Vehicle Parameter Identification Using DE Algorithms

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***Abstract* — In this work an approach to determine vehicle parameters such as mass, moment of inertia, center of gravity, suspension stiffness and damping along with wheel masses is proposed. In this regard bicycle car model of the vehicle is employed, and an objective function (OF) based on acceleration responses of the vehicle is formed. To solve the optimization problem eight versions of Differential Evolution (DE) algorithm as well as genetic algorithm toolbox of Matlab® (R2014b) is employed. It is demonstrated that DE/current to best/1 among the considered algorithms is the best in the sense of minimum norm of relative errors, and the proposed approach achieves to determine unknown parameters with acceptable errors.**

Keywords — vehicle parameter identification; optimization; differential evolution algorithm

I. INTRODUCTION

Vehicle parameters such as mass, moments of inertia, suspension and tire properties should be accurately determined since they are closely related to riding quality, handling, breaking and traction performance of the vehicle. Thus, many works have been conducted on this issue considering vehicle's longitudinal, lateral and vertical dynamics. For example, Venture et al. [1] proposed a method based on multibody model of the car to determine dynamic parameters of chasis, suspensions, tire vertical stiffness and wheel parameters. Furukawa and Dissanayake [2] presented a technique for identifying parameters of an autonomous vehicle using multi-objective optimization. Wesemeier and Isermann [3] used one track model of the vehicle to determine cornering stiffness and center of gravity parameters. The proposed method needs the input variables such as vehicle forward speed, lateral acceleration, yaw rate and slip angle. Khaknejad et al [4] identified mass, yaw moment of inertia, the distance between center of mass and front axle, and the velocity of a sedan car using bicycle model of vehicle and least square estimation with exponential forgetting factor. Wilhelm et al [5] proposed an OF based on the difference of measured and simulated powers. Minimizing this the authors determined mass, rolling resistance and aerodynamic coefficients as well as efficiency of power train of an electric vehicle. Kidambi et al [6] assessed the accuracy and performance of four estimation methods, i.e. recursive least squares with multiple forgetting factors; extended Kalman filtering; a dynamic grade observer; and parallel mass and grade estimation using a longitudinal accelerometer. Rozyn and Zhang [7] used vertical vibration model to predict mass, pitch and roll inertia moments. Their method is based on modal parameter estimation using the free-decay responses of the vehicle and estimation of the system characteristic matrix. Cui and Kurfess [8] determined mass, moments of inertia, suspension and tire stiffness, and center of gravity coordinate of a full car model with nonlinear/hysteresis shock absorber.

In this work, motivated by the works such as [7,8], a simple and efficient approach is presented to determine mass, moments of inertia as well as suspension parameters of a vehicle. To this end bicycle car model is employed, and an OF based on acceleration responses of the vehicle is formed. To solve the relevant optimization problem eight DE algorithms and genetic algorithm (GA) toolbox of Matlab® are tested and compared.

II. THEORY

A. *Vehicle Model*

The vehicle is modeled as shown in Fig. 1., which includes the degrees of freedom such as x body vertical bounce, θ body pitch, x_i (i=1,2) wheel hop. m is body mass, I is pitch moment of inertia, c_i and k_i (i=1,2) are suspension damping and stiffness. k_{ti} is tire vertical stiffness, a_i denotes the longitudinal components of body center of gravity (CoG). y_1 and y_2 are independent road excitations.

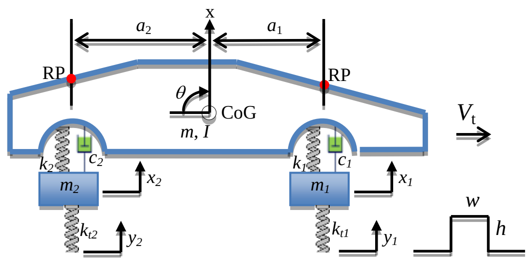

*Figure 1. Bicycle car model (*RP*: Response Point)*

Applying Lagrange's method equations of motion are obtained as follows [9]:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F\} \qquad (1)$$

where

$$[M]=\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix},\ \{x\}=\begin{Bmatrix} x \\ \theta \\ x_1 \\ x_2 \end{Bmatrix} \tag{2a}$$

$$[C]=\begin{bmatrix} c_1+c_2 & a_2c_2-a_1c_1 & -c_1 & -c_2 \\ a_2c_2-a_1c_1 & c_1a_1^2+c_2a_2^2 & a_1c_1 & -a_2c_2 \\ -c_1 & a_1c_1 & c_1 & 0 \\ -c_2 & -a_2c_2 & 0 & c_2 \end{bmatrix} \tag{2b}$$

$$[K]=\begin{bmatrix} k_1+k_2 & a_2k_2-a_1k_1 & -k_1 & -k_2 \\ a_2k_2-a_1k_1 & k_1a_1^2+k_2a_2^2 & a_1k_1 & -a_2k_2 \\ -k_1 & a_1k_1 & k_1+k_{t1} & 0 \\ -k_2 & -a_2k_2 & 0 & k_2+k_{t2} \end{bmatrix} \tag{2c}$$

$$\{F\}=\begin{Bmatrix} 0 \\ 0 \\ y_1k_{t1} \\ y_2k_{t2} \end{Bmatrix} \tag{2d}$$

This equation set can be solved by numerical integration. In this work Newmark beta method is applied.

B. *The Proposed Approach*

It is assumed all vehicle parameters except tire stiffness are unknown. Tire stiffness is based on tire's mechanical properties and environmental characteristics, and is generally determined by experiment [9]. In this work tire stiffness are known beforehand. On the other hand, according to one of the parameter identification approaches some inputs are applied to the system and certain outputs are recorded. These outputs are then used as reference, and the system parameters in the mathematical model are adjusted such that the output of the model agrees with the reference. This process can be modeled as an optimization problem whose OF is based on the difference of the reference data and the model output. In this work the following OF is employed:

$$f(\{v\})=\frac{\sum_{i=1}^{2}\left\|\ddot{z}_i^r-\ddot{z}_i^c\right\|}{\sum_{i=1}^{2}\left\|\ddot{z}_i^r\right\|} \tag{3}$$

where $\{v\}$ is the vector of unknown parameters, i.e.

$$\{v\}=[m\ I\ m_1\ m_2\ k_1\ k_2\ c_1\ c_2\ a_1]^T \tag{4}$$

It is simple to record acceleration response of the vehicle with current instruments. Hence, two response points located along the vertical extensions of front and rear axles are marked, which are demonstrated as RP in Fig. 1. Then the vertical displacement at a RP, i.e. z_i, is defined as follows:

$$\left.\begin{aligned} z_1 &= x-\theta a_1 \\ z_2 &= x+\theta a_2 \end{aligned}\right\} \tag{5}$$

The superscript r and c in Eq.(3) mean reference and computed data, respectively. Overdot indicates derivative with respect to time, i.e. $\ddot{z}=d^2z/dt^2$. As to the input, when the vehicle is moving with constant speed V_t the tires are subject to step like bump with height h and width w (see Fig. 1). By this way all vibration modes of the vehicle can be excited. Hence the response signals are expected to contain sufficient data about the vehicle's dynamic behavior. Recording starts at the instant when the front tire first meets the bump, and endures till the rear tire leaves the bump. For the vehicle speed and bump geometry the following values are considered: V_t=20km/h, w=10cm, h=4cm. Higher vehicle speeds are unnecessary, since the undamped natural frequencies of the vehicle are in the range 0 – 15Hz. The values of w and h may be different, as well. Those given above are determined after some trials. Response of the system to such inputs is obtained solving the Eq. (1). This will be used as reference in parameter identification process. In practice the recorded data includes some noise because of instrumentation and measurement errors. To consider noise effect reference data is contaminated by random numbers at the rate 5%, and the noisy reference data, i.e. $\ddot{z}_i^r$ in Eq.(3), are plotted in Fig. 2. The vehicle parameters considered are [10]: m=505.1 kg, m_1=28.58 kg, m_2=54.3kg, I=651 kgm^2, a_1=1.468 m, a_2=1.098 m, k_1=k_2=15000N/m, k_{t1}=k_{t2}=155900 N/m.

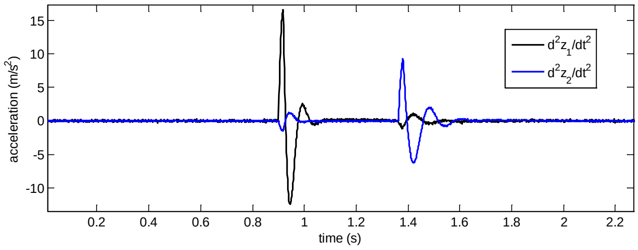

Figure 2. Acceleration responses of the vehicle.

Now the optimization problem to be solved can be defined as follows:

$$\min(f\{v\})\ \text{subject to}\ \{LB\}\le\{v\}\le\{UB\} \tag{6}$$

where {*LB*} and {*UB*} are the vectors including lower and upper boundary values. To solve the Eq.(6) generally population based methods are employed to avoid local solutions. DE is such a method, as well.

III. Differential Evolution Algorithms

DE is a heuristic global optimization method developed by Storn and Price [11].It is easy to use, fast convergent, and requires few control variables compared to GA [11]. Over time various versions of DE have been developed. In this work, the first version of DE along with some others known by the author of this work are considered.

DE starts with creating N candidate points in the search space. Then, at k[th] iteration the i[th] point, x_i^k (i=1,2,…,N), is subject to mutation. To this end, three mutually different random indexes, r_1, r_2, r_3 in the range 1:N are picked up. These indexes are also different from i i.e. $r_1 \neq r_2 \neq r_3 \neq i$. Then mutant vector is generated according to

$$v_i^{k+1} = x_{r_1}^k + F(x_{r_2}^k - x_{r_3}^k) \quad (7)$$

where *F* is a real constant called amplification factor. It can take values in the range 0:2. Using the elements of mutant vector a trial vector, u_i^{k+1} , is generated by crossover. Its j[th] dimension is defined as follows

$$u_{ij}^{k+1} = \begin{cases} v_{ij}^{k+1} & \text{if } (rand_j \leq CR) \text{ or } j = rnbr(i) \\ x_{ij}^k & \text{if } (rand_j > CR) \text{ and } j \neq rnbr(i) \end{cases} \quad (8)$$

where *rand$_j$* is the jth evaluation of a uniform random generator with outcome in the range 0:1 [11]. CR is the crossover constant, rnbr(i) is randomly chosen integer in the range 1:D, and D is the dimension of the problem. Later, selection is applied. If u_i^{k+1} yields a smaller OF value than x_i^k then x_i^{k+1} is set to u_i^{k+1}. Otherwise current x_i^k is set to x_i^{k+1}. In another version of DE the mutant vector is produced as follows [11]:

$$v_i^{k+1} = x_{best}^k + F(x_{r_1}^k + x_{r_2}^k - x_{r_3}^k - x_{r_4}^k) \quad (9)$$

where x_{best}^k is the point with minimum OF. Also, the following strategies are availabe [12]:

$$v_i^{k+1} = x_{best}^k + F(x_{r_1}^k - x_{r_2}^k) \quad (10)$$

$$v_i^{k+1} = x_i^k + F(x_{best}^k - x_i^k) + F(x_{r_1}^k - x_{r_2}^k) \quad (11)$$

$$v_i^{k+1} = x_{r_1}^k + F(x_{r_2}^k - x_{r_3}^k) + F(x_{r_4}^k - x_{r_5}^k) \quad (12)$$

In the work of Karaboğa and Ökdem [13] a slightly different mutation type is given as

$$v_i^{k+1} = x_i^k + K(x_{r_1}^k - x_i^k) + F(x_{r_2}^k - x_{r_3}^k) \quad (13)$$

where K is the combination factor, F is the scaling factor whichk takes a random value between –2 and 2 at each iteration. Das et al [14] advise

$$v_i^{k+1} = x_{r_1}^k + R(x_{r_2}^k - x_{r_3}^k) \quad (14)$$

where R is the scaling factor which takes uniform random value between 0.5 and 1 at each iteration. The authors claim that population diversity is retained by this way as the search progresses. The same authors introduced another version in which scaling factor is reduced linearly at each iteration as

$$R = (R_{\max} - R_{\min}) \frac{K_{\max} - k}{K_{\max}} \quad (15)$$

where R_{max}=1.2, R_{min}=0.4, K_{max} is the maximum allowable number of iterations, and *k* is iteration counter. This way helps explore the whole search space at the earlier iterations. As the search progresses narrower space around the converged region is explored. Table 1 summarizes the DE versions employed in this work. In the table U(a,b) denotes uniform random distribution in the range a:b. For DE1 to DE5 the same CR and F values, which are consistent with the advice in [11], are employed, since the mutant vector generation scheme is similar at each of them. For the other DE versions the values employed in the relevant works are considered.

TABLE I. DE VERSIONS

Name	Relevant Equation	Values of parameters
DE1	(7)	F=0.6, CR=0.9
DE2	(9)	
DE3	(10)	
DE4	(11)	
DE5	(12)	
DE6	(13)	K=0.5, CR=0.8, F∈U(-2,2)
DE7	(14)	CR=0.9, R∈U(0.5,1)
DE8	(14), (15)	CR=0.9

IV. APPLICATION

The same vehicle parameters are employed, and the algorithms were run with the following parameters: population size (PS) 20, maximum number of iterations 50. Generally PS between 5D and 10D is suggested [11]. But here a lower value is chosen to investigate whether the algorithm can achieve to converge to the minimum by lower number of function evaluations. Lower and upper boundaries of the search space are chosen as {*LB*}=0.5{*OV*}, {*U*B}=1.5{*OV*}, where {*OV*} is the vector containing optimum values (OV). One can verify the search space is sufficiently large when compared with the relevant works. On the other hand, each algorithm was run thirty times to obtain statistical results. The mean values and the relative errors (ε) are given in Table II. ||ε|| stands for the norm of relative errors (NoRE), which is a measure of the closeness of the reached point to the optimum. Comparing the results in this context it is clear that DE4 is the best, since it has minimum NoRE. Most of the parameters were identified with errors smaller than 1% by this algorithm. That is, the algorithm can find the optimum point with negligible error. Besides, it is clear that all DE versions are better than GA. To give more chance to the algorithms the PS is increased to 30, and the results in Table III are obtained. Again DE4 is the best, and most of the other DE versions are now closer to the optimum.

V. CONCLUSION

In this work a simple and efficient method is proposed to determine vehicle parameters. To this end, vehicle vertical model is considered, and acceleration responses are employed. An OF based on the difference of reference and computed accelerations is minimized using several DE algorithms. It is concluded that DE4, which is known as DE/current to best/1, is the best among the others, and the proposed approach is efficient to obtain accurate values of vehicle parameters.

TABLE II. OPTIMIZATON RESULTS (PS = 20)

	m	*I*	m_1	m_2	k_1	k_2	c_1	c_2	a_1	‖ε‖
OV	505.10	651.00	28.58	54.30	15e3	15e3	1828.00	1828.00	1.468	
DE1	508.51	655.87	28.73	54.49	15492.59	15981.77	1846.61	1874.05	1.473	
ε (%)	0.67	0.75	0.51	0.36	3.28	6.55	1.02	2.52	0.34	7.91
DE2	504.27	654.80	28.63	54.37	13894.15	15285.92	1848.22	1830.31	1.468	
ε (%)	-0.16	0.58	0.17	0.12	-7.37	1.91	1.11	0.13	0.03	7.72
DE3	509.32	656.31	28.68	54.46	14698.04	15624.85	1841.36	1870.48	1.473	
ε (%)	0.84	0.82	0.36	0.29	-2.01	4.17	0.73	2.32	0.39	5.39
DE4	506.68	653.84	28.64	54.46	15016.74	14894.38	1834.80	1847.71	1.471	
ε (%)	0.31	0.44	0.21	0.30	0.11	-0.70	0.37	1.08	0.20	**1.51**
DE5	518.94	667.16	28.52	54.61	14715.99	15434.35	1840.49	1956.73	1.482	
ε (%)	2.74	2.48	-0.22	0.57	-1.89	2.89	0.68	7.04	0.98	8.78
DE6	515.70	660.64	28.69	54.22	14770.55	14787.06	1866.67	1879.41	1.475	
ε (%)	2.10	1.48	0.39	-0.15	-1.53	-1.42	2.12	2.81	0.51	4.88
DE7	515.49	664.77	28.81	54.89	14903.22	15813.86	1870.68	1933.85	1.483	
ε (%)	2.06	2.12	0.80	1.10	-0.65	5.43	2.34	5.79	0.99	8.96
DE8	519.69	664.39	28.69	55.07	14395.64	15507.00	1867.92	1957.32	1.482	
ε (%)	2.89	2.06	0.37	1.42	-4.03	3.38	2.18	7.07	0.94	9.90
GA	554.54	702.02	29.04	55.41	15338.49	15636.14	1991.57	2000.46	1.509	
ε (%)	9.78	7.84	1.60	2.05	2.26	4.24	8.95	9.44	2.81	19.10

TABLE III. OPTIMIZATON RESULTS (PS = 30)

	m	*I*	m_1	m_2	k_1	k_2	c_1	c_2	a_1	‖ε‖
OV	505.10	651.00	28.58	54.30	15e3	15e3	1828.00	1828.00	1.468	
DE1	508.39	654.82	28.71	54.33	15125.45	14973.91	1841.65	1863.43	1.471	
ε (%)	0.65	0.59	0.46	0.05	0.84	-0.17	0.75	1.94	0.200	2.46
DE2	507.78	653.72	28.71	54.61	14890.57	14341.34	1840.15	1854.23	1.471	
ε (%)	53	42	0.46	0.57	-73	-4.39	0.66	1.43	0.230	4.83
DE3	508.63	655.78	28.67	54.51	15088.73	15078.72	1836.85	1874.58	1.474	
ε (%)	0.7	0.73	0.32	0.39	0.59	0.52	0.48	2.55	0.410	2.97
DE4	505.75	652.58	28.64	54.38	14706.80	14949.88	1830.61	1839.05	1.470	
ε (%)	0.13	0.24	0.23	0.15	-1.96	-0.33	0.14	0.61	0.130	**2.12**
DE5	525.28	657.83	28.41	54.97	14333.51	12771.78	1826.53	1891.05	1.477	
ε (%)	4	1.05	-0.6	1.24	-4.44	-14.85	-0.08	3.45	0.580	16.48
DE6	512.13	657.64	28.70	54.57	14704.53	14863.99	1854.47	1875.84	1.473	
ε (%)	1.39	1.02	0.43	0.5	-1.97	-0.91	1.45	2.62	0.340	4.14
DE7	508.34	654.11	28.70	54.89	13809.60	14463.92	1817.99	1921.59	1.479	
ε (%)	0.64	0.48	0.42	1.08	-7.94	-3.57	-0.55	5.12	0.770	10.24
DE8	512.26	659.86	28.65	54.53	14777.78	15579.17	1835.51	1903.17	1.483	
ε (%)	1.42	1.36	0.26	0.42	-1.48	3.86	0.41	4.11	1.120	6.27
GA	562.34	682.64	28.84	55.06	14517.00	14283.09	1901.54	2063.85	1.518	
ε (%)	11.33	4.86	0.90	1.39	-3.22	-4.78	4.02	12.90	3.410	19.55

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