

# Finite Element Modelling of Flexible Pavement

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**Abstract—** In this paper, finite element modeling of flexible pavement is discussed and general suggestions on using this method to determine structural criteria of flexible pavement is given. The paper suggests using linear elastic modeling in determination of flexible pavement expected fatigue and rutting damage life due to the low stress and strain levels caused by traffic loading. The paper also suggests the use of falling weight deflectometer in obtaining elastic moduli of different flexible pavement layers rather than coring, triaxial test, or California bearing ratio tests. The paper also discusses the different types of finite element modeling used to model the structural behavior of the pavement layers under traffic loadings. Furthermore, the tyre-pavement contact area, tyre pressure, primary response parameters and load equivalency factors were also discussed in this paper.

**Keywords—** *Falling weight deflectometer, finite element analysis, flexible pavements, structural behaviour, traffic loading.*

## 1. INTRODUCTION

Finite element method (FEM) is one of the powerful techniques used in various engineering fields. The aim of this method is to develop a numerical solution for complex engineering problems. It divided the complex structure into tiny small parts, then it analysis the small parts and by summing the solution of the small parts the general approximate solution of the complex structure can be obtained. The most attractive part of this method is its ability to solve very complex engineering problems especially with the aid of computer software. The computer software is required to solve the resulted big matrixes during the analysis using this method.

Although, it is not easy to define the exact time when this method was firstly introduced, it is well known that this method was introduced to solve the complex elastic and structural problems in civil and aeronautical engineering. Its development can be traced back to china in the early 1940s by the work of A. Hrennikoff [1] and R. Courant[2]. At that time, finite

element method was called the matrix theory in structural analysis and several literature starts to discuss this method and to improve it. Although, this method was great in solving complex problems, it ended up with matrixes of hundreds of rows and columns. Although, this kind of matrixes is solvable in theory, it not possible to be solved without the aid of computer. In 1960s the electronic computers were introduced and soon many researchers start to develop computer programs that are able to solve engineering problems using the finite element method. This program was developed more in 1980s, where a graphical representations of the problems were introduced and this decreases the human errors in formatting the finite element problems. Since then until now, programs are developing more and more to solve more complex engineering problems with higher accuracy.

It is well-known that flexible pavements are complex engineering structures due to the various layers, materials and boundary conditions they contain. Thus, this paper suggests a proper methodology of using finite element method to analyse flexible pavements and to acquire the data needed to perform finite element analysis.

## 2. INTRODUCTION FOR PAVEMENT ANALYSIS MODELS

Flexible pavement is a complicated structure that is made of different materials with different properties. Some of these materials behave in an elastic manner, some viscoelastic and some have a plastic behaviour. That is why a perfect mathematical model is almost impossible to achieve.

The structural analysis of the pavement has been developed significantly in the recent years. It started by using the linear elastic modelling that has been developed by Boussinesq [3]. The linear elastic model is a simple analytical model in which the material is assumed to be isotropic and homogenous. In this model, the horizontal, vertical, normal and shear stresses can be computed by assuming that the material is completely elastic with a linear relationship between stress and strain. In this model the material can be described completely by using two elastic

parameters such as the elastic modulus and Poisson's ratio.

After that, the Boussinesq model has been improved by the work of Burmister (1945). Burmister extended Boussinesq's theory from a single layer to a multilayer elastic model [4]. Burmister introduced the use of the multilayer elastic theory to analyse pavement structures. His model represented the different layers with different elasticity modulus and Poisson's ratio. He started his solution for two layered pavement structure and then extended his solution to three layered pavement structure [5].

However, the linear elastic models have some weaknesses; for example, it is a poor model to represent the real stress and strain behaviour, except at a low stress level and for small strain value. That was the reason for developing a lot of nonlinear models during the last forty years. Some of these models were experimentally based and some were based on the theoretical principles. Some of these models were more accurate than the others [6].

One of these nonlinear models is the model developed by Duncan and Chang (1970). Their nonlinear elastic model is based on hooks law to related strain to stress increments, and based on the Coulomb failure criterion [7]. This model was then more enhanced by Rodríguez-Roa [8]. Rodríguez-Roa showed that the model could predict the granular soil behaviour with sufficient accuracy in conditions when the soil mass was not about to fail [8].

After that, more advanced models called the elastoplastic models were developed. These models could predict the behaviour of the soil at high stress levels more precisely. Furthermore, it could describe the relation between the shear and volumetric strains in the soil more accurately.

In this paper, it is suggested that, the materials to be assumed to have an elastic behaviour. This model is sufficient since the values of stresses and strains that to be calculated at normal flexible pavement design and traffic is very small and as mentioned earlier, this model can predict the soil behaviour for small stress levels and small strain values.

### 3. ELASTIC MODULUS DETERMINATION

The traditional methods to calculate the modulus of elasticity for different pavement layers are as follows:

- 1- Plate loading test.
- 2- Triaxial compression test.
- 3- California bearing ratio (CBR) test.

The determination of the modulus of elasticity using the plate loading test and triaxial compression test has the following disadvantages:

- 1- The process can be time consuming especially when there are many samples to be examined.
- 2- The process may involve significant expenses.
- 3- Most importantly, there is a limitation on the numbers of the samples that can be taken from the pavement [6].

The determination of the elasticity modulus using the CBR test is one of the traditional methods. There are many equations developed to relate the modulus of elasticity to the CBR values.

Some of these equations are as follows:

$$E = 10 \times \text{CBR} \% \quad [9]$$

$$E = 10 \times \text{CBR}\%^{0.73} \quad [10]$$

$$E = 17.6 \times \text{CBR}\%^{0.64} \quad [11]$$

Where E stands for the modulus in MPa.

Regardless of these studies, it has been proven that there is no relation between the CBR value and the modulus of elasticity of the material. Moreover, the CBR test is not suitable for granular materials with large particle size. Therefore, using the CBR test value to predict the modulus of elasticity value can lead to significant errors [6].

Due to the above mentioned reasons, it is suggested that the falling weight deflectometer is used to determine the modulus of elasticity of the different pavement layers. The Falling Weight Deflectometer (FWD) is a non-destructive testing device used usually for pavement evolution including the evaluation of the pavement layer modulus of elasticity (Figure 1). The FWD is designed to transmit a loading pulse to pavement by dropping a certain weight on the surface of the pavement. The load pulse creates a certain deflection on the pavement structure. The deflection is collected using deflection sensors installed on the FWD. By using a method called backcalculation method, the modulus of elasticity for the pavement layers can be measured.

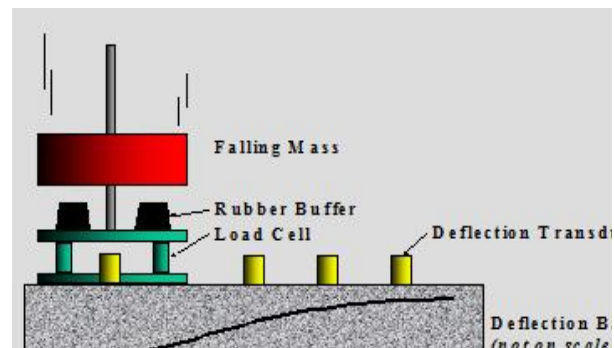


Figure 1: Falling Weight Deflectometer (FWD) (Dynatest)

### 4. FINITE ELEMENT METHOD

Each pavement layer in the finite element analysis is considered to be a solid continuum. The solid continuum domain is divided into sub domains. Each sub domain is divided into a number of finite

elements. These finite elements assemble to the whole problem during the analysis. These elements are connected by common nodes at their common ends. The analysis gives an approximate solution of the problem for different boundary conditions and under various loading types using the energy or stiffness formulation [12].

The finite element models have been used intensively in pavement analysis and design. There are three types of models which have been used for the multi-layered pavement structure analysis and design as follows:

- 1- Three-dimensional (3-D).
- 2- Plane strain (2-D).
- 3- Axisymmetric [12].

The axisymmetric model is used in determining the load equivalency factor of single axle with single tire since it is adequate and reduces the analysis time. However, for single axle with dual tires and multi-axes plane strain and 3-D finite element analysis can be used.

The finite element analysis has been confirmed to be suitable for the applications of complex pavement problems. Even though the use of the finite elements three-dimensional models can solve all the problems that can be solved using the two-dimensional models, the three dimensional models are very expensive to be used in terms of computational time and data preparation [13]. Thus, the two dimensional model was used in this study for single axle with single and dual tires.

For solving the finite element problem, a stiffness matrix needs to be derived. For the stiffness matrix derivation, there are three factors that should be considered: the geometry of elements, the material properties of elements, and the degrees of freedom allowed for the nodes to move. The solution gives the stresses and strains at the integration points, and displacements at the nodal periods [14].

#### 5. TYRE-PAVEMENT CONTACT AREA

One of the important parameters for pavement analysis is the tyre-pavement contact area. Tyre-Pavement contact area is one of the complex issues about pavement analysis, since the contact area is affected by the type of tyre. There are a lot of studies investigated the effects of different representation of tyre-pavement contact area [15-26]. Unfortunately, there are a lot of inconsistencies in the data collected by different researchers who studied the pressure distribution between the pavement and tyre [27].

However, the contact area can be assumed to be in the shape of two half circles with a rectangular area between them as shown in Figure 2. This shape which consists of a rectangular and two half circles can be simplified as recommended by Huang (2004). The simplified shape will have an equivalent area equals to  $0.5227 L^2$  and a width of  $0.6 L$  [28]. This assumption was taken for the plain strain model. For

the axisymmetric model this assumption cannot be used since the shape of this contact area is not axisymmetric. Since some of the models that were used in this study are axisymmetric, a circular contact area having an equivalent area of  $0.5227 L^2$  is used, and rectangular area with  $0.5227 L^2$  are suggested and used for the 3-D finite element analysis. This assumption has been used before for the multilayer elastic theory. Although it is not completely correct, the error incurred is believed to be small [28].

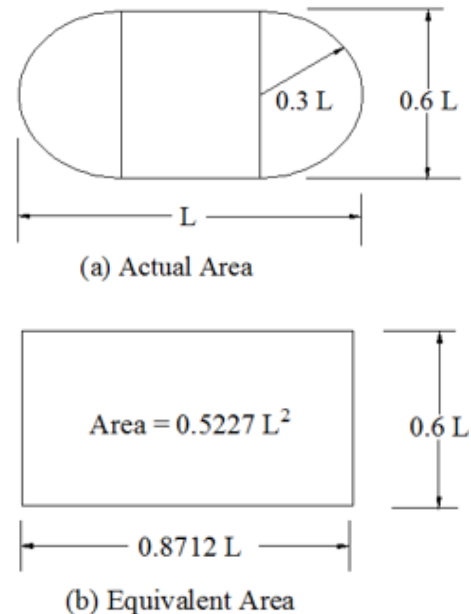


Figure 2: Tyre-Pavement Contact Area [28]

#### 6. TYRE PRESSURE

Tyre pressure has a significant effect on the stresses developed in the pavement, specially the top layers such as surfacing and base layers [29]. It is worth mentioning that truck tyre pressure has increased in 1995 to nearly 750 kPa from approximately 555 kPa in the 1960s [30]. The main reason for this increment is the development of the tyre construction technology. Tyres become thicker and stronger, which allow them to sustain higher inflation pressure and increase their load carrying capacity.

The existing methods for pavement design generally use tyre inflation pressure of 520 kPa. Some of the mechanistic-empirical design methods also verified the applicability of using 700 kPa inflation pressure for pavement design [29].

#### 7. PRIMARY RESPONSE PARAMETERS

The flexible pavement is a layered structure that consists of at least three layers: at the top is the asphalt layer, under the asphalt layer is the aggregate base layer, and lastly is the compacted natural soil layer. Figure 3 shows the stress distribution in the pavement structure under the axle load in usual conditions. Moving wheels which rapidly shifts from tension to compression creates stress. This shifting

between the tension to compression at the top and bottom of the pavement structure causes fatigue cracking to develop. According to the elastic theory, the maximum tensile strain will be developed at the bottom of the asphalt layer [27]. Hence, most of the pavement design models concentrate on the tensile strain at the bottom of the asphalt layer to predict the performance of the asphalt layer under the fatigue criteria. In addition to that, rutting is caused by the cumulative vertical compressive strain on the top of the pavement materials as well as on the subgrade layer [31]. These two parameters are the most important parameters that are needed to calculate load equivalency factor under fatigue and rutting criteria.

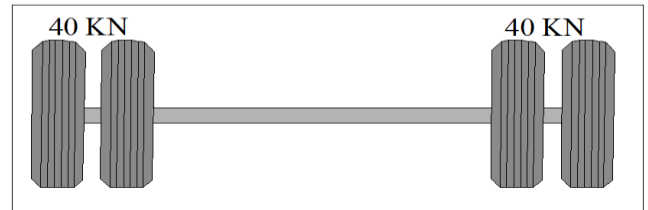


Figure 4 Standard Axle Load

After completing the finite element models of the pavement structure, it is then possible to determine the load equivalency factor based on both the fatigue and rutting criteria. This is determined by using maximum tensile strain at the underside of the asphalt layer under the standard single axle load, maximum tensile strain at the underside of asphalt layer for arbitrary load, maximum vertical compressive strain at the top of subgrade under the standard single axle load, and the maximum vertical compressive strain at the top of subgrade for arbitrary load, as shown in equations 3 and 4.

**Load equivalency factors based on fatigue criteria**

The ratio of  $N_{fs}$  to  $N_{fa}$  with the same material is the Equivalent Axle Load Factor (EALF) on the basis of fatigue failure [35] is as follows:

$$EALF = \left( \frac{N_{fs}}{N_{fa}} \right) = \frac{[f1 (\epsilon_{ts})^{-f2} (E)^{-f3}]}{[f1 (\epsilon_{ta})^{-f2} (E)^{-f3}]} = \left( \frac{\epsilon_{ts}}{\epsilon_{ta}} \right)^{-f2} \quad (3)$$

$N_{fs}$  = number of repetitions to failure of standard load  
 $N_{fa}$  = number of repetitions to failure of arbitrary load  
 $\epsilon_{ts}$  = maximum tensile strain at the underside of asphalt layer under the standard single axle load

$\epsilon_{ta}$  = maximum tensile strain at the underside of asphalt layer under arbitrary load

$f1, f2, f3$  = Constants equal to 0.0796, 3.291 and 0.854.

**Load equivalency factors based on rutting criteria**

The ratio of  $N_{fs}$  to  $N_{fa}$  with the same material is the EALF on the basis of the rutting failure [35] as follows:

$$EALF = \left( \frac{N_{fs}}{N_{fa}} \right) = \frac{[f4 (\epsilon_{cs})^{-f5}]}{[f4 (\epsilon_{ca})^{-f5}]} = \left( \frac{\epsilon_{cs}}{\epsilon_{ca}} \right)^{-f5} \quad (4)$$

$N_{fs}$  = number of repetitions to failure of standard load.  
 $N_{fa}$  = number of repetitions to failure of arbitrary load.  
 $\epsilon_{cs}$  = maximum compressive strain under the standard single axle load.

$\epsilon_{ca}$  = maximum compressive strain under arbitrary load.

$f4, f5$  = Constants equal to 1.365E-9, 4.477.

9. SUMMARY:

This paper discussed the use of finite element analysis for flexible pavement and design.

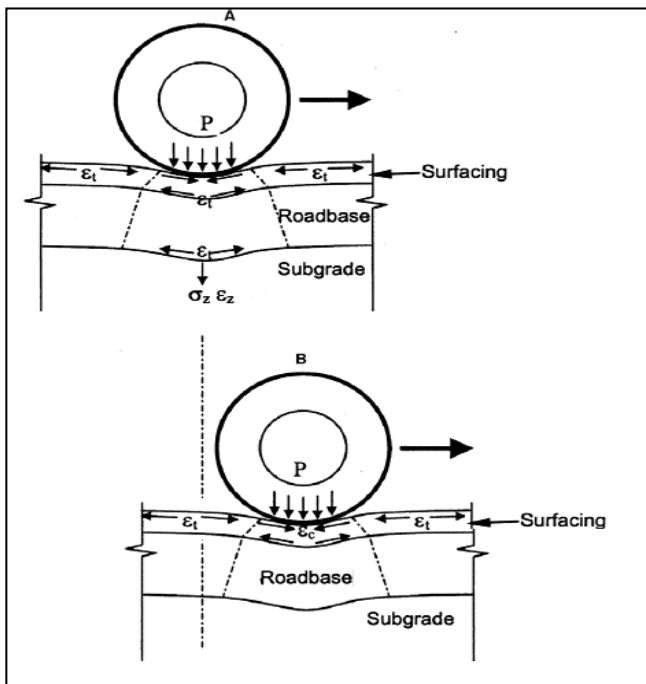


Figure 3: Stress Distribution in Pavement Structure Flexible pavement response affected by uniform load P; successive wheel positions A and B cause cyclic loading that result in compressive stress  $\sigma_x$  and strain  $\epsilon_x$  on subgrade, and tensile strain  $\epsilon_t$  and compressive strain  $\epsilon_c$  in pavement [32]

8. LOAD EQUIVALENCY FACTORS

Pavement design is based on traffic loading, which is determined by the number of repetitions of the standard axle load, generally 18-kip (80 kN) single axle load applied on the pavement by two sets of dual tyres as shown in Figure 4.

This is usually known as the equivalent single axle load (ESAL) [33]. One way of describing the damage caused by a certain axle load is by using the axle load equivalence factor. The load equivalency factor is described as the damage per pass to the pavement structure by a certain axle load in question relative to a standard axle load [34].

1- The paper suggests the use of linear elastic modelling for normal pavement structure with normal traffic loading.

2- The paper suggests the use of axisymmetric modelling for single axle with single tyre confirmation, while plain strain and three-dimensional modelling for dual and tandem axles.

3- The paper also suggests the use of falling weight deflectometer rather than coring, tri-axle test, or California bearing ratio tests in determination of elasticity moduli of pavement layers.

4- The paper highlights the different methods of representing tyre pavement contacts area.

5- The paper highlights the main response parameters used to calculate load equivalency factors based on fatigue and rutting criteria, that is to say maximum tensile strain at the bottom of the asphaltic layer, and maximum compressive strain the top of the subgrade layer.

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