and the graphs ris.3-4 which shows the frequencies of own flexural oscillation ω_{mn} the curved sections of the steel pipe according to the velocity of the flowing fluid at different thicknesses of the shell.

The flow velocity u, varies in the range of real velocity of the liquid flowing in the pipes (to $25\frac{м}{c}$), has little effect on the natural frequency of the curved sections of steel pipe for all investigated a shell vibration modes ($m=1,2,3$,4 при $n=1,2,3$). The oscillation frequency ω_{mn} decreasing the flow rate increases from 0 to 25 $\frac{м}{c}$ not more than 10%.

Table 1. Natural frequency depending on the velocity of the flowing fluid

$\frac{r}{R}=\frac{1}{20}$, $\frac{h}{r}=\frac{1}{60}$, $\mu=23$		ω_{mn} (Hz) at a flow rate of fluid flowing in $\frac{m}{s}$		
Mode shape	Frequencies	$u=0$	$u=20$	$u=40$
$m=1$	ω_{11}	26,46	21,01	17,25
	ω_{12}	21,01	20,45	17,74
	ω_{13}	22,92	22,72	20,55
$m=2$	ω_{21}	13,39	12,83	10,42
	ω_{22}	16,67	15,82	12,51
	ω_{23}	18,68	18,44	16,2
$m=3$	ω_{31}	13,02	12,29	9,28
	ω_{32}	16,43	15,61	12,32
	ω_{33}	18,34	18,17	15,83
$m=4$	ω_{41}	19,47	19,33	14,21
	ω_{42}	20,12	20,06	12,97
	ω_{43}	21,36	21,22	11,38

For each of the above sections of the pipeline largest natural oscillation frequencies are the first form ω_{1n} at $m=1$. In the absence of deformation of the contour of the cross sections of the pipe – i.e., a pipe varies as the beam pipe section. At $m=2$ curvature parameter section of the pipeline μ and $\frac{h}{r}$ significantly affects the natural frequencies.

The smaller the curvature of the pipe and thinner than its wall, the lower are its natural frequency ω_{mn} virtually all forms of. **Table 2. The natural frequencies depending on the velocity of the flowing fluid**

$\frac{r}{R}=\frac{1}{20}$, $\frac{h}{r}=\frac{1}{60}$, $\mu=11{,}5$		ω_{mn} (Hz) at a flow rate of fluid flowing in $\frac{м}{c}$		
Mode shape	Frequencies	$u=0$	$u=20$	$u=40$
$m=1$	ω_{11}	55,34	53,47	51,13
	ω_{12}	56,05	55,27	52,39
	ω_{13}	60,56	59,99	57,18
$m=2$	ω_{21}	36,26	34,09	28,57
	ω_{22}	44,60	43,78	40,35
	ω_{23}	51,67	50,52	47,06
$m=3$	ω_{31}	35,02	33,01	26,51
	ω_{32}	43,11	43,50	39,22
	ω_{33}	50,03	49,63	46,58
$m=4$	ω_{41}	53,01	50,31	47,44
	ω_{42}	54,95	52,05	48,50
	ω_{43}	55,82	53,92	49,99

For the dynamic analysis of the pipeline are the most important envelope waveform ($m=2$ and 3). With increasing curvature of the pipeline, then there is a relationship $\frac{r}{R}$, at constant relative thickness ($\frac{h}{r}=const$) frequencies ω_{mn} own flexural vibrations increase. Also, by increasing the relative thickness of ($\frac{h}{r}$, at a constant curvature of the tube) the natural frequencies of flexural vibrations increases.

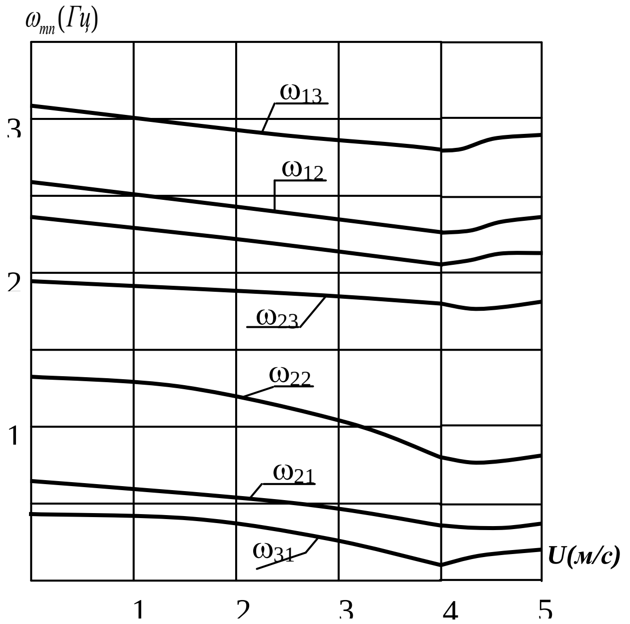

Figure 3. Changing the Eigen frequencies of bending vibrations of the speed of the flowing fluid (h=0.001).

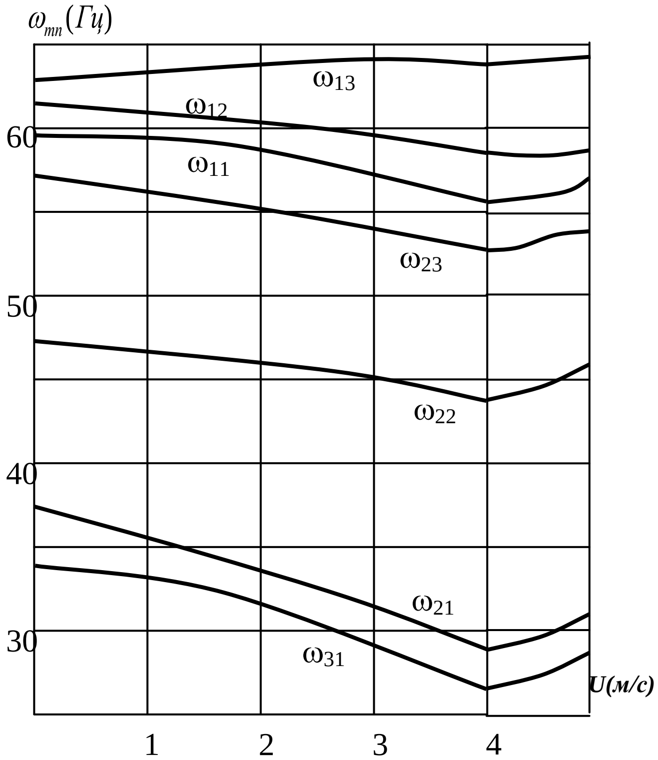

Figure 4. Changing the Eigen frequencies of bending vibrations of the speed of the flowing fluid (h=0.005).

Thus, the greater the curvature of the pipe, the more rigid it becomes, and the thicker the pipe wall, the more rigid it is.

Conclusions.

1. Based on the theory of nonlinear shells for the task, the technique of determining the natural frequencies of the thin-walled curved portions of large diameter pipe under the influence of internal hydrostatic pressure and the hydrostatic pressure caused by the fluid motion. Application of this method in dynamic calculations of pipelines to avoid any dangerous resonance phenomenon.

2. The system of differential equations of motion and curvilinear toroidal shell based on a variational principle and system of differential equations of motion of the toroidal shell reduced to a Mathieu equation.

3. The flow velocity u, varies in the range of real velocity of the liquid flowing in the pipeline (to $25\frac{м}{c}$), has little effect on the natural frequency of the curved sections of steel pipe for all investigated a shell vibration modes ($m=1,2,3,4$ при $n=1,2,3$). the oscillation frequency ω_{mn} decreasing the flow rate increases from 0 to $25\frac{м}{c}$ not more than 10%. for each of the above sections of the pipeline largest natural oscillation frequencies are the first form ω_{1n} at m=1. Thus, the greater the curvature of the pipe, the more rigid it becomes, and the thicker the pipe wall, the more rigid it is.

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